SOME MODIFICATIONS OF FRACTAL IMAGE CODING

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Abstract

In this paper some modifications of fractal image coding are presented. Proposed methods are based on correlation coefficients computing as an alternative approach to searching of similarity between blocks. The convergence speed of decoding process is faster then convergence speed of standard method. The convergence process with modified start conditions of decoding process are analysed and verified on gray scale static images too.

Keywords

fractal image coding, correlation, self-similarity

1. Introduction

Fractals are mathematical objects with a high degree of redundancy in the sence that they are recursively made of transformed copies of either themselves or parts of themselves. These objects were first labeled as an Iterated Sequences and they were rediscovered thanks to the pioneering work of Mandelbrot [2] . Barnsley was the first who proposed the notion of Fractal Image Compression, by which real-life objects or images would be modeled by deterministic fractal objects-attractors of sets of two-dimensional affine transformations [3] . The mathematical theories of Iterated Function Systems (IFS) and Recurrent Iterated Function Systems along with the Collage Theorem [6], constitute the broad foundations of fractal image compression.

In this paper we are concerned with block-based image coding schemes which can compress any digital monochrome image. The principles of Fractal Block Coding were originally published in the doctoral dissertation of A. Jacquin a student of Barnsley's. Fractal block image coding is based on block similarity in an image[3]. In this coding method the distance between the image blocks is used for searching of the most self-similar ones [1]. The root-mean square distortion between the image blocks is used for blocks self-similarity searching, in general.

In this paper the correlations between image blocks have been selected for detection of their similarity and some decoding problems have been analysed. Some experimental results for proposed method based on correlations and modification of decoding method are presented too.

2. Fractal image block coding algorithm

2.1 Encoding algorithm

Encoding algorithm consist of the three basic steps
- Image partitionig into the image blocks
- Calculation of the blocks similarity
- Calculation of the fractal coding parameters.

In the first step the orignal image is divided into the non-overlapped R_i (Range) blocks, that cover all image, and overlapped D_i (Domain) blocks. The size of D_i blocks is bigger than size of R_i blocks.

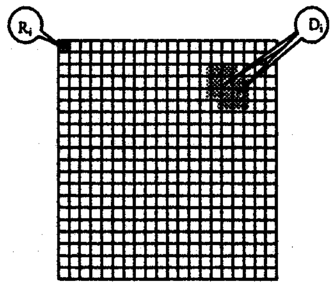

Fig.1 Decomposition of image to R and D blocks

In the next step for each R_i block the most similar D_i block is searched. Due to the size of D_i blocks is bigger than size of R_i blocks we have to reduce the size of D_i to equal size of R_i blocks by using contraction. Reduced D_i block is denoted as B_i

$$B_i = \mu(D_i), \tag{1}$$

where $\mu()$ - is operation of contraction e.g. subsampling or averaging of D_i block.

After contraction of D_i blocks the most similar B_i block is searched for each R_i block by using the metrics which describes distance between R_i and B_i blocks.

In standard fractal image coding algorithm Euclidian distance is used for calculation of distance. It means that parameter d in the following equation should be minimal for most similar blocks

$$d^2(R,B)=\sum_{i=1}^{n}(r_i-b_i)^2, \tag{2}$$

where n is number of pixel in R_i and B_i block.

For reconstruction of R_i blocks from the most similar B_i blocks we can used the following transformation

$$R_i = s*B_i + o*\underline{1}, \tag{3}$$

where s- offset
o- brightness.

The transformation parameters can be expressed in the following form [1]

$$s=\frac{\left[n\sum_{i=1}^{n}r_ib_i-\sum_{i-1}^{n}r_i\sum_{i=1}^{n}b_i\right]}{\left[n\sum_{i=1}^{n}r_i^2-\left(\sum_{n=1}^{n}r_i\right)^2\right]}, \tag{4}$$

$$o=\frac{1}{n}\left[\sum_{i=1}^{n}b_i-s\sum_{i=1}^{n}r_i\right]. \tag{5}$$

After calculation of transformation parameter for each R_i and B_i blocks we obtain the set of fractal coding parameters in the following form

$$R_i \approx (x,y,s,o,I) \tag{6}$$

where

x,y- coordinates of position of the most similar D_i block in image (e.g. coordinates of its top left corner)

s,o- transformation parameters

I- izometry, that means of position of the most similar D_i block in sense its rotation and reflection (8 position).

2.2 Decoding algorithm

Decoding of original image is an iterative process of reconstruction $\hat{R}_i$ blocks from the set of fractal coding parameters by using Eq.(3). The decoding process of each R_i block can be described in following form [4]

$$\begin{aligned}
\hat{R}_i^1 &= s*B_i + o*\underline{1}\\
\hat{R}_i^2 &= s*(s*B_i + o*\underline{1}) + o*\underline{1}\\
&\vdots\\
\hat{R}_i^k &= s^k*B_i + \left(o\sum_{p=0}^{k-1}s^p\right)*\underline{1}
\end{aligned} \tag{7}$$

The important question in decoding process is the start condition. It means the form of starting image at the decoder side. In standard decoding algorithm we can consider, that the image frame at the decoder side is zero. Result of decoding process after k-th iteration is the reconstructed image called attractor.

3. Modification of standard fractal image block coding and decoding algorithm

3.1 Modification of standard fractal image block coding algorithm

In standard fractal image block coding algorithm the Euclidian distance is used for similarity of R_i and D_i blocks searching. The alternative metrics has been used by Fisher, Y. that was called sup metric[1].

Another approach for block similarity calculation is correlation between R_i and B_i blocks. Similarity of R_i and B_i blocks in original image can be expressed by using correlation coefficient. Before computing of correlation coefficient between R_i block and all B_i blocks we have to calculate the following statistic parameters of R_i and B_i blocks [5]

- mean block values

$$\bar{r}=\frac{\sum_{i=1}^{n}r_i}{n} \qquad \bar{b}=\frac{\sum_{i=1}^{n}b_i}{n} \tag{8}$$

- standard block deviation

$$S_r=\sqrt{\frac{\sum_{i=1}^{n}(r_i-\bar{r})^2}{n-1}} \qquad S_b=\sqrt{\frac{\sum_{i=1}^{n}(b_i-\bar{b})^2}{n-1}} \tag{9}$$

➢ covariance between blocks

$$\mathrm{cov}\,rb = \frac{\sum_{i=1}^{n} r_i b_i}{n} - \bar{r}.\bar{b} \qquad (10)$$

Then the correlation coefficient r_c for R_i and B_i blocks of original image can be expressed in the form

$$r_c = \frac{\mathrm{cov}\,rb}{S_r . S_b} \qquad (11)$$

The outcomes from using of correlation instead calculation of Euclidian distance between image blocks should in faster reconstruction of image at the decoder side than reconstruction of image in standard decoding algorithm.

3.2 Modification of standard fractal image decoding algorithm

The aim of standard fractal image decoding algorithm modification was acceleration of image reconstruction process at the decoder side respectively faster convergence to the attractor. Output from the standard encoding process can be described by the coding matrix, in which each row consists of the set of fractal coding parameters (x,y,s,o,I) for each R_i blocks in an original image (Fig.2)

R_1 :	x_1	y_1	s_1	o_1	I_1
R_2 :	x_2	y_2	s_2	o_2	I_2
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
R_N :	x_N	y_N	s_N	o_N	I_N

N- number of R_i blocks in original image

Fig.2. The form of standard coding matrix

In the standard decoding algorithm we consider that starting image frame at the decoder side is zero. To accelerate process of image reconstruction the modificated start condition have been used. The starting image frame at the decoder side was replaced by mean brightness value of original image. It means that the coding matrix from Fig.2 have been extended by one row with information about mean brightness value of original image. The coding matrix then has the following form (Fig.3)

	Mean value				
R_1 :	x_1	y_1	s_1	o_1	I_1
R_2 :	x_2	y_2	s_2	o_2	I_2
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
R_N :	x_N	y_N	s_N	o_N	I_N

N- number of R_i blocks in original image

Fig.3. The form of modified coding matrix

4. Experimental results

In computer simulations the modification of fractal image coding and decoding algorithm has been verified in sense of image quality reconstruction and speed of convergence to attractor. In the fractal image coding algorithm the correlation coefficients have been used for block similarity searching instead of the Euclidian distance calculation. The decoding algorithm was modified in sense of starting image frame at the decoder side. For the quality of reconstructed image PSNR in [dB] has been used. The experimental results for modified fractal coding algorithm are shown in Tab.1. In the first coloum of table is the quality of reconstructed image by using standard fractal coding algorithm and in the second coloum is the quality of reconstructed image by using modified fractal coding algorithm with correlations.

Tab.1

1.iteration	5,42	16,62
2.iteration	8,81	25,51
3.iteration	12,56	27,76
4.iteration	16,29	27,95
5.iteration	19,59	27,98
6.iteration	22,12	27,98
7.iteration	23,71	27,98
8.iteration	24,65	27,98

Speed of convergence the both methods to the attractor is shown on Fig.4.

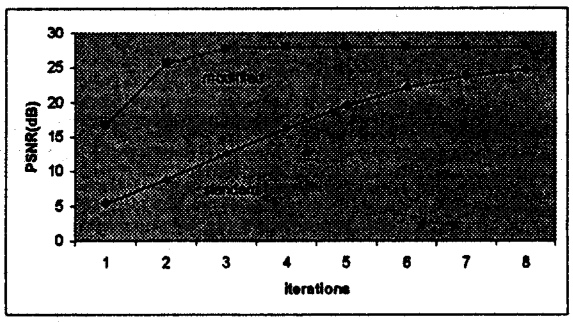

Fig.4 Speed of convergence to the attractor the standard and modified coding algorithm.

First three iterations in process of image reconstruction at the decoder side is shown on Fig.5. At the left side of Fig.4 the process of image reconstruction by using modified decoding algorithm is shown and the right side ones is shown the process of image reconstruction by using standard decoding algorithm.

Fig.5 Process of image reconstruction by using modified and standard coding algorithm.

The experimental results for modified fractal decoding algorithm with the modification of starting image frame at the decoder side are shown in Tab.2. The structure of Tab.2 is the same as a Tab.1.

Tab.2

1.iteration	5,42	19,76
2.iteration	8,81	24,12
3.iteration	12,56	24,13
4.iteration	16,29	24,13
5.iteration	19,59	24,14
6.iteration	22,12	24,16
7.iteration	23,71	24,17
8.iteration	24,65	24,18

Speed of convergence the standard and modified decoding algorithm is shown on Fig.6.

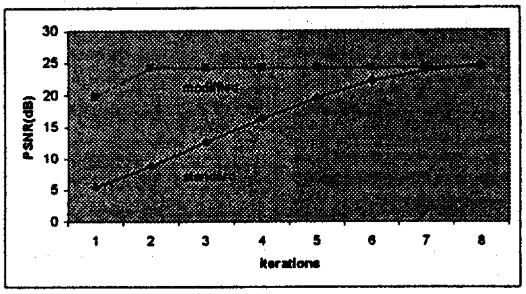

Fig.6 Speed of convergence to the attractor the standard and modified decoding algorithm

Comparison of image reconstruction process for modified decoding algorithm (left side) and standard decoding algorithm (right side) is shown on Fig.7.

Fig.7 Process of image reconstruction by using modified and standard decoding algorithm

5. Conclusion

In this paper some modification the fractal image coding and decoding algorithm has been analysed. In the standard fractal image coding algorithm the criteria for similarity of blocks was modified. In the standard fractal image decoding algorithm the starting conditions was modified too.

Outcomes from modifications are faster image convergence to the attractor.

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