Semilogarithmic Nonuniform Vector Quantization of Two-Dimensional Laplacean Source for Small Variance Dynamics

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Abstract. *In this paper high dynamic range nonuniform two-dimensional vector quantization model for Laplacean source was provided. Semilogarithmic A-law compression characteristic was used as radial scalar compression characteristic of two-dimensional vector quantization. Optimal number value of concentric quantization domains (amplitude levels) is expressed in the function of parameter A. Exact distortion analysis with obtained closed form expressions is provided. It has been shown that proposed model provides high signal-to-quantization noise ratio (SQNR) values in wide range of variances, and overachieves quality obtained by scalar A-law quantization at the same bit rate, so it can be used in various switching and adaptation implementations for realization of high quality signal compression.*

Keywords

Laplacean source, A-law, two-dimensional vector quantization, nonuniform quantization.

1. Introduction

The quantization technique based on dividing a large set of points (vectors) into groups having approximately the same number of points closest to them and represented by its centroid point is called vector quantization (VQ). The set of discrete amplitude levels is quantized jointly instead each sample being quantized separately [1]. Data points are represented by the index of their closest centroid, so it commonly occurred data have low error, and rare data high error. The data is compressed, because a lower-space vector requires less storage space. Only the index of the codeword in the codebook is sent instead of the quantized values. This conserves space and achieves more compression. VQ allows the modeling of probability density functions by the distribution of prototype vectors, which is powerful, especially for identifying the density of large and high-dimensioned data. This is why VQ is suitable for lossy data compression.

Since VQ have a higher degree of freedom for choosing the reconstruction values and the decision regions they provide better performances (higher SQNR for the same bit-rate) compared to scalar quantizers. However VQ are, in general case, more complex than scalar quantizers, with the increase of quantizer dimension. The simplest VQ are two-dimensional VQ.

VQ of Laplacean source has been previously considered in the literature [2]-[6]. Proposed encoder design for switching piecewise uniform vector quantization of the memoryless two-dimensional Laplacean source was analyzed in [2]. In [5] and [7], a geometric approach was taken into consideration. Lattice quantization was applied to define a high dynamic range vector quantization model in [5].

However none of these papers has considered optimizing concentric quantization domains (amplitude levels) in the function of parameter *A*. In order to improve performance, we have provided two models of quantization considering *A* parameter. Also none of previously published papers considers analysis in the area of small parameter *A* values [3], [5]. Usually higher values of *A* parameter are considered, i. e., $A = 48269$ in [8], while more complex analysis should be carried out for the cases of smaller *A* parameter values, like in this paper.

Comparison with scalar *A*-law quantization has shown that proposed model provides high SQNR values in wide range of variances, and reaches better quality for over 2 dB than obtained by scalar A-law quantization at same bit rate, so it can be used in various switching and adaptation implementations for realization of high quality signal compression.

At the beginning we give the description a of two dimensional vector quantizer, also we define compression function. A distortion analysis with obtained closed form expressions is provided. After that we introduce two models of quantizer construction considering optimization of amplitude levels in the function of parameter *A*. At the end, we compared the performance of proposed vector quantizer with performance of the well-known semi-logarithmic *A*-law scalar quantizer [9]-[12].

2. Semilogarithmic Quantization Model of two Dimensions

At the quantizer input is a 2-dimensional vector $\boldsymbol{x} = [x_1, x_2]^T$ consisting of two independent and identically distributed variables with Laplacian distribution with zero mean and unit variance σ,

$$f(x_1) = \frac{1}{\sqrt{2}\sigma}\exp\left(-\frac{\sqrt{2}|x_1|}{\sigma}\right),$$

$$f(x_2) = \frac{1}{\sqrt{2}\sigma}\exp\left(-\frac{\sqrt{2}|x_2|}{\sigma}\right), \tag{1}$$

$$f(x_1, x_2) = f(x_1)f(x_2) = \frac{1}{2\sigma^2}\exp\left(-\frac{\sqrt{2}|x_2| + |x_2|}{\sigma}\right).$$

In two-dimensional x_1x_2 system, probability density function (*pdf*) given by equation (1) represents a square line. This square surface representing dynamic range of a two dimensional vector quantizer, can be partitioned into L concentric domains as shown in Fig. 1. In the case of nonuniform vector quantization, these concentric domains are of unequal width.

The number of output points in each domain is denoted by N_i, where $N = \sum_{i=1}^{L} N_i$ represents the total number of output points. Every concetric domain is partitioned into rectangular cells. As shown in [3], [5], it is considered to be $N_1 = N_2 = \ldots = N_L$, and optimization is carried out over L.

Since computational complexity is a function of quantizer design complexity, the proposed technique provides lower computational complexity compared to the model presented in [4]. Namely, VQ model presented in [4] is more complex, since it can be observed as a model consisting of L*k uniform quantizers and k nonuniform quantizers (L is the number of concentric domains and k denotes the number of quantizers in corresponding switching quan-tizer). Our newly proposed VQ can be observed as model consisting of one uniform quantizers and one nonuniform quantizer. However, as known, SQ (scalar quantizer) models, consisting of a non-uniform quantizer, have even lower complexity.

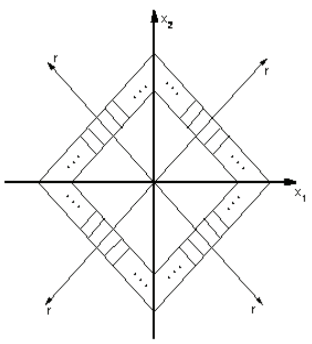

Fig. 1. Two-dimensional space partitioning.

The radius r of input vector $\boldsymbol{x}$ is defined as $r = |x_1| + |x_2|$ and it is also a random variable that has a probability density function [5], [13]

$$f_2(r) = \frac{2r}{\sigma^2}\exp\left(-\frac{\sqrt{2}r}{\sigma}\right), \; r \geq 0. \tag{2}$$

Optimal compression function used in 2-dimensional vector quantization is:

$$h(r) = \begin{cases} \dfrac{Ar}{1 + \ln A}, & 0 \leq r \leq \dfrac{r_{\max}}{A}; \\[2ex] r_{\max}\dfrac{1 + \ln\left(\dfrac{Ar}{r_{\max}}\right)}{1 + \ln A}, & \dfrac{r_{\max}}{A} \leq r \leq r_{\max}. \end{cases} \tag{3}$$

This compression function consists of two parts: linear and logarithmic. $r_1 = r_{\max}/A$ is the border between these two parts. $r_{\max}$ is the maximal range of quantizer.

During quantization an irreversible error is made, which is expressed by distortion. Total distortion D consists of granular distortion D_g in granular region and overload distortion D_o in overload region:

$$D = D_g + D_o. \tag{4}$$

Granular distortion D_g consists of two parts: distortion in linear part D_{g1} and distortion in logarithmic part D_{g2}. D_{g1} is calculated as:

$$D_{g1} = \frac{\Delta_1^2}{12}P_1 \tag{5}$$

where

$$P_1 = \int_0^{r_1} f_2(r)dr$$

is the probability that an input vector $\boldsymbol{x}$ belongs to area S_1 that includes all cells from the linear part

$$\Delta_1^2 = \frac{s_1}{N_1} = \frac{2r_1^2}{N_1} \text{ where } N_1 = L_1^2 \text{ and } L_1 = L/(1 + \ln A).$$

D_{g2} can be calculated using Bennet integral as:

$$D_{g2} = \frac{\Delta_2^2}{12}\int_{r_1}^{r_{\max}} \frac{f_2(r)}{[h'(r)]^2}dr \tag{6}$$

where $\Delta_2 = \frac{r_{\max}}{L}$. Overload distortion is defined as [5]

$$D_0 = \frac{1}{n}\int_{r_{\max}}^{\infty}\left[(r - m_L)^2 + \frac{1}{12}n\Delta_L^2\right]f_n(r)dr \tag{7}$$

where

$$m_L = r_{\max} - \frac{\Delta L}{2} \text{ and } \Delta L = \frac{1 + \ln A}{L}r_{\max}.$$

In our case the number of dimensions is 2 ($n = 2$).

Applying simple mathematic calculation, we can obtain the expressions for D_{g1}, D_{g2} and D_0 in closed form:

$$D_{g1}=\frac{r_1^2}{6N_1}\left[1-e^{-\frac{\sqrt{2}}{\sigma}r_1}-\frac{r_1\sqrt{2}}{\sigma}e^{-\frac{\sqrt{2}}{\sigma}r_1}\right],$$

$$D_{g2}=\frac{(1+\ln A)^2}{6L^2\sigma^2}\left[e^{-\frac{\sqrt{2}}{\sigma}r_1}\left(r_1^3\frac{\sigma}{\sqrt{2}}+\frac{3\sigma^2r_1^2}{2}+\frac{3\sigma^3r_1}{\sqrt{2}}+\frac{3\sigma^4}{2}\right)-\right.$$
$$\left.-e^{-\frac{\sqrt{2}}{\sigma}r_{\max}}\left(r_{\max}^3\frac{\sigma}{\sqrt{2}}+\frac{3\sigma^2r_{\max}^2}{2}+\frac{3\sigma^3r_{\max}}{\sqrt{2}}+\frac{3\sigma^4}{2}\right)\right],$$

$$D_0=\frac{1}{\sigma}\left[e^{-\frac{\sqrt{2}}{\sigma}r_{\max}}\left(\frac{r_{\max}^3}{\sqrt{2}}+\frac{3}{2}\sigma r_{\max}^2+\frac{3}{\sqrt{2}}\sigma^2r_{\max}+\frac{3}{2}\sigma^3\right)-\right.$$
$$-e^{-\frac{\sqrt{2}}{\sigma}r_{\max}}\left(\sqrt{2}r_{\max}^2+2\sigma r_{\max}+\sqrt{2}\sigma^2\right)+ \tag{8}$$
$$\left.+\left(m_L^2+\frac{\Delta_L^2}{6}\right)e^{-\frac{\sqrt{2}}{\sigma}r_{\max}}\left(\frac{r_{\max}}{\sqrt{2}}+\frac{\sigma}{2}\right)\right].$$

Exact distortion analysis has been obtained in the form of closed form expressions in (8). In previous papers [3], [5] an approximation was made, because the influence of the linear part of characteristic on the distortion was not taken into account. Namely for higher values of A parameter distortion D_{g1} was ignored. Since our analysis covers full range of A parameters, distortion D_{g1} was taken into account, and expression (8) could be applied more accurately for smaller values of A.

In the function of A parameter values optimal number of amplitude levels can be expressed through two models [5]:

I model ($A \geq 20$)

$$L=\frac{\sqrt{(1+\ln A)N}}{2}.$$

II model ($A < 20$)

$$L=\sqrt{N}.$$

In order to compare our 2-dimensional vector quantizer with another already defined quantizer we define signal-to-quantization noise ratio (*SQNR*) as:

$$SQNR=10\log_{10}\frac{\sigma^2}{D}. \tag{9}$$

3. Performance and Discussion of Results

The comparison of *SQNR* values in the function of input variances for *A*-law scalar quantizer with $N = 16$ levels, and for 2-dimensional vector quantizer with $N = 256$ levels (for high value of parameter A, A = 87.6) is given in Fig. 2. Scalar quantization size of codebook is defined with $N = 2^R$, while with vector quantization bit rate is $R = ½ \log_2 N$. The comparison corresponds to the same bit rate, so signal quality is comparable.

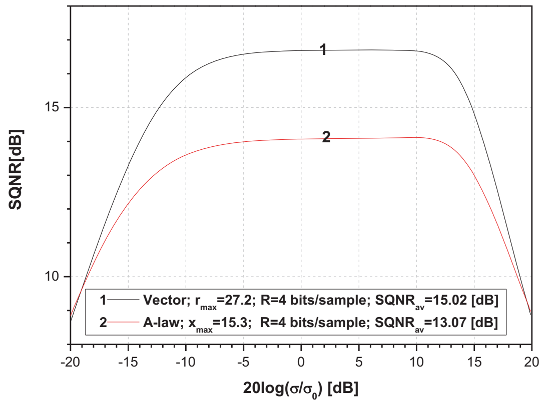

Fig. 2. Comparison of *SQNR* between *A*-law scalar quantizer and 2-dimensional vector quantizer for $A = 87.6$.

From Fig. 2 we can see that the proposed vector quantizer implementation has two main advantages. Firstly, higher values of *SQNR* are achieved with proposed 2-dimensional vector quantizer implementation than with the scalar one, over the wide range of input variance, so it is very suitable for non-adaptive quantization. Secondly, 2-dimensional vector quantizer's maximal *SQNR* value overachieves maximal *SQNR* value for A-law scalar quantizer by 2.60 dB. Because of that vector quantizer could provide higher quality performances during adaptation process.

In Tab. 1 the average *SQNR* values for proposed vector quantizer implementation and A-law scalar quantizer are presented for various ranges of normalized input signal variances (A = 87.6). We can see that with variance range dynamics decrease, level of quality achieved with proposed model arises compared to scalar A-law quantization model.

down bound [dB]	up bound [dB]	width [dB]	$SQNR_{av}$ [dB] (vector quantizer)	$SQNR_{av}$ [dB] (scalar quantizer)
-20	20	40	15.02	13.07
-15	15	30	16.22	13.82
-10	10	20	16.62	14.05
-5	5	10	16.69	14.09

Tab. 1. Average value of SQNR for vector quantizer and for A-law scalar quantizer for different widths of input signal variances (A=87.6).

Table 2 shows percentage of distortion D_{g1} in the overall distortion D for different values of parameter A. It can be seen, that for smaller values of A parameter the influence of D_{g1} on overall performances increases, so our proposed model has more general properties than approximation from [5,8,9], (where the influence of the linear part of characteristic on the distortion was not taken into account).

A	D_{g1}	D	D_{g1}/D*100[%]
10	0.00125	0.011960	10.451
100	0.000088	0.030749	0.286
1000	0.000001	0.061068	0.001

Tab. 2. Percentage of distortion D_{g1} in the overall distortion D for different values of parameter A.

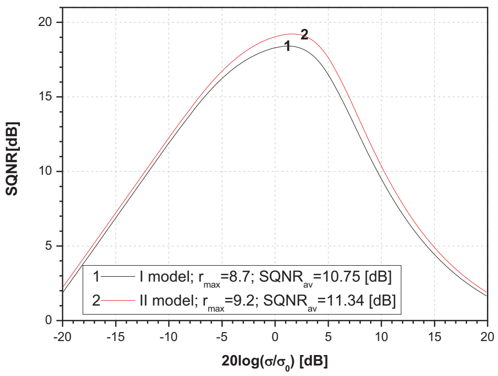

Fig. 3. Comparison of SQNR for proposed vector quantizer when L is calculated according to the first and second model for A = 10 and R = 4 bits/sample.

From Fig.3 we can see SQNR values for smaller values of the parameter A (A < 20). Namely *SQNR* values for the proposed vector quantizer implementation for the case when L is calculated according to the second model overachieve *SQNR* values for the case when L is calculated according to the first model in the complete range of normalized signal variances.

In Fig. 4 *SQNR* values through the input normalized variances are given for A-law scalar quantizer model and proposed 2-dimensional vector quantizer, for small value of parameter A (A = 10).

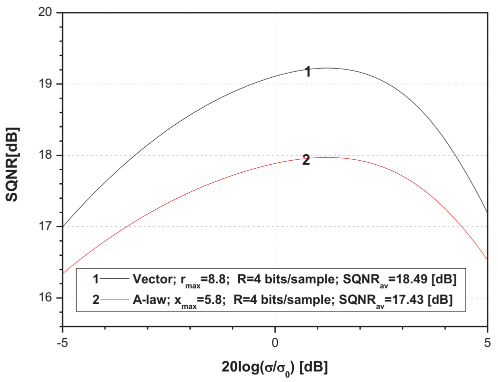

Fig. 4. Comparison of SQNR between A-law scalar quantizer and 2-dimensional vector quantizer for A = 10.

We can conclude that in the area of small A values (for input signals of small variance dynamics), proposed 2-dimensional vector quantizer achieves higher *SQNR* values in the observed variance range over *SQNR* values, that can be achieved by using A-law scalar quantizer. Also presented 2-dimensional vector quantizer achieves higher maximal *SQNR* value so could reach better performance during adaptation.

4. Conclusions

An exact distortion analysis with obtained closed form expressions is provided for the high dynamic range nonuniform two-dimensional vector quantization model for Laplacean source. As radial scalar compression characteristic of two-dimensional vector quantization, semilogarithmic A-law compression characteristic was considered. Concentric quantization domains number (number of amplitude levels) is optimized in the function of parameter A. Two models of quantization considering A parameter are provided. Comparing with previously obtained results by scalar A-law quantization it has been shown that proposed model provides high SQNR values in wide range of variances, and reaches better quality for over 2 dB (for $A \geq 20$) and over 1 dB (for $A < 20$) at the same bit rate. Also the proposed model can be used in various switching and adaptation implementations.

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