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Procedia Engineering 142 (2016) 146 – 153

Procedia Engineering

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Sustainable Development of Civil, Urban and Transportation Engineering Conference

Inspection Based Probabilistic Modeling of Fatigue Crack Progression

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Abstract

Attention to the fatigue cracks in steel structures and bridges has been paid for a long time. Fatigue crack damage depends on a number of stress range cycles. Three sizes are important for the characteristics of the propagation of fatigue cracks - the initial size, detectable size and acceptable size. The theoretical model of fatigue crack progression in paper is based on a linear fracture mechanics. When determining the required degree of reliability, it is possible to specify the time of the first inspection of the construction which will focus on the fatigue damage. Using a conditional probability, times for subsequent inspections can be determined. For probabilistic calculation of fatigue crack progression was used the original and new probabilistic methods - the Direct Optimized Probabilistic Calculation ("DOProC"), which uses a purely numerical approach without any simulation or approximation techniques. The algorithm of the probabilistic calculation was applied in the FCProbCalc code ("Fatigue Crack Probabilistic Calculation"), using which is possible to carry out the probabilistic modelling of propagation of fatigue cracks in a user friendly environment very effectively.

Peer-review under responsibility of the organizing committee of CUTE 2016

Keywords: probabilistic methods; fatigue crack; inspection of structure; DOProC; Direct Optimized Probabilistic Calculation; FCProbCalc; reliability assessment; random variable; probability of failure

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Peer-review under responsibility of the organizing committee of CUTE 2016
doi:10.1016/j.proeng.2016.02.025

1. Introduction

Probabilistic and stochastic methods [20] are used in engineering tasks [9, 28]. The advantage is, in particular, that the calculation or reliability assessment of building constructions takes into account the random nature of input quantities [2]. These approaches need a rather big database of input quantities obtained by numerical modelling [8, 11, 12], laboratory measurements [24] or directly in site [3, 22, 35].

This paper describes the use and application of the original probabilistic method which is under development now: the Direct Optimized Probabilistic Calculation ("DOProC"). The method uses a purely numerical approach without any simulation or approximation techniques [6, 7]. In comparison with other probabilistic methods the method provides more accurate solutions to the probabilistic tasks, and, in some cases, results in considerably faster completion of computations. Such solution entails a small numerical error only and minor inaccuracies, the reason being discretizing of input and output quantities. The algorithm enables easy application in solving engineering problems, e.g. associated with the assessment of the structural reliability [21].

DOProC can be used now to solve efficiently a number of probabilistic computations. DOProC has proved to be a good solution, for instance, in probabilistic analyses of fatigue crack propagation in steel structures subject to cyclical loads, such as [27, 36]. Theoretical backgrounds were described in detail e.g. in [13], where a particular attention being paid to fatigue cracks from the edge and from the surface (see Fig. 1). Similarly to other probabilistic analysis [4, 5, 10, 32], the probabilistic calculation is used as a basis for proposing a system of inspections of the construction.

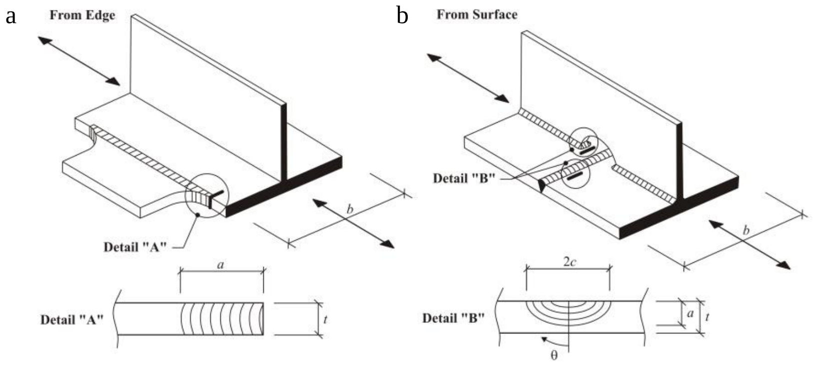


Fig. 1. Characteristic propagation of cracks from the outer edge (a) and from the surface (b)

2. Formulating the probability task

Reliability of the bearing structure [17] has been significantly influenced by degradation resulting, in particular, from the fatigue of the basic materials [23, 29, 30, 31]. In order to describe the propagation of the crack, the linear elastic fracture mechanics [1] is typically applied. This method uses Paris-Erdogan's law [26] and defines relation between propagation rate of the crack size a, and range of the stress rate coefficient, ΔK, in the face of the crack:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C.\left(\Delta K\right)^{m} , \tag{1}$$

where C, m are material constants and N is the number of loading cycles.

The fatigue crack will propagate in a stable way only if the initial crack a_0 exists in the place where the stress is concentrated. This place is located at the edge or on the surface of the element.

The primary assumption is that the primary design should take into account the effects of the extreme loading and the fatigue resistance should be assessed then. This means, the safety margin Z in the probability task is:

$$Z(R,E) = R - E\,, \tag{2}$$

where R is the random resistance of the element and E is the random variable effect of the extreme load. If such element is subject to the operating load, following cases can be occurred:

- Safe service life – the fatigue effects do not degrade the element by means of the fatigue crack,
- Acceptable failure rate – the fatigue effects degrade the element and decrease the load-bearing capacity of the element,
- Acceptable failure rate - fatigue effects are expressed as stress changes. This approach is more demonstrative than previous ones and has been preferred.

The calculation model of the fatigue crack propagation defines the stress when the maximum acceptable crack results in the constant resistance of the structure, R, that corresponds to the stress in the yield point f_y.

When using (1), the condition for the acceptable crack length, a_{ac}, is:

$$N = \frac{1}{C}\int_{a_0}^{a_{ac}} \frac{\mathrm{d}a}{(\Delta K)^m} > N_{\mathrm{tot}}\ , \tag{3}$$

where N is the number of cycles needed to increase the crack from the initiation size a_0 to the acceptable crack size a_{ac}, and N_{tot} is the number of cycles throughout the service life.

The equation for the propagation of the crack size (1) needs to be modified for this purpose. The range of the stress rate coefficient, ΔK, at the constant stress range, $\Delta\sigma$, is:

$$\Delta K = \Delta\sigma.\sqrt{\pi.a}.F(a)\ , \tag{4}$$

where $F(a)$ is the calibration function which represents propagation of the crack (for instance, from the edge see [15]). After the change of the number of cycles from N_1 to N_2, the crack will propagate from the length a_1 to a_2. Having modified (1) and using (4), the following formula will be achieved:

$$\int_{a_1}^{a_2} \frac{\mathrm{d}a}{\left[\sqrt{\pi.a}.F(a)\right]^m} = \int_{N_1}^{N_2} C.(\Delta\sigma)^m \,\mathrm{d}N\ . \tag{5}$$

If the length of the crack a_1 equals to the initial length a_0 (this is the assumed size of the initiation crack in the probabilistic approach) and if a_2 equals to the final acceptable crack length a_{ac} (This is the acceptable crack size which replaces the critical crack size a_{cr} if the crack results in a brittle fracture. In order to calculate the phenomenon (10) – see below, size a_2 can be equal to the size of the detectable crack a_d), then the left-hand side of the equation (5) can be regarded as the resistance of the structure - R:

$$R(a_{ac}) = \int_{a_0}^{a_{ac}} \frac{\mathrm{d}a}{\left[\sqrt{\pi.a}.F(a)\right]^m}\ . \tag{6}$$

Similarly, it is possible to define the cumulated effect of loads E (random variable effects of the extreme load), that is equal to the right side of eq. (5) assuming that $N_1 = N_0$ and $N_2 = N$:

$$E = \int_{N_0}^{N} C.(\Delta\sigma)^m \,\mathrm{d}N = C.(\Delta\sigma)^m.(N_0 - N)\ , \tag{7}$$

where N is the total number of stress peak range $\Delta\sigma$, when the crack size increases from a_0 to a_{ac} and N_0 represents the number of load cycles in the time of the fatigue crack initiation (it is typically equal to zero).

It is possible to define a safety margin Z:

$$Z(\mathbf{X}) = R(a_{ac}) - E(N) \ , \tag{8}$$

where X is a vector of random variables, which includes mechanical properties, geometry of the structure, load effects and dimensions of the fatigue crack. The analysis of the safety margin Z gives a probability of failure p_f:

$$p_f = P(RF(\mathbf{X}) < 0) = P(R(a_{ac}) - E(N) < 0) \ . \tag{9}$$

3. Inspection times

While the fatigue crack is propagating, it is possible to define three random phenomena that are related to the growth of the fatigue crack and may occur in any time, t, during the service life of the structure. Then:

- $U(t)$ phenomenon: No fatigue crack failure has not been revealed within the t time and the fatigue crack size $a(t)$ has not reached the detectable crack size, a_d. This means:

$$a(t) < a_d \ , \tag{10}$$

- $D(t)$ phenomenon: A fatigue crack failure has been revealed within the t time and the fatigue crack size $a(t)$ is still below the acceptable crack size a_{ac}. This means:

$$a_d \le a(t) < a_{ac} \ , \tag{11}$$

- $F_{(t)}$ phenomenon: A failure has been revealed within the t time and the fatigue crack size $a(t)$ has reached the acceptable crack size a_{ac}. This means:

$$a(t) \ge a_{ac} \ . \tag{12}$$

Using the phenomena above, it is possible to define probability for their occurrence in any t time. Those three phenomena cover the complete spectrum of phenomena that might occur in the t time. This means:

$$P(U(t)) + P(D(t)) + P(F(t)) = 1 \ . \tag{13}$$

Because it is not certain in the probabilistic calculation whether the initial crack exists and what the initial crack size is and because other inaccuracies influence the modelling of the crack propagation, a specialized inspection is necessary to check the size of the detectable crack in a specific period of time. The factor which influences most the time of inspection is the acceptable size of the fatigue crack a_{ac}.

The probabilistic calculation is carried out in time steps where one step typically equals to one year of the service life of the construction. When the probability of failure $P(F(t))$ reaches the designed failure probability p_d, an inspection should be carried out in order to find out fatigue cracks, if any, in the construction element. The inspection in the t time may result in any of the three mentioned phenomena. The inspection provides information about conditions of the construction. Such conditions can be taken into account when carrying out further probabilistic calculations.

If no fatigue cracks are found, the analysis of inspection results gives conditional probability during occurrence. Using the inspection results for the t time, it is possible to define the probability of the mentioned phenomena in another time: $T > t_I$. For that purpose, the conditional probability should be taken into consideration. In order to determine the time for the next inspection, it is necessary to define the conditional probabilities $P\big(F(T)\big|U(t_I)\big)$ and $P\big(F(T)\big|D(t_I)\big)$, which can be expressed using the full probability law (details – see, for instance, [16, 18]), as follows:

$$P\left(F_{(T)}\middle|U_{(t_I)}\right)=\frac{P\left(F_{(T)}\right)-P\left(F_{(t_I)}\right)-P\left(D_{(t_I)}\right).P\left(F_{(T)}\middle|D_{(t_I)}\right)}{P\left(U_{(t_I)}\right)}\ , \tag{14}$$

where $T > t_I$.

If re-distribution of stress from a point that is weakened by the crack is not taken into account, the crack propagation crack is usually rather high in the practical range of detectable values. If a fatigue crack is found during the inspection, it is necessary to monitor the safe growth of the crack or to take actions that will slow down or stop further propagation of the fatigue crack.

Those approaches which are based on the calculated DOProC probability of three basic phenomena, (10) to (12), using the safety margin Z defined in (8), for each year of operation of the construction were included into FCProbCalc code ("Fatigue Crack Probability Calculation"), which has been developed using the aforementioned techniques [14, 19]. By means of FCProbCalc, it is possible to carry out the probabilistic modelling of propagation of fatigue cracks in a user friendly environment and to propose a system of regular inspections which should reveal damage to the structure.

4. DOProC probabilistic calculation

The reference probabilistic calculation in FCProbCalc included the probabilistic assessment of a steel/reinforced concrete bridge from on the highway from [13] in a point where a longitudinal beam connects to a transversal beam (For more details see [15]). The input quantities were determined deterministically or stochastically using parametric probability distributions (see Tab. 1 and 2). The required reliability [34] was described by the reliability index $\beta = 2$ which corresponded to the designed probability of failure $p_d = 0.02277$.

Tab. 1. Overview of variable input quantities expressed in a histogram with parametric probability distribution

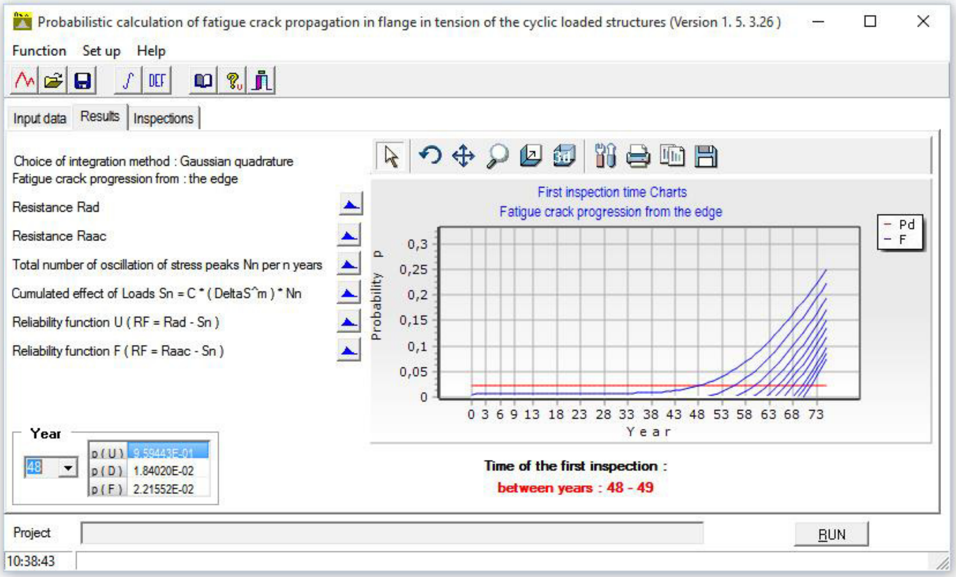
Quantity	*Type*	*Mean value*	*Standard deviation*
Oscillation of stress peaks $\Delta\sigma$ [MPa]	Normal	30	3
Number of oscillation of stress peaks per year N [-]	Normal	10^6	10^5
Yield point f_y [MPa]	Lognormal	280	28
Nominal stress in the flange plate σ [MPa]	Normal	200	20
Initial size of the crack a_0 [mm]	Lognormal	0.2	0.05
Smallest measurable size of the crack a_d [mm]	Normal	10	0.6

The probabilistic calculation was carried out for fatigue cracks propagating from the edge and surface. If a period of time is specified and the time step is 1 year, it is possible to determine resistance of the construction R pursuant to (6), load effects, E, pursuant to (7), as well as the probability of elemental phenomena, U, D and F, pursuant to (10) through (12) and (14), which are the basis for specification of inspection times.

Fig. 2 shows results of the probabilistic modelling of a fatigue crack from the edge. The curves describe dependence of the probability of failure, p_f, on time of operation of the bridge structure. When the probability of failure exceeds the specified designed probability, p_d, the inspection should be performed. It was decided that the first inspection of the bridge should take place after 48 years of operation. This inspection will focus on growth of the fatigue crack on the edge.

Tab. 2. Overview of input quantities expressed in a deterministic way

Quantity	Value
Material constant m	3
Material constant C	$2.2.10^{-13}$
Width of the flange plate bf [mm]	400
Thickness of the flange plate tf [mm]	25
Designed probability of failure pd	0.02277



Choice of integration method : Gaussian quadrature
Fatigue crack progression from : the edge
Resistance Rad
Resistance Raac
Total number of oscillation of stress peaks Nn per n years
Cumulated effect of Loads Sn = C * (DeltaS^m) * Nn
Reliability function U (RF = Rad - Sn)
Reliability function F (RF = Raac - Sn)

Year 48

p (U)	9.59443E-01
p (D)	1.84020E-02
p (F)	2.21552E-02

Time of the first inspection :
between years : 48 - 49

Fig. 2. FCProbCalc desktop with resulting probability of failure events focused for the fatigue crack from the edge; the first inspection of the bridge should take place after 48 years of operation (Gaussian quadrature was used for numerical integration)

Tab. 3. Calculated times for the first and subsequent inspections of the bridge structure

Inspection No.	Time of inspection in years Failure crack from the edge	 Failure crack from the surface
1.	48	109
2.	55	122
3.	59	130
4.	62	136
5.	64	141
6.	66	145
7.	68	not applicable
8.	69	not applicable
9.	70	not applicable
10.	71	not applicable

In the study was analyse probabilistic calculation of fatigue crack progression from the surface also. It follows from the comparison of times for the first inspections which focus on the fatigue damage by the both types of the fatigue cracks (after 48 years of operation for the edge crack and after 109 years of operation for the surface crack) that the fatigue cracks propagate from the surface with a considerably lower speed that the fatigue cracks which initiate at the edge. Tab. 3 lists the proposed times for the resulting inspections specified using the Gauss quadrature with numerical integration used in calculation of (6) and (7).

5. Conclusion

This paper discusses development of the DOProC probabilistic method and its use in the reliability assessment of the constructions. A particular attention is paid to the theory and practical aspects of the probabilistic assessment of the constructions which are subject to fatigue and tend to create fatigue cracks. The result of this method is similar to other probabilistic approaches: proposal of a system of regular inspections of the construction.

Those computations were applied in FCProbCalc which was used for the mathematical modelling of propagation of fatigue cracks from the edge and surface. A probabilistic reliability assessment of the constructions was also performed in this software – it was based on the exact definition of the permissible size of the fatigue crack. The probabilities were obtained for three basic phenomena which are related to propagation of the fatigue cracks. On the basis of those data, the probability of failure can be calculated for each year of operation of the construction. When determining the required degree of reliability, it is possible to specify the time of the first inspection of the construction which will focus on the fatigue damage. Using a conditional probability, times for subsequent inspections can be determined.

The methods and application can considerably improve estimation of maintenance costs for the structures and bridges subject to cyclical loads.

If this methodology is developed further, the goal of investigations seems to be, in particular, application of Bayesian networks [25, 33] in the computational model which describes propagation of fatigue cracks.

Acknowledgements

This project has been completed thanks to the financial support provided to VSB-Technical University of Ostrava by the Czech Ministry of Education, Youth and Sports from the budget for conceptual development of science, research and innovations for the years 2015 and 2016.

Appendix A.

A restricted version of the computational modules in FCProbCalc and other software applications based on the DOProC method can be downloaded at http://www.fast.vsb.cz/popv.

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