Two-Dimensional Forward Scattering: Comparisons of Approximate and Exact Solutions

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Abstract. *Various methods for analyses of scattering are mentioned and new approximate relationships are derived. Experimental results for thin wire and several numerical simulations of forward scattering using approximate estimations, physical optics and exact solutions for two-dimensional scattering are presented both for far and near fields. That allows not only accuracy analyses but also conclusions about scattering and total fields in the presence of objects, which are important for many applications such as communications, bistatic and multistatic radars and electromagnetic compatibility.*

Keywords

Forward scattering, radar cross section, physical optics, modal solutions, two dimensional scattering.

1. Introduction

A scattering of electromagnetic waves is very important for various applications such as communications, bistatic and multistatic radars and electromagnetic compatibility [1] - [19]. Antenna far fields are usually considered for telecommunications and radars, when antenna radiation pattern and gain should be evaluated. On the other hand, the knowledge of near fields as well as far fields are often required from electromagnetic compatibility (EMC) point of view, when electromagnetic interference (EMI) such as wind turbine effects [14] electromagnetic susceptibility (EMS) and safety levels with respect to human exposure to radiofrequency electromagnetic fields are analyzed.

A variety of methods can be used for calculations of scattering such as exact solutions using vector partial differential equations or vector integral equations for the scattered fields and approximate methods, which use geometrical optics (GO) including various modifications of geometrical theory of diffraction, physical optics (PO), or physical theory of diffraction (PTD). The scattered field could be exactly obtained by classical modal solutions such as Mie solutions for scattering from a sphere [1] - [3]. Exact solutions could be used for several relatively simple cases. Even if analytical solutions have limited practical applications they will always be fundamental and essential tools for making advances in electromagnetic modeling phenomena. The integral formulation of Maxwell's equations is exact and the numerical solution of these equations then represents an “approximate solution to an exact formulation”. The standard solution technique is the method of moments (MOM). The MOM has been successfully used for a number of different problems such as finite circular cylinders or wind turbines [13], [14]. A parallel algorithm was used for scattering computation of a plane model with a fuselage of more than 1 000 wavelengths [15]. Exact solutions using vector partial differential equations can be used for numerical simulations of general scattering problems using finite element methods (FEM) or finite difference time domain (FDTD), which could be generally very time consuming. The approximate methods could be much faster and especially PTD could offer reliable results [6] - [8]. This paper briefly explains some methods for scattering analyses and provides new approximate relationships and numerous numerical simulations for two-dimensional scattering. The other numerical simulations for two-dimensional scattering could be found in [16]. Experimental results for thin wire are given as well. That allows comparisons of approximate and exact solutions and experiments for two-dimensional forward scattering.

2. Forward Scattering

The total field E due to scattering object is calculated as the sum of the incident wave E^i and the scattered field E^s

$$E = E_i + E_s. \tag{1}$$

Two major modern approximate approaches to diffraction theory are PTD [6] and geometrical theory of diffraction (GTD) and its modified versions, the uniform theory of diffraction (UTD) developed at Ohio State University and the similar uniform asymptotic theory of diffraction (UAT). These approaches are valid, each yields a ray description of the field, each has its advantages, and the two have now been cross-fertilizing each other for half a century. Like many good theories PTD is much easier to apply than to explain. PTD has played a key role in the development of modern low-radar-reflectivity weapons systems, functioning both as a design tool and as a conceptual framework. Moreover, for forward scattering the improved version of PTD [6], [7] could be used. According to [6] and [8] the forward scattering could be considered as a shadow radiation, which does not depend on the whole shape of the scattering object, and is completely determined by the size and the geometry of the shadow contour. Its power equals the total power incident on a scattering object, and it does not depend on the reflection coefficients. This quantity can be considered to be the diffraction limit of the reduction of scattering by application of absorbing coatings on a scattering object. That actually means that the shadow radiation contains a component in the form of the edge waves diverging from the shadow contour. The PO [6], [8] gives a useful approximation for the forward scattering (shadow radiation), E^{sh}, with harmonic time dependence exp(jωt) for the wave field

$$E^{sh} = \frac{-j}{\lambda} \iint_A E^i \frac{\exp(-jkr)}{r} dS \tag{2}$$

where λ is the wavelength, $k = 2\pi/\lambda$, A is the object projected area in plane yz and r is the distance between the observation point P (on axis x) and the integration point. For greater distances R between the observation and the object projected area A (far-filed zone), the following approximation can be easily derived [8]

$$E^{sh}_{FF} = \frac{-j}{\lambda} E^i \frac{A \exp(-jkR)}{R}$$

$$\text{for } R > d_{max}^2/2\lambda \tag{3}$$

where d_{max} is the maximum dimension of area A. That is, the greater the scattering the greater is the forward scatter and the darker is the shadow [3]. The forward scattering is bigger for higher frequencies according to (3) but the null-to-null beam width of the scattering is very narrow [3]

$$\theta_0 \approx 57\lambda/d_{max}\,[\text{deg}] \tag{4}$$

For smaller distances, R, equation (2) can be approximated by

$$E^{sh} \cong \frac{-j}{\lambda} E^i \frac{\exp(-jkR)}{R} \times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[\frac{-j\pi}{\lambda R}\left(y^2+z^2\right)\right] dydz \tag{5}$$

Using stationary phase method [4] the following equation can be obtained for small distances R

$$E^{sh}_{NF} = -E^i \exp(-jkR)$$

$$\text{for } R < d_{min}^2/2\lambda \tag{6}$$

where d_{min} is the minimum dimension of area A. That means, at a finite distance from the scattering object, the shadow radiation can be considered as a wave beam that asymptotically cancels the incident field for $\lambda \to 0$. Some more remarks concerning near-field forward scattering could be found in [17] - [19].

The radar cross section (RCS) can be defined [1] - [4]

$$\sigma = 4\pi \lim_{R\to\infty} R^2 \frac{\left|E^s\right|^2}{\left|E^i\right|^2} \tag{7}$$

where R is a range between the transmitting antenna and an observation point.

3. Two-Dimensional Scattering

If d_1 and d_2 are dimensions of area in planes xy and xz, respectively, then for targets with very long dimension $d_2 >> d_1$ (or possibly two dimensional targets), equation (5) can be approximated by

$$E^{sh}_{2D} \cong \frac{E^i \exp(-jkR)}{\sqrt{\lambda R}} \exp\left(-j\frac{3\pi}{4}\right) \times \int_{-d_1/2}^{d_1/2} \exp\left(\frac{-j\pi}{\lambda R} y^2\right) dy \tag{8}$$

Numerical calculations using (8) are relatively easy (comparing with numerical simulations of vector partial differential equations or vector integral equations). Nevertheless, that could be approximated by the following estimation

$$E^{sh}_{AP} \cong \frac{E^i d_1 \exp(-jkR)}{\sqrt{\lambda R}} \exp\left(-j\frac{3\pi}{4}\right)$$

$$\text{for } R > d_1^2/2\lambda \text{ and } R < d_2^2/2\lambda. \tag{9}$$

A source radiating in the presence of a conducting cylinder is one of the simplest problems for which an exact solution can be obtained [1] - [4]. Let the incident wave is a linearly z-polarized plane wave in x direction as is shown in Fig. 1

$$E^i_z = E_0 \exp(-jk\rho\cos\phi)\,. \tag{10}$$

The total field with the conducting cylinder is given by (1), where the scattered field is given by

$$E^s_z = E_0 \sum_{n=-\infty}^{\infty} j^{-n} a_n H_n^{(2)}(k\rho) \exp(jn\phi) \tag{11}$$

where

$$a_n = \frac{-J_n(ka)}{H_n^{(2)}(ka)}. \tag{12}$$

$J_n(x)$ is the Bessel function of the first kind and $H_n^{(2)}(x)$ is the Hankel function of the second kind. For numerical simulations the limits of n is taken according to the following estimation (that could be justified considering asymptotic expansions for large orders [20])

$$N = 1.08\,ka + 17. \tag{13}$$

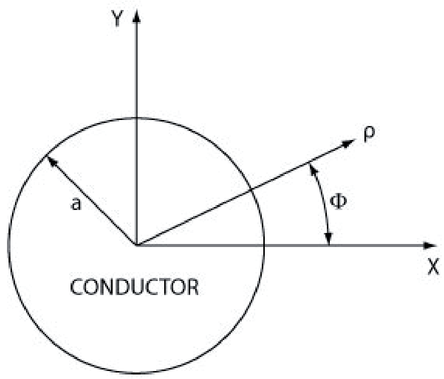

Fig. 1. A linearly *z*-polarized plane wave incident upon a conducting cylinder.

The radar cross section ("echoing width") can be defined [1] - [4] for two-dimensional scattering

$$\sigma = 2\pi \lim_{R\to\infty} R \frac{\left|E^s\right|^2}{\left|E^i\right|^2}. \tag{14}$$

Above equations allow accuracy analyses as well as a derivation of various conclusions about scattering and total fields in the presence of scattering objects, which are important for several applications such as communications, radars and electromagnetic compatibility. Various examples of the relative scattering E_z^s/E_z^i and total fields E/E_z^i according to (11) and (1) have been numerically simulated using Matlab. They are shown in Fig. 2 - 5. Both cylindrical radius a and distance ρ are given in wavelengths.

Fig. 2 and 3 show variations of relative forward scattering (___), total field (. . .), two-dimensional PO approximations PO_2 (– – –) according to (8), estimations AP (+) according to (9) and near-field approximation PO_N (.) according to (6). Arrows indicate the $d^2/2\lambda$ limit. Fig. 3a and 3b clearly demonstrate that the forward radiation can be considered as a wave beam that asymptotically cancels the incident field in the shadow region for $\lambda\to 0$ (for electrically greater objects) at a finite distance from the scattering object. Estimations (9) are very useful considering amplitudes as is shown in Fig. 2a to 2c.

Estimations (6) are rather rough but they approach the correct values for $\lambda\to 0$. However, phase estimations according to (6) and (9) shown in Fig. 3a to 3c are rather rough and therefore the total field calculations would offer smaller values.

Fig. 2 and 3 clearly demonstrate that two-dimensional PO approximations according to (8) provide very good approximations of amplitudes and acceptable approximations of phases and therefore the total field calculations would offer slightly different values.

Fig. 4 demonstrates that a forward scattering (shadow radiation) is smaller than an incident field. Fig. 5 clearly shows that the total field in the shadow region created by incident wave and forward scattering (shadow radiation) is zero for very small distances. It could be slightly greater than an incident field for greater distances. That corresponds to results [17] - [19]. The total field in the shadow region at forward scattering nearby regions could be slightly greater (in the order of 1 dB) due to diffraction.

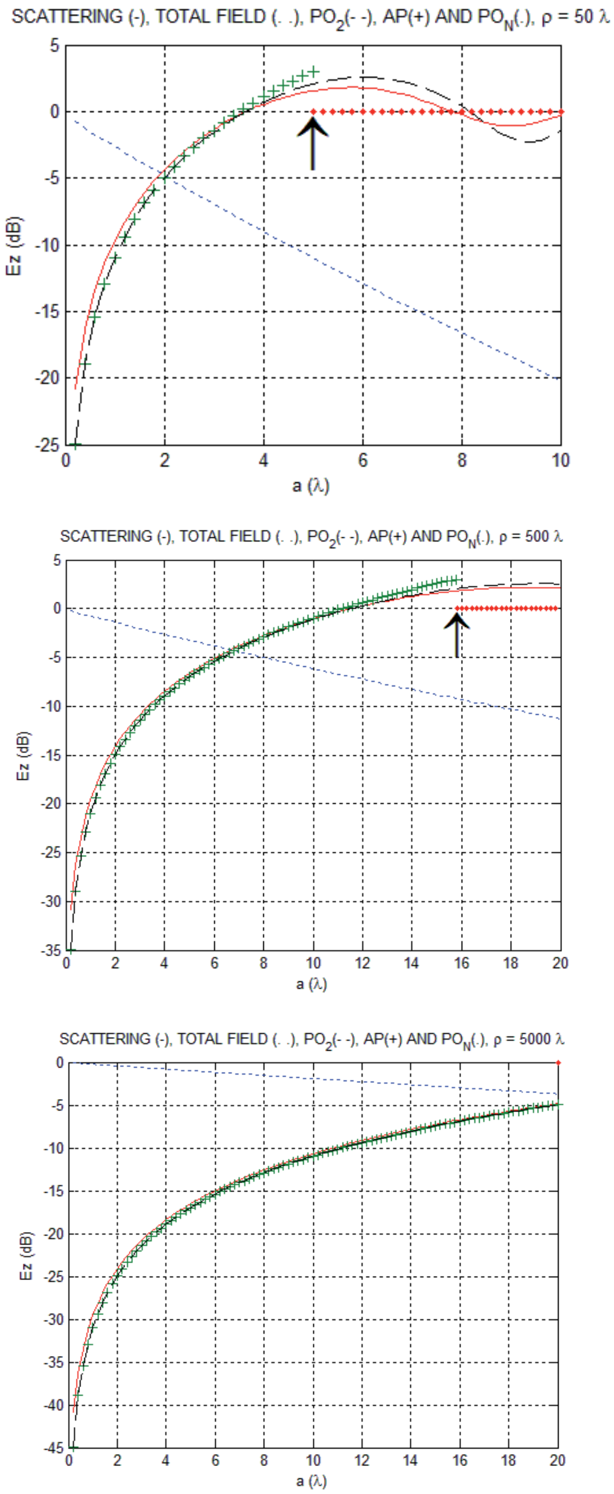

Fig. 2. Relative forward scattering (___), total field (. . .), PO (– – –), AP (+) and PO_N (.) a) for ρ = 50 λ, b) for ρ = 500 λ, and c) for ρ = 5 000 λ.

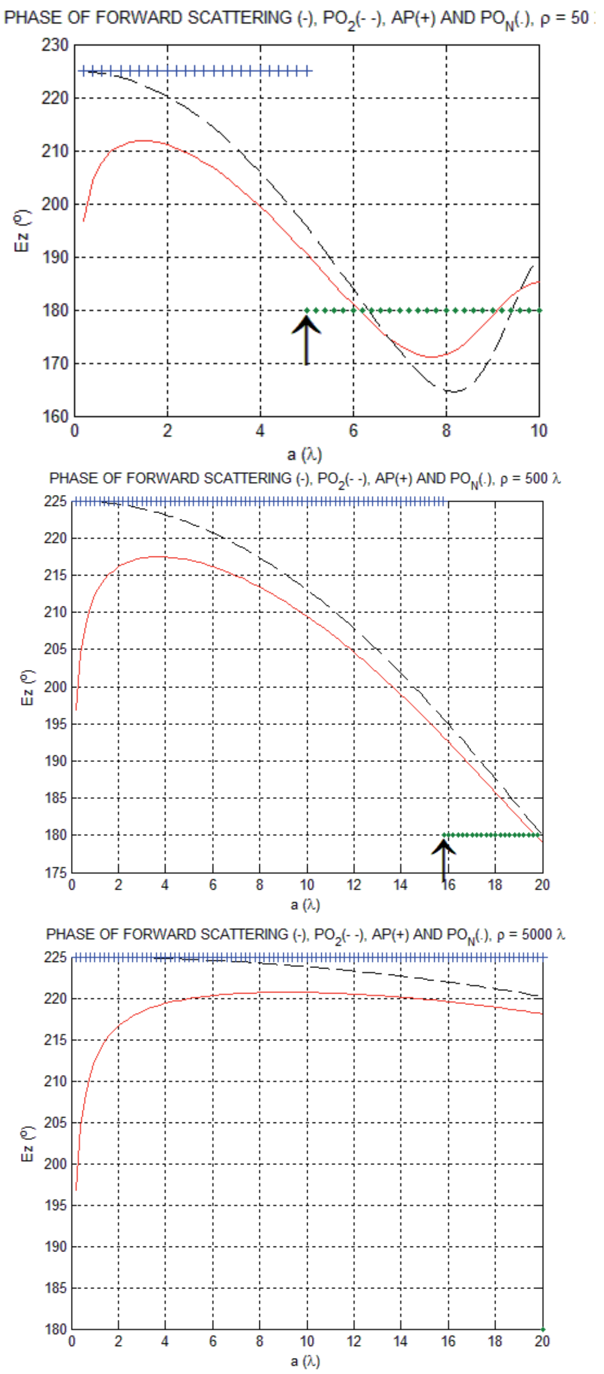

Fig. 3. Phase of forward scattering (___), total field (. . .), PO (– – –), AP (+) and PO_N (.) a) for ρ = 50 λ, b) for ρ = 500 λ, and c) for ρ = 5 000 λ.

The same conclusions could be obtained considering analyses [14]. The above results clearly demonstrate well known conclusions about the forward scattering and total fields [1] - [6] and [8]. Moreover, they demonstrate notice [12] about upsetting information concerning powerful radar signals or communication antennas in view of an electromagnetic compatibility or microwave safety levels on the human population in medium distances given by some "independent experts" that forward scattering could create total fields, which are by tens dB greater than incident field.

According to [10], where calculations of measured RCS have been performed using interferences of incident and scattered signals, there are great measurement deviations from the expected square frequency dependence. That could be partly explained by the fact that in the shadow region, at a finite distance from the scattering object, this radiation can be considered as a wave beam that asymptotically cancels the incident field for λ→0 as above simple approximations and various examples validate and the greater intensification of total field cannot be considered. Moreover, various errors are created due to multipaths, external signals and changes of forward RCS as the shadow contour is changing during the flight.

4. Measurements

A brass wire of 250 mm length and 4 mm diameter has been measured in an anechoic chamber. An antenna transmitted approximately 39 dBm signal with f = 32.9 GHz, the distance between transmitting antenna and the wire was 1.9 m. The distance between receiving antenna and the wire was 2.2 m for measurement of forward scattering. Several measurements both for a linearly z-polarized plane (E_z^i) and transversely to z (H_z^i) wave have been done.

The forward scattering measurements for the linearly z-polarized plane wave (E_z^i) give signal by 2 dB greater than measurements without the brass wire.

The forward scattering measurements for the transversely polarized plane wave (H_z^i) give approximately the same signal as measurements without the brass wire.

Fig. 6 shows variations of $D = 10 \log \sigma/kal^2$ for the perfectly conducting cylinder with radius of a using exact solution, where kal^2 is the PO approximation of backscattering (ϕ = 180 deg.). PO means the PO forward scattering approximations, E 180 indicates that incident electric field E_z^i is parallel to cylinder axis and ϕ = 180 degrees, H indicates that incident magnetic field H_z^i is parallel to cylinder axis (exact solution relationships can be found in [1] - [4]) etc. That clearly demonstrates effect of polarization and forward and back scattering.

It is obvious that measurements for the linearly z-polarized plane wave (E_z^i) are affected by the wire much more than measurements for the transversely polarized plane wave (H_z^i). Moreover, the forward scattering is bigger for higher frequencies according to (3) but the null-to-null beam width of the scattering is very narrow.

To compare the measurements and theoretical results, Fig. 7 displays the total field for more general problem of a current filament parallel to a conducting cylinder [1]. The distance between the filament and the cylinder axis is ρ' = 210 λ and the distance between the observation point and the cylinder axis is ρ = 240 λ (plane-wave incidence is the special case, when ρ'→∞).

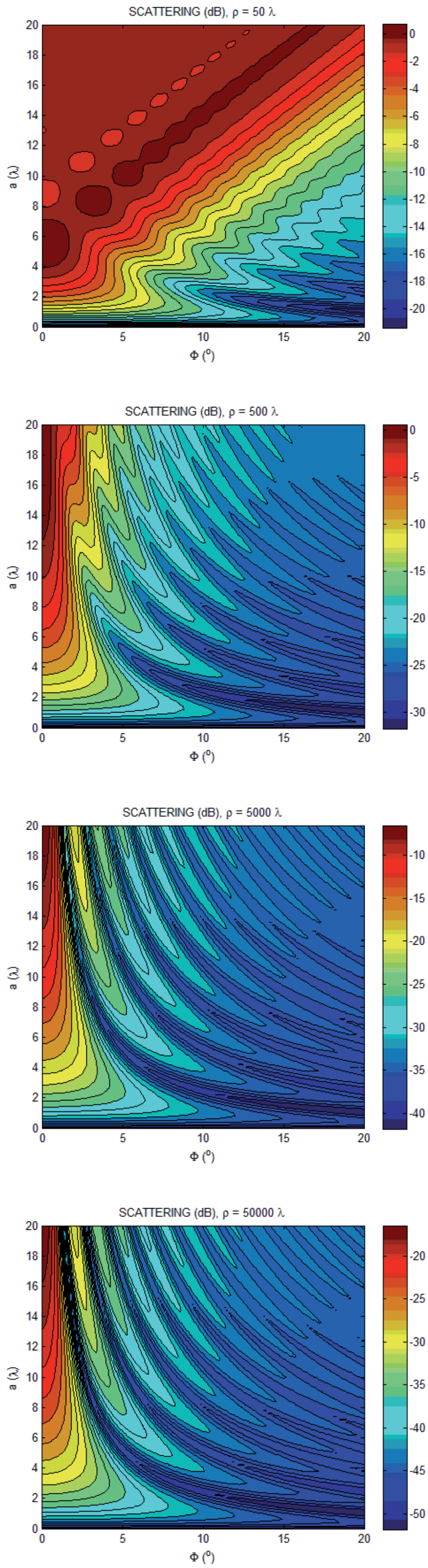

Fig. 4. Relative forward scattering (dB) a) for ρ = 50 λ, b) for ρ = 500 λ, c) for ρ = 5 000 λ, and d) for ρ = 50 000 λ.

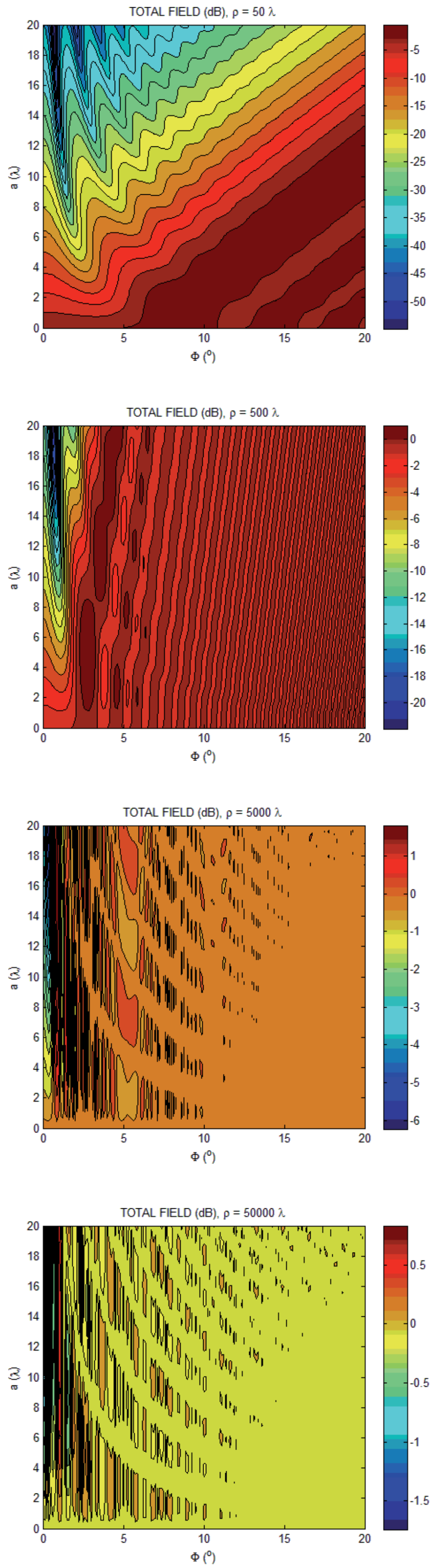

Fig. 5. Total field a) for ρ = 50 λ, b) for ρ = 500 λ, c) for ρ = 5 000 λ, and d) for ρ = 50 000 λ.

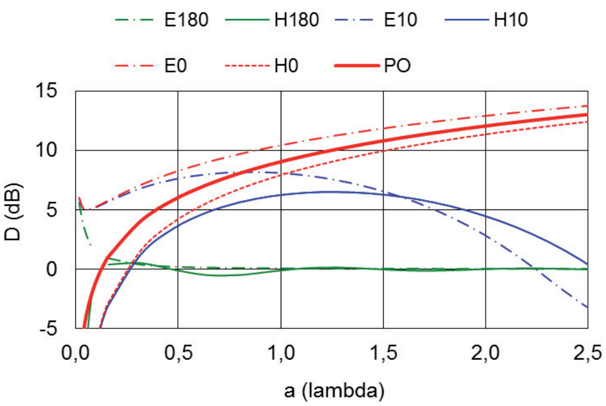

Fig. 6. Variations of $D = 10 \log \sigma/kal^2$ for the perfectly conducting cylinder with radius of a (PO means the PO forward scattering approximations, E 180 indicates that incident electric field is parallel to cylinder axis and ϕ = 180 degrees etc.).

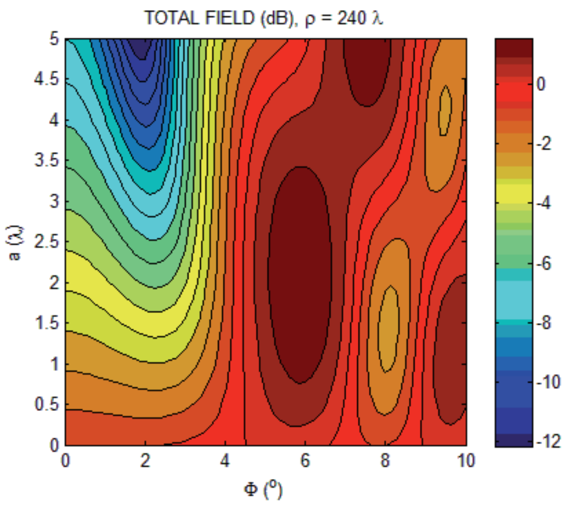

Fig. 7. Total field of a current filament parallel to a conducting cylinder ρ' = 210 λ and ρ = 240 λ.

The discrepancies between measurements and theoretically predicted results could be explained considering the fact that exact solution assumes two-dimensional scattering and the finite length wire was measured with very long dimension $d_2 >> d_1 = 2a$. The assumption about plane wave is clearly fulfilled for d_1 but for d_2 is not valid, and therefore the differences are created. The total field for two-dimensional scattering qualitatively corresponds to results [17] - [19].

5. Conclusion

The electromagnetic wave scattering is very important for various applications such as communications, bistatic and multistatic radars and electromagnetic compatibility. Antenna far fields are usually considered for telecommunications and radars, when antenna radiation pattern and gain should be evaluated. On the other hand, the knowledge of near fields as well as far fields are often required from electromagnetic compatibility (EMC) point of view, when electromagnetic interference (EMI) such as wind turbine effects, electromagnetic susceptibility (EMS) and safety levels with respect to human exposure to radiofrequency electromagnetic fields are analyzed.

The paper briefly explains some methods for analyses of scattering, derives new approximate relationships and gives numerous results of numerical simulations of forward scattering. Some experimental results are briefly described and compared with exact solutions. That allows not only the comparison of approximate and exact solutions but also various conclusions about scattering and total fields in the presence of scattered objects.

The following conclusions could be drawn:

- Estimations (9) are very useful considering amplitudes. However, phase estimations are rather rough and therefore the total field calculations would offer slightly smaller values. Estimations (3) are rather rough but they approach the correct values for $\lambda \to 0$.
- The PO approximation of forward scattering according to equation (8) could be used especially for objects much greater than a wavelength, when diffraction effects are very small.
- The quadratic phase effect, when the scattered fields will be substantially diminished, should be considered for small distances for the PO approximation according to equation (8).
- The forward scattering power is equal to the total power incident on a scattering object, which is much greater than a wavelength, and it does not depend on the object reflection coefficients.
- The forward radiation can be considered as a wave beam that asymptotically cancels the incident field in the shadow region for $\lambda \to 0$ at a finite distance from the scattering object according to equation (3).
- The forward scattering is bigger for higher frequencies but the null-to-null beam width is very narrow.
- The total field in the shadow region created by incident wave and forward scattering (shadow radiation) is zero for very small distances. It could be slightly greater (in the order of 1 dB) than an incident field for greater distances.
- The total field in the shadow region could be slightly greater (in the order of 1 dB) due to diffraction at nearby regions.

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