THE DISCRETE SYSTEM AND ITS MODELS FOR DIGITAL SIGNAL PROCESSORS

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Abstract

The discrete system is mostly described with the help of the system transfer function and the zero input response is usually neglected. In this paper the state-space equations express the total response of the discrete system, and they are arranged to a digital signal processor implementation.

Keywords:
LTI discrete system, signal flow graphs, canonic forms, digital signal processor.

1. Introduction

Difference equations play much the same role for discrete systems that differencial equations play for continuous systems. The linear constant-coefficient difference equation of the sth order can be used as a model of the linear-time invariant (LTI) discrete system

$$b_s y(n+s) + b_{s-1} y(n+s-1) + \ldots + b_1 y(n+1) + b_0 y(n) =$$

$$= a_s x(n+s) + a_{s-1} x(n+s-1) + \ldots + a_1 y(n+1) + a_0 x(n) \quad ,(1)$$

where $y(n)$ denotes the output discrete signal, and $x(n)$ is the input signal of discrete system, a_i, b_i are constants. The initial state of the discrete system is defined by means of the initial conditions as $y(0), y(1),.., y(s-1)$, and $x(0)$, $x(1), x(2),.., x(s-1)$. The difference equation (1) describes the external behaviour of LTI system. Such a description reduces the discrete system to " a black box ", which internal mechanisms are ignored. The state - space description of the system includes not only its external but also its internal behaviour and may often be obtained by direct application of the physical or the other fundamental laws. The state description has an eminent importance for the understanding, the analysis and the simulation of discrete systems.

2. State-Space Representation of the Linear Discrete System

The equation (1) describes the relation between input and output signals, but the internal structure of the system is not known, and the internal state can not be defined. The difference equation of the sth order (1) can be arranged to the state vector matrix equations of the first order, or simply, **discrete state dynamic equations**

$$\mathbf{v}(n+1) = \mathbf{A}\,\mathbf{v}(n) + \mathbf{B}\,x(n)$$

$$y(n) = \mathbf{C}\,\mathbf{v}(n) + \mathbf{D}\,x(n) \quad . \qquad (2)$$

Note that the state vector is defined as $\mathbf{v}(n) = [v_1(n)\, v_2(n) \ldots v_{s-1}(n)\, v_s(n)]^T$, $y(n)$ resp. $x(n)$ are the output, resp. input signals. It is clear that matrix equations (2) are convenient in the case of a computer solution. If somebody wants to process the input data in line procedure, then he must use a special hardware, which for example is a digital signal processor (DSP). Therefore, the state equations (2) can be solved with the help of the unilateral z-transfom and arranged into a more suitable form

$$v_1(n) = v_2(n-1) + v_1(0)\,\delta(n) \quad ,$$

$$v_2(n) = v_3(n-1) + v_2(0)\,\delta(n) \quad ,$$

$$v_3(n) = v_4(n-1) + v_3(0)\,\delta(n) \quad ,$$

$$v_4(n) = v_5(n-1) + v_4(0)\,\delta(n) \quad ,$$

$$v_{s-1}(n) = v_s(n-1) + v_{s-1}(0)\,\delta(n) \quad , \qquad (3)$$

$$v_s(n) = \frac{1}{b_s} x(n-1) - \frac{b_{s-1}}{b_s} v_s(n-1) \ldots$$

$$\ldots - \frac{b_0}{b_s} v_1(n-1) + v_s(0)\delta(n) \quad ,$$

$$y(n) = a_s v_s(n+1) + a_{s-1} v_s(n) + \ldots$$

$$\ldots + a_1 v_2(n) + a_0 v_1(n) \quad . \qquad (4)$$

Figure 1. shows the signal flow-graph (SFG) according to eguations (3) and (4). This structure is called the **canonic form** 2. The SGF is useful for computing the transfer function from some source nodes to a sink node.

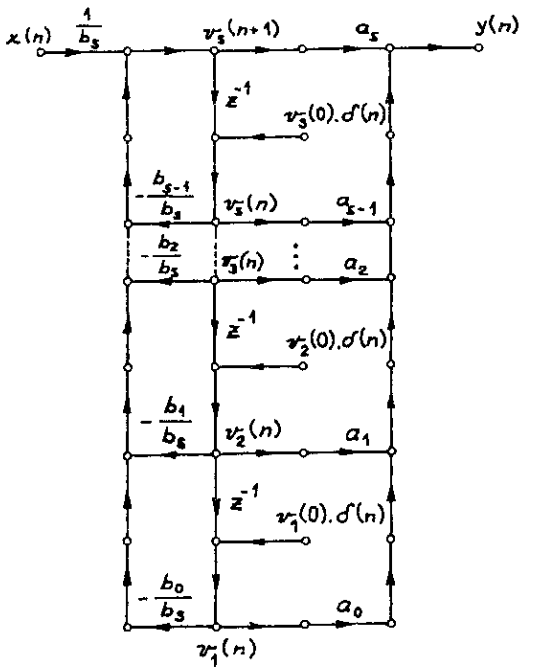

Figure 1
Signal flow-graph of the canonic form 2 structure ($\delta(n$ is meant as the unit-impulse signal).

To do this, it is convenient to use **Mason's gain formula.** If $Y(z)$, resp. $X(z)$ are the z-transform of $y(n)$, resp. $x(n)$, then the solution of the difference equation (3) can be written in the form

$$Y(z) = H(z)X(z) + \sum_{i=1}^{s} H_{2i}(z)\,v_i(0)\,. \tag{5}$$

The function H(z) is known as **the system transfer function,** and $v_i(0)$, $i = 1, 2, .., s$ are initial conditions of the state-space representation. The first term of the equation (5) is called **the zero state response** (ZSR). The total response $Y(z)$ can be considered as the sum of the ZSR and **the zero input response** (ZIR) (the remaining term of (5)). The ZSR depends only on the input signal $x(n)$, $n \geq 0$ and the ZIR is determined on the initial system state $v_i(0)$. The transfer functions in the (5) are

$$H(z) = \frac{A_s(z)}{B_s(z)} =$$

$$= \frac{a_s z^s + a_{s-1} z^{s-1} + \ldots + a_2 z^2 + a_1 z + a_0}{b_s z^s + b_{s-1} z^{s-1} + \ldots + b_2 z^2 + b_1 z + b_0} \quad . \tag{6}$$

$$H_{2i}(z) = \frac{1}{B_s(z)} + \sum_{j=1}^{i} (a_{i-j} b_s - b_{i-j} a_s) z^{s-j+1} \quad . \tag{7}$$

These functions can be also obtained with the help of Mason's gain formula.

There is the other possibility to arrange the state dynamic equations (2). This structure is known as **the canonic form 1.** The partial state values are determined by a combination of input and output signals as

$$v_1(n) = a_{s-1}x(n-1) - b_{s-1}y(n-1) + v_2(n-1) + v_1(0)\,\delta(n) \quad ,$$

$$v_2(n) = a_{s-2}x(n-1) - b_{s-2}y(n-1) + v_3(n-1) + v_2(0)\,\delta(n) \quad ,$$

$$v_3(n) = a_{s-3}x(n-1) - b_{s-3}y(n-1) + v_4(n-1) + v_3(0)\,\delta(n) \quad ,$$

$$\ldots\ldots\ldots\ldots \tag{8a}$$

$$v_{s-1}(n) = a_1 x(n-1) - b_1 y(n-1) + v_s(n-1) + v_{s-1}(0)\,\delta(n) \quad ,$$

$$v_s(n) = a_0 x(n-1) - b_0 y(n-1) + v_s(0)\,\delta(n) \quad ,$$

$$y(n) = \frac{1}{b_s} v_1(n) + \frac{a_s}{bs} x(n) \tag{8b}$$

Figure 2. depicts the SFG of the form 1, which is obtained by drawing the equations (8).

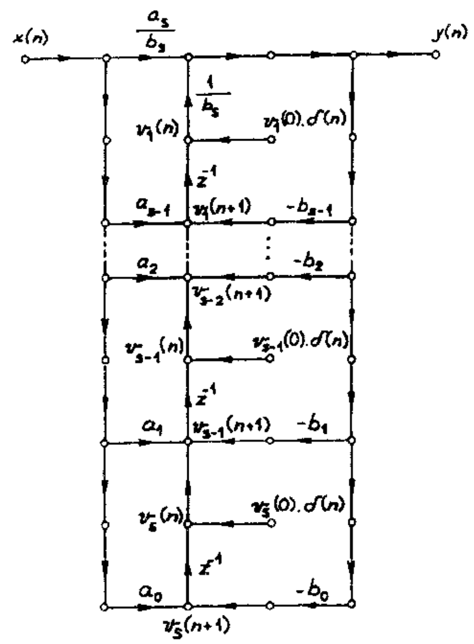

Figure 2
Signal flow-graph of the canonic form 1 structure.

The solution of the difference equation (3) can be written in the other form as

$$Y(z) = H(z)X(z) + \sum_{i=1}^{s} H_{1i}(z)v_i(0) \quad . \tag{9}$$

The system transfer function H(z) has the same form as the equation (6). The following transfer functions are defined with the help of this expression

$$H_{1i}(z) = \frac{z^{s-i+1}}{b_s(z)}\,, i = 1, 2, 3, \ldots, s \quad . \tag{10}$$

To check the correct result of the transfer function (10) the Mason's gain formula can be used.

3. Conclusion

The paper describes two canonic structures of the discrete system state-space representation. Author connectes these forms with the difference equation (3) and adds the possibility of the analysis of nonzero initial conditions. There is a typical use of these structures in a digital filter implementation. This procedure can be applied to the development and the description of the other discrete system models.

4. References

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