ANALYSIS OF WIRE ANTENNAS BY MOMENT METHODS

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Abstract

In the presented submission, various moment methods for computing the current distribution and the input impedance of wire antennas are rewieved and compared. The use of various basis and weighting functions is discussed. At each method, computational requirements and accuracy are investigated.

Keywords

cylindrical antennas, moment methods

1. Introduction

All the important technical parameters of antennas such as gain, input impedance or directivity pattern can be easily computed if the current distribution on the antenna surface is known. Unfortunately, computation of current distribution makes troubles because integral equations have to be solved.

There are two basic approaches to the solution of integral equations - iterative and moment ones. Iterative methods come from a rough approximation of the current distribution (e.g. sine wave on the dipole antenna) that is iteratively precisioned. On the other hand, moment methods transform integral equations to a set of simultaneous linear equations those are solved by matrix operations.

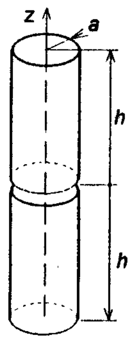

Fig.1 Wire antenna

Presented paper is focused in the moment analysis of wire antennas. In all the cases, antennas are supposed to be circular cylinders of radius a and length $2h$. The antenna axis is situated to the axis z (fig.1) of the cylindrical coordinate system (r, ρ, z). Cylinders occur in the vacuum ($\mu = \mu_0$, $\varepsilon = \varepsilon_0$, $\sigma = 0$) and do not exhibit any losses.

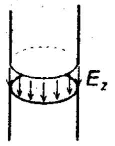

Fig.2 Excitating electrical field between antenna terminals

In the middle of the cylinder (z=0), there is a short gap. In the gap, a hypothetical harmonic generator is assumed such as the excitating electrical field can be azimuthally symmetric. The voltage across the gap

$$V = -\int_{gap} E_z dz \qquad (1.1)$$

is supposed to be 1V. In (1.1), E_z is the z-component of the excitating electrical field intensity on the interpolated antenna surface (fig.2). Outside the gap, E_z is zero because of the perfect conductivity of the cylinder.

Section 2 of the paper describes the nature of moment methods and various basis and weighting functions are introduced. Section 3 deals with the wire antennas (their diameter is negligible in comparison with the wavelength).

All the theoretical conclusions are illustrated by results of computer simulations those have been performed in matlab 4.2 and MathCAD Plus 5.0.

2. Methods of moments

Assume a general integral equation

$$\int_a^b f(z,\xi)\, d\xi = g(z) \qquad (2.1)$$

where f is an unknown function, $\langle a,b \rangle$ is the integration interval and g is a known function describing sources. The moment method solution of (2.1) consists in 3 steps:

1. The unknown function f is approximated by a linear combination of known basis functions f_n and unknown coefficients c_n

$$f \approx \tilde{f} = \sum_{n=1}^{N} c_n f_n \,. \qquad (2.2)$$

2. The approximation of the unknown function $\tilde{f}$ is put back to the solved equation (2.1). After that, the summation and integration are swapped. This yields

$$\sum_{n=1}^{N} c_n \int_a^b f_n(z,\xi)\, d\xi = g(z) + R(z) \,. \qquad (2.3)$$

Here, $R(z)$ is the residuum which expresses the fact that the approximation $\tilde{f}$ does not fulfil the relation (2.1) exactly. Equation (2.3) is one equation for N unknown coefficients c_n

3. The approximation $\tilde{f}$ most accurately fulfils (2.1) if the residuum R is minimal. Hence, the residuum is minimized by the method of weighted residua: product of the weighting function w and the residuum R integrated over the region of interest $<a,b>$ have to be zero [1]. If N weighting functions are used then the set of N simultaneous linear equations for N unknown coefficients c_n appear

$$\int_a^b w_m(z) R(z)\, dz = 0 \qquad m = 0,1,\ldots N \qquad (2.4a)$$

$$\sum_{n=1}^{N} c_n \int_a^b w_m(z) \int_a^b f_n(z,\xi)\, d\xi\, dz = \int_a^b w_m(z) g(z)\, dz . \qquad (2.4b)$$

Both basis and weighting functions have to be linearly independent on the interval $<a,b>$.

2.1 Basis functions

Basis functions can be global or local ones. Global basis functions are defined on all the region of interest $<a,b>$. E.g., system of functions

$$f_n(z) = \cos(\pi n z / h) \qquad (2.5)$$

is on $<a,b>$ linearly independent and coefficients c_n in the approximation

$$f(z) \approx \tilde{f}(z) = \sum_{n=1}^{N} c_n f_n = \sum_{n=1}^{N} c_n \cos(\pi n z / h) \qquad (2.6)$$

have got then meaning of Fourier coefficients.

Approximation based on the global basis functions is called the single-basis approximation.

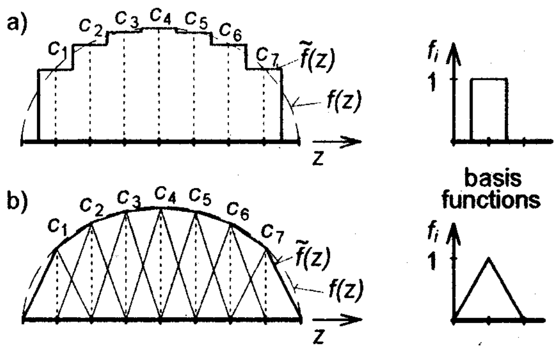

Fig.3 Multi-basis approximations
a) piece-wise constant
b) piece-wise linear

Local basis functions are defined on all the region too but each of them is non-zero only on a sub-region of the interval of interest $<a,b>$ as can be seen in fig.3. If basis functions are normed then coefficients c_n have got the meaning of nodal values of the computed function f (fig. 3). Approximation based on the local basis functions is called the multi-basis approximation.

2.2. Weighting functions

Point matching and Galerkin's methods are the most common ways of residuum minimization.

Point matching (or collocation) uses Dirac pulses as weighting functions

$$w_m(z) = \delta(z - z_m) . \qquad (2.7)$$

Point matching method exhibits very low computational requirements because one integration is eliminated in (2.4) thanks to the filtering property of δ-pulses

$$\sum_{n=1}^{N} c_n \int_a^b f_n(z_m, \xi)\, d\xi = g(z_m) . \qquad (2.8)$$

On the other hand, residuum minimization is related only to the matching points z_m.

In Galerkin's method, weighting functions are identical with the basis ones

$$w_m(z) = f_m(z) . \qquad (2.9)$$

Galerkin's method exhibits higher computational requirements in comparison with point matching because one of integrations is not eliminated in this case. On the other hand, the residuum minimization is performed with all the points $z \in <a,b>$.

3. Wire antennas

Assume the cylindrical antenna of fig.1. Then, the radiated electromagnetic field can be expressed in terms of vectorial and scalar potentials $\mathbf{A}$ and φ respectively. Potentials have to fulfil inhomogennous wave equations [4]

$$\frac{\partial^2 A_z(z)}{\partial z^2} + k^2 A_z(z) = -\mu_0 J_z(z) \qquad (3.1a)$$

$$\frac{\partial^2 \varphi(z)}{\partial z^2} + k^2 \varphi(z) = -\frac{\rho(z)}{\varepsilon_0} . \qquad (3.1b)$$

Here, J_z is the z-component of the current density [A.m^{-2}] impressed to the antenna by the source, ρ is the volume charge density [C.m^{-3}] on the antenna, A_z is the z-component of the vectorial potential and φ is the scalar potential, $k=2\pi/\lambda$ is the wavenumber and λ is the wavelength.

The current flowing on the antenna causes charge accumulation at the ends of the antenna cylinder. This fact can be described by the equation of continuity [4]

$$\frac{\partial J_z(z)}{\partial z} + j\omega \rho(z) = 0 . \qquad (3.2a)$$

If radius of the antenna cylinder is much smaller than the wavelength $a \ll \lambda$ then the current and charge can be assumed to be concentrated in the axis of the cylinder [1] and solving (3.1) yields [4]

$$A_z(z) = \frac{\mu}{4\pi}\int_{2h} I_z(\xi)\frac{e^{-jkR(z,\xi)}}{R(z,\xi)}d\xi \qquad (3.2b)$$

$$\varphi(z) = \frac{1}{4\pi\varepsilon}\int_{2h}\sigma(\xi)\frac{e^{-jkR(z,\xi)}}{R(z,\xi)}d\xi . \qquad (3.2c)$$

Here, $I_z(\xi)$ is the current [A] flowing in the axis of wire, $\sigma(\xi)$ denotes the length charge density [$C.m^{-1}$] on the axis of wire, $R(z,\xi)$ is the distance between the location ξ of electromagnetic field sources $I_z(\xi)$ and $\sigma(\xi)$ and the location z potentials $A(z)$ and $\varphi(z)$.

On the base of $A(z)$ and $\varphi(z)$, electrical intensity of the field radiated by the antenna can be computed [4]

$$E_z^s(z) = -j\omega A_z(z) - \frac{\partial\varphi(z)}{\partial z} . \qquad (3.2d)$$

Electrical intensity has to fulfil the boundary condition on the antenna surface S

$$E_z^i + E_z^s = 0 \qquad \text{on } S ; \qquad (3.2e)$$

E_z^i denotes electrical intensity of the incident wave.

If the current distribution on the antenna is to be computed then the set of equations (3.2) has to be solved.

Fulfilling the boundary condition (3.2e) requires computation of electrical intensity (and consequently potentials) on the surface of the wire. That is why the distance R is described by the equation

$$R(z,\xi) = \sqrt{a^2 + (z-\xi)^2} . \qquad (3.3)$$

In the following paragraphs, the use of various basis and weighting functions for the solution of (3.2) by moment methods is discussed.

3.1 Piece-wise constant approximation, point matching

Segmentation of the antenna is depicted in fig.4. Lower bounds of segments are signed by „-", upper ones by „+". Lower bound of the first segment and upper bound of the last segment are shifted from the ends of the antenna to fulfil condition $I(-h)=I(h)=0$. Segments' lengths are $\Delta=2\alpha$.

Putting piece-wise constant approximation to the integral equations (3.2b,c) yields

$$A_z(z) \approx \frac{\mu}{4\pi}\sum_{n=1}^{N} I_n \int_{-h+(n-0.5)\Delta}^{-h+(n+0.5)\Delta}\frac{e^{-jkR(z,\xi)}}{R(z,\xi)}d\xi \qquad (3.4b)$$

$$\varphi(z) \approx \frac{1}{4\pi\varepsilon}\sum_{n=1}^{N}\sigma_n \int_{-h+(n-0.5)\Delta}^{-h+(n+0.5)\Delta}\frac{e^{-jkR(z,\xi)}}{R(z,\xi)}d\xi . \qquad (3.4c)$$

Here, I_n and σ_n are nodal values of current and charge density distributions.

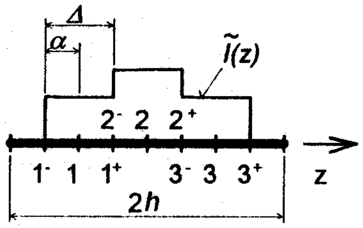

Fig.4 Piece-wise constant approximation

Since first derivative of the piece-wise approximation is zero on the constant sections and does not exist on their borders, (3.2a) and (3.2d) are rewritten in terms of finite differences. If the fact that $I_n = I_z(-h+n\Delta)$ is considered then the continuity equation can be expressed as (3.5a)

$$\frac{I_z(-h+(n+1)\Delta) - I_z(-h+n\Delta)}{\Delta} + j\omega\sigma(-h+(n+0.5)\Delta) \approx 0$$

and the relation for computing electrical intensity is then of the form

$$E_z^s(-h+n\Delta) \approx -j\omega A_z(-h+n\Delta) - \qquad (3.5d)$$

$$-\frac{\varphi(-h+(n+0.5)\Delta) - \varphi(-h+(n-0.5)\Delta)}{\Delta} .$$

Relations (3.5a) and (3.5d) show that Dirac pulses for point matching have to be placed to the middle of segments for the vectorial potential (3.5b)

$$A_z(-h+m\Delta) \approx \frac{\mu}{4\pi}\sum_{n=1}^{N} I_n \int_{-h+(n-0.5)\Delta}^{-h+(n+0.5)\Delta}\frac{e^{-jkR(-h+m\Delta,\xi)}}{R(-h+m\Delta,\xi)}d\xi$$

and to borders of segments for the scalar potential

$$\varphi(-h+(m+0.5)\Delta) \approx \qquad (3.5c)$$

$$\approx \frac{1}{4\pi\varepsilon}\sum_{n=1}^{N}\sigma_{n^+}\int_{-h+n\Delta}^{-h+(n+1)\Delta}\frac{e^{-jkR(-h+(m+0.5)\Delta,\xi)}}{R(-h+(m+0.5)\Delta,\xi)}d\xi$$

In (3.5c), $\sigma_{n+} = \sigma(-h+(n+0.5)\Delta)$.

Now, (3.5) can be rewritten more compactly

$$\sigma_{n^+} \approx \frac{-1}{j\omega}\left[\frac{I_{n+1} - I_n}{\Delta}\right] \qquad (3.6a)$$

$$A_z(m) \approx \frac{\mu}{4\pi}\sum_{n=1}^{N} I_n \int_{\Delta_n}\frac{e^{-jkR(m,\xi)}}{R(m,\xi)}d\xi \qquad (3.6b)$$

$$\varphi(m^+) \approx \frac{1}{4\pi\varepsilon}\sum_{n=1}^{N}\sigma_{n^+}\int_{\Delta_{n+}}\frac{e^{-jkR(m^+,\xi)}}{R(m^+,\xi)}d\xi \qquad (3.6c)$$

$$-E_z^i(m) \approx -j\omega A_z(m) - \frac{\varphi(m^+)-\varphi(m^-)}{\Delta} \qquad (3.6d)$$

In (3.6d), the boundary condition (3.2e) is included.

Now, let's have a look at the continuity theorem (3.6a); it expresses the fact that segments of the antenna can be replaced by elementary electrical dipoles (fig.5). Taking this idea in mind, submission of nth segment to the scalar potential can be computed on the base of (3.6c) as

$$\varphi(m^+) = \frac{1}{j\omega\varepsilon}\left[I_n\int_{\Delta_{n+}}\frac{e^{-jkR}}{4\pi R}d\xi - I_n\int_{\Delta_{n-}}\frac{e^{-jkR}}{4\pi R}d\xi\right]\frac{1}{\Delta}. \qquad (3.7)$$

Putting (3.7) and (3.6b) to (3.6d) and multiplying both sides of eqn. by Δ yields

$$\mathbf{E}_z^i\Delta = \mathbf{ZI} \qquad (3.8)$$

where

$$Z_{mn} = j\omega\mu\Delta\int_{\Delta_n}\frac{e^{-jkR(m,\xi)}}{4\pi R(m,\xi)}d\xi + \qquad (3.9)$$

$$+\frac{1}{j\omega\varepsilon}\left[\int_{\Delta_{n+}}\frac{e^{-jkR(m+,\xi)}}{4\pi R(m^+,\xi)}d\xi - \int_{\Delta_{n-}}\frac{e^{-jkR(m+,\xi)}}{4\pi R(m^+,\xi)}d\xi\right]\frac{1}{\Delta} -$$

$$-\frac{1}{j\omega\varepsilon}\left[\int_{\Delta_{n+}}\frac{e^{-jkR(m-,\xi)}}{4\pi R(m^-,\xi)}d\xi - \int_{\Delta_{n-}}\frac{e^{-jkR(m-,\xi)}}{4\pi R(m^-,\xi)}d\xi\right]\frac{1}{\Delta}$$

denotes submission of current and charge on nth segment to the voltage induced on the mth segment.

Since electrical intensity is zero on all the segments except of the source gap, elements of voltage vector are zero except of the gap-segment corresponding element which equals 1. Then, (3.8) provides the current distribution I. Ratio of input voltage and input current give then the input impedance of analyzed antenna.

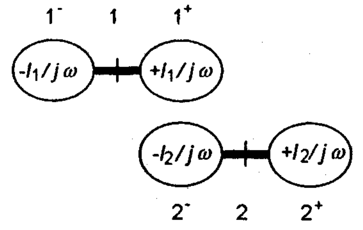

Fig.5 Antenna as a set of elementary electrical dipoles

The described algorithm can be very easily programmed. First, all the necessary integrals

$$\psi = \int_{\Delta_n}\left[\exp(-jkR)/4\pi R\right]d\xi$$

are computed. In matlab syntax:

```
for m = 1:(N+2)
 z = (m-1)*del;                     % del = segment length
 psi(m) = quad8('g',-del/2,+del/2,1e-5,0,z,a,k);
end
```

In the `quad8` function (numerical integration based on adaptive recursive Newton Cotes 8 panel rule), `g` is the integrated function, `-del/2` and `+del/2` are limits of integration, `1e-5` is required maximal error of integration and `z, a, k` are parameters. The integrated function `g` is defined as:

```
function out=g( ksi, z, a, k)
R = sqrt( a^2 + (z-ksi)^2); % ksi ∈ (-Δ/2; +Δ/2)
out = exp(-j*k*R)/(4*pi*R); % Green's function
```

Now, the impedance matrix can be built up, if length of all segments is the same then distances (m^+,n^+), (m^-,n^-) equal.

```
for m = 1:N                              % impedance matrix
 for n = m:N
  dist = abs(m-n);   % source-destination distance
  hlp = 2*psi(1+dist) - psi(1+abs(dist-1)) -
        psi(1+abs(dist+1));
  Z(m,n) = j*omega*mi*del*psi(1+dist) +
        hlp/(j*omega*epsilon*del);        % eqn. 3.9
  Z(n,m) = Z(m,n);              % matrix is symmetrical
 end
end
```

In the above list, `mi` and `epsilon` are permeability μ_0 and permittivity ε_0, `omega` is the circular frequency ω and `j` denotes imaginary unit.

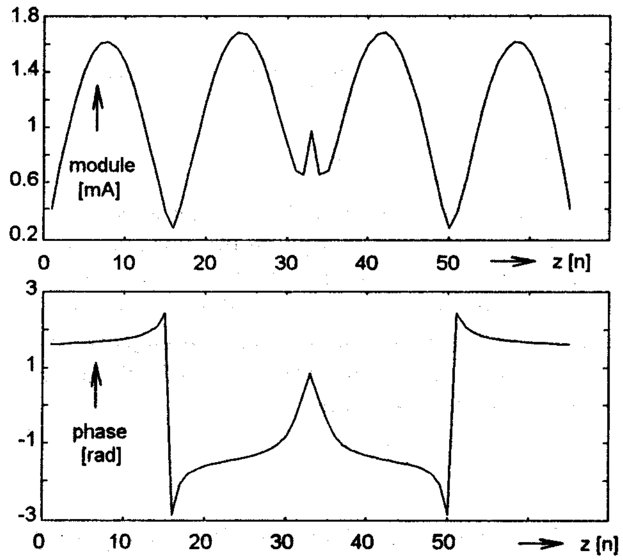

Fig.6 Current distribution on the symetrical dipole. Piece-wise constant approximation, point matching. Length of dipole 2λ, diameter 0.001588 λ, number of segments 64.

At the end, impedance matrix is inverted and its respective column gives the nodal values of the searched current distribution (fig.5).

An example of the analysis results is depicted on fig.6; module and phase of the current distribution of the dipole $h = \lambda$ and $a = 0.001588\lambda$ is plotted there.

3.2 Piece-wise linear approximation, point matching

The piece-wise linear approximation that will be used for the antenna analysis is depicted in fig.7. It is formed by shape functions $N_n^{(m)}$; function $N_n^{(m)}$ is non-zero on *m*th segment and reaches value 1 in the nod *n*. Hence, the current approximation can be expressed as

$$\tilde{I}(z) \approx I_1\left[N_1^{(1)}(z)+N_1^{(2)}(z)\right]+I_2\left[N_2^{(2)}(z)+N_2^{(3)}(z)\right]+\dots \qquad (3.10)$$

where I_i are nodal values of current distribution. Shape functions can be expressed as (3.11)

$$N_1^{(1)}(z)=\frac{z-z_0}{z_1-z_0} \quad N_1^{(2)}(z)=\frac{z_2-z}{z_2-z_1} \quad N_1^{(2)}(z)=\frac{z_2-z}{z_2-z_1}$$

etc.

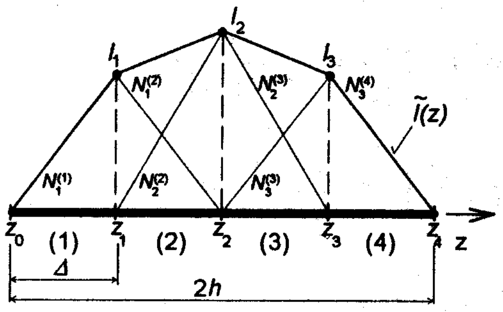

Fig.7 Piece-wise linear approximation

All the shape functions related to the same nod form the basis function, e.g.

$$N_1 = N_1^{(1)} + N_1^{(2)}. \qquad (3.12)$$

Symbols in (3.10-12) are explained in fig.7. Using basis functions, (3.10) can be redefined as

$$\tilde{I}(z) \approx \sum_{n=1}^{N} N_n I_n. \qquad (3.13)$$

Putting (3.13) to the continuity theorem (3.2a) yields

$$\sigma_{n^+} \approx \frac{-1}{j\omega}\left[\frac{I_{n+1}-I_n}{\Delta}\right] \qquad (3.14)$$

which is the same relation that has been reached when piece-wise constant approximation has been used (3.6a). Hence, the submission of charges to the electrical field (second term of eqn. 3.9) has to be computed by the same way as it has been described in the previous paragraph.

Putting (3.13) to (3.2b) gives

$$A_z(z) = \frac{\mu}{4\pi}\sum_{n=1}^{N} I_n \int_{2h} N_n \frac{e^{-jkR(z,\xi)}}{R(z,\xi)}\, d\xi. \qquad (3.15)$$

Submission of the current I_n to the vectorial potential at $z=m\Delta$ can be then expressed as

$$A(m\Delta) = \frac{\mu}{4\pi}\left[\int_{z_{n-1}}^{z_n} \frac{\xi - z_{n-1}}{\Delta}\frac{e^{-jkR(m\Delta,\xi)}}{R(m\Delta,\xi)}\, d\xi + \right. \qquad (3.16)$$

$$\left. + \int_{z_n}^{z_{n+1}} \frac{z_{n+1}-\xi}{\Delta}\frac{e^{-jkR(m\Delta,\xi)}}{R(m\Delta,\xi)}\, d\xi \right] I_n.$$

Replacing first term of (3.9) by (3.16) yields the final algorithm

$$Z_{mn} = j\omega\mu\Delta \int_{z_{n-1}}^{z_n} \frac{\xi - z_{n-1}}{\Delta}\frac{e^{-jkR(m\Delta,\xi)}}{4\pi R(m\Delta,\xi)}\, d\xi +$$

$$+ j\omega\mu\Delta \int_{z_n}^{z_{n+1}} \frac{z_{n+1}-\xi}{\Delta}\frac{e^{-jkR(m\Delta,\xi)}}{4\pi R(m\Delta,\xi)}\, d\xi + \qquad (3.17)$$

$$+ \frac{1}{j\omega\varepsilon}\left[\int_{\Delta_{n+}} \frac{e^{-jkR(m+,\xi)}}{4\pi R(m^+,\xi)}\, d\xi - \int_{\Delta_{n-}} \frac{e^{-jkR(m+,\xi)}}{4\pi R(m^+,\xi)}\, d\xi\right]\frac{1}{\Delta} -$$

$$- \frac{1}{j\omega\varepsilon}\left[\int_{\Delta_{+n}} \frac{e^{-jkR(m-,\xi)}}{4\pi R(m^-,\xi)}\, d\xi - \int_{\Delta_{n-}} \frac{e^{-jkR(m-,\xi)}}{4\pi R(m^-,\xi)}\, d\xi\right]\frac{1}{\Delta}$$

The kernel of the program written by the matlab syntax follows; first, all the necessary integrals

$$\varsigma = \int_{\Delta_n} \left[\exp(-jkR)/4\pi R\right] d\xi$$

$$\psi = \int_{\Delta_n} N_n^{(n)} \frac{\exp(-jkR)}{4\pi R}\, d\xi + \int_{\Delta_{n+1}} N_n^{(n+1)} \frac{\exp(-jkR)}{4\pi R}\, d\xi$$

are computed:

```
for m =1:(N-1)
 z = (m-1)*del;
 hlp1 = quad8('g1',-del,0,1e-5,0,z,-del,0,a,k);
 hlp2 = quad8('g2',0,+del,1e-5,0,z,0,+del,a,k);
 psi(m) = hlp1 + hlp2;
 ksi(m) = quad8('g',-del/2,+del/2,1e-5,0,z,a,k);
end;
```

Functions g1, g2 and g are given by the following definitions:

```
function out = g1( ksi, z, ksiDn, ksiUp, a, k)
R = sqrt( a^2 + (z-ksi)^2);     % ksi∈(ksiDn,ksiUp)
out = ((ksi-ksiDn)/(ksiUp-ksiDn))*
      exp(-j*k*R)/(4*pi*R);

function out = g2( ksi, z, ksiDn, ksiUp, a, k)
R = sqrt( a^2 + (z-ksi)^2);     % ksi∈(ksiDn,ksiUp)
out = ((ksiUp-ksi)/(ksiUp-ksiDn))*
      exp(-j*k*R)/(4*pi*R);

function out=g( ksi, z, a, k)
R = sqrt( a^2 + (z-ksi)^2);
out = exp(-j*k*R)/(4*pi*R);
```

At this moment, impedance matrix can be built up.

```
for m = 1:(N-1)
 for n = m:(N-1)
  dist = abs(m-n);
  hlp1 = j*omega*mi*del*psi(1+dist);
  hlp2 = (2*ksi(1+dist) - ksi(1+abs(dist-1)) -
       ksi(1+abs(dist+1)))/(j*omega*epsilon*del);
  Z(m,n) = hlp1 + hlp2;
  Z(n,m) =Z(m,n);
 end
end
```

Finally, inversion of impedance matrix yields the searched current distribution.

3.3 Piece-wise linear approximation, Galerkin's method

In the previous paragraph, putting piece-wise linear approximation to the continuity theorem has led us to the situation that has been identical with the piece-wise constant approximation. Now, let's perform the substitution on the symbolic level only

$$\sigma(z) = -\frac{1}{j\omega}\sum_{n=1}^{N}\frac{\partial N_n(z)}{\partial z} I_n \,. \qquad (3.18)$$

Putting (3.18) to (3.2c) gives us

$$\varphi(z) = -\frac{1}{j\omega\varepsilon}\sum_{n=1}^{N} I_n \frac{\partial N_n}{\partial z}\int_{2h}\frac{e^{-jkR(z,\xi)}}{4\pi R(z,\xi)}d\xi \,. \qquad (3.19)$$

Substituting (3.18), (3.19) and (3.15) to (3.2d)

$$E_z^s(z) = -j\omega A_z(z) - \frac{\partial\varphi(z)}{\partial z}$$

with considering the boundary condition (3.2e) yields

$$R(z) = j\omega\mu\sum_{n=1}^{N} I_n \int_{2h} N_n \frac{e^{-jkR(z,\xi)}}{4\pi R(z,\xi)}d\xi + \qquad (3.20)$$

$$+\frac{1}{j\omega\varepsilon}\frac{\partial}{\partial z}\sum_{n=1}^{N} I_n \frac{\partial N_n}{\partial z}\int_{2h}\frac{e^{-jkR(z,\xi)}}{4\pi R(z,\xi)}d\xi - E_z^i(z) \,.$$

Now, residuum (3.20) can be minimized by the use of Galerkin's method

$$\int_{2h} N_m(z)R(z)\,dz = \Im_1 + \Im_2 + \Im_3 = 0 \,, \qquad (3.21)$$

where

$$\Im_1 = j\omega\mu\sum_{n=1}^{N} I_n \int_{2h} N_m(z)\left[\int_{2h} N_n(\xi)\frac{e^{-jkR(z,\xi)}}{4\pi R(z,\xi)}d\xi\right]dz \qquad (3.22a)$$

$$\Im_2 = \frac{1}{j\omega\varepsilon}\sum_{n=1}^{N} I_n \int_{2h}\left\{N_m(z)\frac{\partial}{\partial z}\left[\frac{\partial N_n}{\partial z}\int_{2h}\frac{e^{-jkR(z,\xi)}}{4\pi R(z,\xi)}d\xi\right]\right\}dz$$

$$\Im_3 = -\int_{2h} N_m(z)E_z^i(z)\,dz \,. \qquad (3.22c)$$

Rather problematic derivations in (3.22b) can be overcome by the use of *per partes* integration

$$\Im_2 = \frac{1}{j\omega\varepsilon}\sum_{n=1}^{N} I_n \left\{\left[N_m(z)\frac{\partial N_n}{\partial z}\int_{2h}\frac{e^{-jkR(z,\xi)}}{4\pi R(z,\xi)}d\xi\right]_{-h}^{+h} - \qquad (3.23)\right.$$

$$\left. - \int_{2h}\left[\frac{\partial N_m}{\partial z}\frac{\partial N_n}{\partial z}\int_{2h}\frac{e^{-jkR(z,\xi)}}{4\pi R(z,\xi)}d\xi\right]dz\right\} \,.$$

Assuming infinitesimally close gap in the middle of dipole that contains the electrical field source

$$E_z^i = 1 \quad z = 0$$
$$= 0 \quad z \neq 0$$

gives

$$\Im_3 = 1 \quad m = (N/2) - 1$$
$$= 0 \quad m \neq (N/2) - 1 \,. \qquad (3.24)$$

Resultant relation for the computation of the impedance matrix (the fact that $\partial N_n^{(m)}/\partial z = \pm 1/\Delta$ has been considered) has got following notation

$$j\omega\mu\Delta\left\{\int_{z_{m-1}}^{z_m} N_m^{(m)}\left[\int_{z_{n-1}}^{z_n} N_n^{(n)}\frac{e^{-jkR(z,\xi)}}{4\pi R(z,\xi)}d\xi + \int_{z_n}^{z_{n+1}} N_n^{(n+1)}\frac{e^{-jkR(z,\xi)}}{4\pi R(z,\xi)}d\xi\right]dz\right.$$

$$\left. + \int_{z_m}^{z_{m+1}} N_m^{(m+1)}\left[\int_{z_{n-1}}^{z_n} N_n^{(n)}\frac{e^{-jkR(z,\xi)}}{4\pi R(z,\xi)}d\xi + \int_{z_n}^{z_{n+1}} N_n^{(n+1)}\frac{e^{-jkR(z,\xi)}}{4\pi R(z,\xi)}d\xi\right]dz\right\} +$$

$$+\frac{1}{j\omega\varepsilon}\left\{\int_{z_{m-1}}^{z_m}\left[\int_{z_{n-1}}^{z_n}\frac{e^{-jkR(z,\xi)}}{4\pi R(z,\xi)}d\xi - \int_{z_n}^{z_{n+1}}\frac{e^{-jkR(z,\xi)}}{4\pi R(z,\xi)}d\xi\right]dz\right. -$$

$$\left. - \int_{z_m}^{z_{m+1}}\left[\int_{z_{n-1}}^{z_n}\frac{e^{-jkR(z,\xi)}}{4\pi R(z,\xi)}d\xi - \int_{z_n}^{z_{n+1}}\frac{e^{-jkR(z,\xi)}}{4\pi R(z,\xi)}d\xi\right]\right\}\frac{1}{\Delta} = Z_{mn} \qquad (3.25)$$

The algorithm (3.25) can be expressed in matlab syntax. In the first step, integrals (3.25) are computed

```
for m=1:(N-1)
 hlp1 = quad8('vct1',0,del,1e-5,0,0,(m-1)*del,
        m*del,(m+1)*del,a,k);
 hlp2 = quad8('vct2',del,2*del,1e-5,0,2*del,
        (m-1)*delta,m*del,(m+1)*delta,a,k);
 psi(m) = hlp1 + hlp2; % 1st+2nd integ. of (3.25)
 hlp1 = quad8('scl',0,del,1e-5,0,(m-1)*del,m*del,
        (m+1)*del,a,k);
 hlp2 = quad8('scl',del,2*del,1e-5,0,(m-1)*del,
        m*del,(m+1)*del,a,k);
 ksi(m) = hlp1-hlp2;    % 3rd-4th integ. of (3.25)
end;
```

Fuctions appearing in the above program are defined as:

```
function out=vct1(z,zDn,ksiDn,ksiMd,ksiUp,a,k)
del = ksiUp - ksiDn;
h1 = quad8('g1',ksiDn,ksiMd,1e-5,0,z,ksiDn,ksiMd,
     a,k);
h2 = quad8('g2',ksiMd,ksiUp,1e-5,0,z,ksiMd,ksiUp,
     a,k);
out = (h1 + h2)*(z-zDn)/del;

function out=vct2(z,zUp,ksiDn,ksiMd,ksiUp,a,k)
del = ksiUp - ksiDn;
h1 = quad8('g1',ksiDn,ksiMd,1e-5,0,z,ksiDn,ksiMd,
     a, k);
h2 = quad8('g2',ksiMd,ksiUp,1e-5,0,z,ksiMd,ksiUp,
     a, k);
out = (h1 + h2)*(zUp-z)/del;

function out=scl(z,ksiDn,ksiMd,ksiUp,a,k)
h1 = quad8('g',ksiDn,ksiMd,1e-5,0,z,a,k);
h2 = quad8('g',ksiMd,ksiUp,1e-5,0,z,a,k);
out = h1 - h2;
```

Functions `g1`, `g2` and `g` have the same body as in the previous paragraph.

Now, building-up the impedance matrix is simple:

```
for m = 1:(N-1)
 for n = m:(N-1)
  dist = abs(m-n);
  Z(m,n) = j*omega*mi*del*psi(1+dist) +
           ksi(1+dist)/(j*omega*epsilon*del);
 end
end
```

Since the used version of matlab has exhibited problems in evaluating double integrals, numerical results of this algorithm have been computed by MathCAD Plus.

3.4 Global cosine approximation, point matching

For the global approximation, let's re-arrange the set of initial equations (3.2). The electrical intensity that is expressed by (3.2d)

$$E_z^s(z) = -j\omega A_z(z) - \frac{\partial \varphi(z)}{\partial z}$$

will be evaluated by substituting A_z from (3.2b)

$$A_z(z) = \frac{\mu}{4\pi}\int_{2h} I_z(\xi)\frac{e^{-jkR(z,\xi)}}{R(z,\xi)}d\xi$$

and φ from Lorentz gauge condition [4]

$$\varphi = -\frac{1}{j\omega\mu\varepsilon}\frac{\partial A_z(z)}{\partial z} \qquad (3.26)$$

which yields

$$E_z(z) = \frac{1}{j\omega\varepsilon}\left[\frac{\partial^2}{\partial z^2}\int_{2h} I(\xi)\frac{e^{-jkR(z,\xi)}}{4\pi R(z,\xi)}d\xi + \right. \qquad (3.27)$$

$$\left. +k^2\int_{2h} I(\xi)\frac{e^{-jkR(z,\xi)}}{4\pi R(z,\xi)}d\xi\right]$$

Since Lorentz gauge condition provides relation of current and charge distributions by the equation of continuity, (3.27) completed by boundary conditions is equivalent to (3.2).

For wire antennas, order of integration and derivation in (3.27) can be swapped [4]

$$E_z(z) = \frac{1}{j\omega\varepsilon}\int_{2h} I(\xi)\left[\frac{\partial^2}{\partial z^2}\frac{e^{-jkR(z,\xi)}}{4\pi R(z,\xi)} + k^2\frac{e^{-jkR(z,\xi)}}{4\pi R(z,\xi)}\right]d\xi .$$

Computing content of the bracket in (3.28) yields

$$E_z(z) = \int_{2h} I(\xi)\left[(1+jkR)(2R^2-3a^2)+k^2R^2a^2\right]\frac{e^{-jkR(z,\xi)}}{4\pi R^5(z,\xi)}d\xi$$

Relation (3.29) is initial equation for the analysis.

Since the current distribution of a symmetrical dipole is an even function, the distribution can be approximated by cosine terms of Fourier series [2], pp.122-129

$$I(z) = \sum_{n=1}^{N} I_n \cos\left[(2n-1)\frac{\pi z}{2h}\right]. \qquad (3.30)$$

In addition, (3.30) automatically fulfils $I(\pm h)=0$.

Substituting (3.30) to (3.29) and collocation in arbitrary N-1 points on the antenna surface gives

$$\sum_{n=1}^{N} I_n \int_{2h} \cos\left[(2n-1)\frac{\pi\xi}{2h}\right]\left[(1+jkR(z_m,\xi))(2R^2(z_m,\xi)-3a^2)+\right.$$

$$\left.+k^2R(z_m,\xi)^2a^2\right]\frac{e^{-jkR(z_m,\xi)}}{4\pi R^5(z_m,\xi)}d\xi = 0$$

$$m = 1, 2 \ldots N\text{-}1 . \qquad (3.31a)$$

Collocation in the source point z=0 (all the cosines equal one) leads to

$$\sum_{n=1}^{N} I_n = 1 \qquad (3.31b)$$

(input voltage has been again considered 1V). Unfortunately, this approach does not enable direct computation of input impedance. Hence, only the current distribution has been investigated (fig.8).

Computing the matrix equation (3.31) gives Fourier coefficients of the current distribution I_n. Program in the matlab syntax follows:

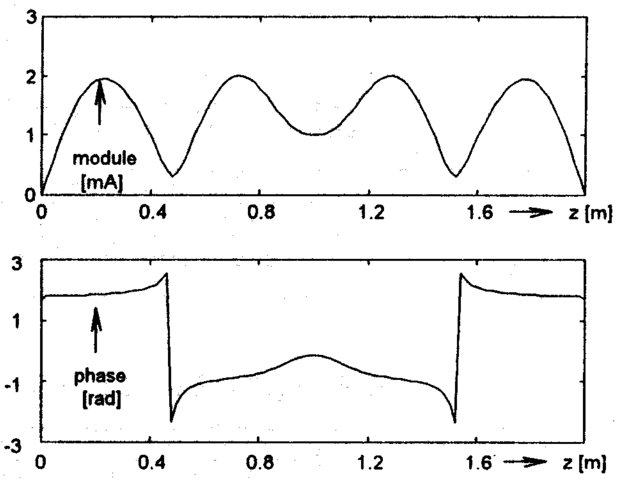

Fig.8 Current distribution on the symetrical dipole. Cosine series approximation, point matching. Length of dipole 2λ, diameter 0.001588 λ, number of segments 6

```
for m=1:N
 z = del*((2*m+1)/2);
 for n=1:N
  if m==N C(m,n)=1 else
   C(m,n)=quad8('cc',-h,+h,1e-5,0,z,n,h,a,k);
  end;
  end;
end;

function out=cc( ksi, z, n, h, a, k)
R = sqrt(a^2+(z-ksi)^2);
hlp1 = ((1+j*k*R)*(2*R^2-3*a^2)+k^2*R^2*a^2)*
       exp(-j*k*R)/(4*pi*R^5);
hlp2 = cos((2*n-1)*pi*ksi/(2*h));
out = hlp1 * hlp2;
```

Inversion of **C** yields Fourier coefficients of the current distribution at fig.8.

5. Conclusions

Presented paper has introduced several moment methods that can be used for numerical analysis of wire antennas.

By each multi-base method, input impedanace of the antenna $h = 0.25\ \lambda$ and $a = 0.001588\ \lambda$ has been computed (tab.1). It can be seen that no dramatical differences have appeared among the methods (tab.1).

If presented methods are compared from the point of view of computational requirements then number of numerical integrations has to be investigated especially since numerical integration consumes most of the computational power (tab.2). This fact is illustarated by time that has been required by specified computer for performing respective algorithm.

It can be concluded, with respect to tab.1 and tab.2 that piece-wise constant approximations and point matching give respectable results although their computational requirements are very small.

Single-basia approximation has exhibited higher computational requirements than multi-basis ones. In addition, evaluation of the input impedance by this method requires additional computations. Current distribution computed by the single-basis approximation have been approximatelly the same as those provided by multi-basis approximations.

7. References

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Tab.1 Input impedance of symetrical dipole ($2h=0.5\lambda$, $a=0.001588\lambda$) computed by discussed methods. Value taken from King-Middleton is $(83.6 + j41.3)\Omega$.

approximation	minimization	N = 8	N = 16	N = 32
constant	point matching	82.3 + j38.5	84.6 + j40.6	86.5 + j43.2
linear	point matching	80.4 + j35.3	84.0 + j39.3	86.2 + j42.8
linear	Galerkin	82.5 + j39.3	86.2 + j43.7	87.9 + j45.6

Tab.2 Comparison of computational requirements of discussed methods.

operations	constant	linear	linear
	point match.	point match.	Galerkin
I()dx	N+2	3(N-1)	0
II()dxdy	0	0	N-1
time [s]*	7	18	392

* Time necessary for performing respective algorithm for 32 segments; AC486DX2, 66MHz, 32MB RAM.