Compression of Vehicle-Driving Data by Means of Orthogonal Bases

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***Abstract*—The paper deals with application of orthogonal bases in signal approximation with the aim of data compression in a vehicle driving simulator. Three different bases are tested: Discrete Fourier Basis, Discrete Cosine Basis, and Slepian Basis. Quality of signal approximation error is assessed in terms of global squared errors. Thus obtained numerical results suggest that Slepian Basis affords the sparsest representation of signals tested in this study. Therefore, a considerable reduction of required memory can be accomplished.**

***Keywords*—signal, approximation, compression, Slepian sequences, DPSS.**

I. Introduction

Modern flight or vehicle-driving simulators are capable of simulating complex scenarios under constant or varying conditions, and with a large number of input parameters. The human operator reacts to various stimuli, and, in turn, responds by input devices such as a steering wheel or pedals. Detailed analysis of such human-machine feedback loop is intimately connected with the need of collecting and storing large quantities of data in an efficient way.

This work aims at achieving a sparse representation of real data from a vehicle driving simulator. The simulator records discrete-time signals; therefore, this work will restrict the discussion and analysis to discrete (sampled) time domain.

The paper is divided as follows. Section II. introduces preliminaries necessary for subsequent sections, such as notation, mathematical tools, and introduction to the Slepian Basis. Section III. presents comparison of Slepian Basis with other bases using the real measured signal.

II. Discrete Prolate Spheroidal Sequencies

Discrete Prolate Spheroidal Sequencies (DPSS), also referred to as Slepian Sequencies, constitute the discrete analogy to continuous-time Prolate Spheroidal Wave Functions (PSWF), also called Slepian Functions.

A. Mathematical Background

Generally, the Slepian Sequencies have a number of interesting properties; we will list only those which are vital for our analysis:

1) Bandlimited signals: A signal $f(t)$ is referred to as bandlimited, if its Fourier Transform $F(\omega)$ is of the form

$$F(\omega) = 0 \Leftrightarrow |\omega| > 2\pi W. \tag{1}$$

Here the W denotes the bandlimit of the signal. Note that every discrete sampled signal is bandlimited due to the Shannon-Nyquist theorem associated with reconstruction formula which employs bandlimited sinc functions [1].

If not mentioned explicitly, all transforms, operations, signals etc. in this work are discrete.

2) The Norm of a vector in ℓ^2 space

$$\|f(k)\| = \left[\sum_{k=a}^{b} |f(k)|^2\right]^{1/2} \tag{2}$$

The Norm of a vector in ℓ^2 space is defined by (2). A function belongs to the Hilbert Space ℓ^2 of discrete functions if the sum (2) is finite.

3) The Scalar Product in ℓ^2

$$(f, g) = \sum_{k=a}^{b} f(k)\overline{g(k)} \tag{3}$$

Scalar Product of two functions is defined by (3), where one of the functions is complex conjugate (marked by overline). This fact comes into effect if we deal with complex-valued sequences; however, Slepian Basis comprises real sequencies.

Aforementioned definitions are closely related to orthogonal and orthonormal bases. In an orthogonal basis $\{e_n(k)\}_{n=0}^{K-1}$ the every pair of non-identic sequences on interval (a, b) yields zero scalar product

$$(e_m, e_n) = 0 \quad \text{if} \quad m \neq n, \tag{4}$$

and the scalar products of identical sequences are non-zero

$$\|e_n(k)\| \neq 0. \tag{5}$$

We use the term orthonormal base if the basis is orthogonal and its elements $\{e_n(k)\}_{n=0}^{K-1}$ have unit norm $\|e_n\| = 1$. [2][3]

Discrete Fourier Basis, Discrete Cosine Basis, or Discrete Slepian Basis are all examples of orthogonal/orthonormal bases. Slepian Base is illustrated in the Figure 1.

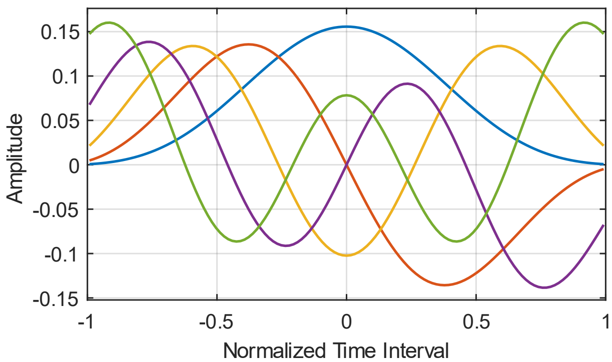

Figure 1: The Slepian Orthonormal Base for $K = 127$ samples. Sequences with orders from 0 to 4 are displayed.

B. *Definition*

Slepian Sequences are defined by the following convolutional equation

$$\sum_{l=1}^{K} \frac{\sin[2\pi W(k-l)]}{\pi(k-l)} \psi_n(K,W,l) = \\ = \lambda_n(K,W)\psi(K,W,k) \qquad (6)$$

where k stands for the discrete time, K is the number of samples in the time domain, ψ_n is nth Slepian sequence, λ_n is its eigenvalue, and W is normalised bandwidth (a real number between 0 and ½). If we have normalized frequency domain, the value 1 corresponds to the sampling frequency, and in turn the maximal frequency admissible in signal spectrum is ½. [4][5]

The number of samples in time domain K is a parameter of the Slepian basis; it determines both the number and length of Slepian sequences. In this work is expected usage of the full Slepian Basis comprising K sequences, unless stated otherwise.

C. *Spectrum*

The measured signal $\{f(k)\}_{k=0}^{K-1}$ can be transformed into the Slepian Spectrum which is expressed by coefficients $\{a_n\}_{n=0}^{K-1}$. Considering a discrete-time sequence $f(k)$, we can obtain coefficients a_n from the second part of the double orthogonality of Slepian Sequences [4][6] as follows:

$$a_n = \sum_{k=1}^{K} f(k)\psi_n(k) = \lambda_n \sum_{k=-\infty}^{\infty} f(k)\psi_n(k) \qquad (7)$$

Example of a Slepian Spectrum is made on a simple periodic signal

$$f(t) = 3\sin 2\pi t + \frac{3}{2}\sin 6\pi t + \frac{3}{4}\sin 10\pi t \qquad (8)$$

which is illustrated in the Figure 2. The Slepian spectrum is illustrated in the Figure 3 and for comparison, there is also the Fourier Spectrum in the Figure 4.

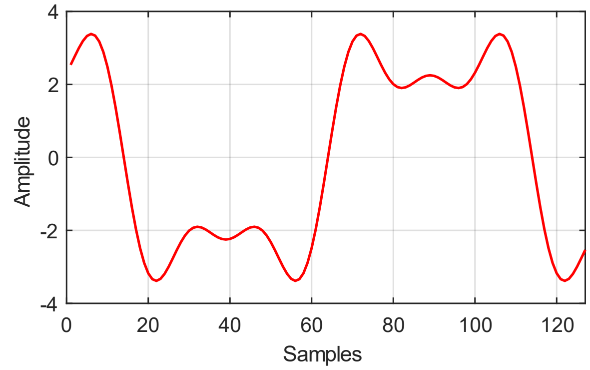

Figure 2: Waveform of the test signal (8).

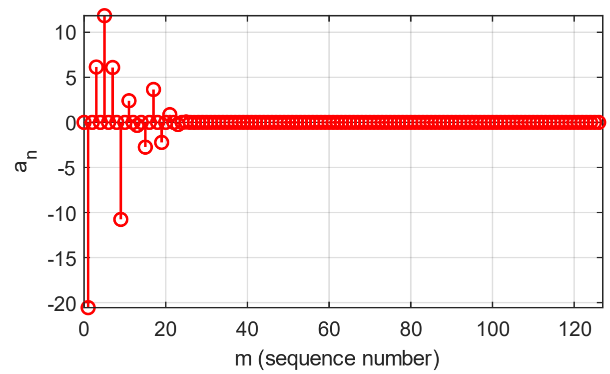

Figure 3: Slepian Spectrum of a harmonic the test signal (8).

The signal was deliberately sampled in such a way that one dicrete period of the signal does not contain an integer multiple of the fundamental frequency 2π rad/s. We can see that spectral coefficients are negligible for the 20th and higher orders of Slepain spectrum. That is also valid for the Fourier Spectrum, although Fourier Spectrum is symmetric and complex.

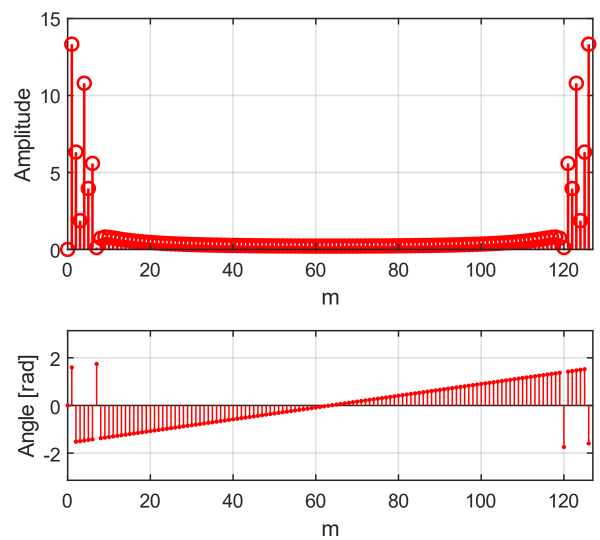

Figure 4: Fourier Spectrum of the periodic signal (8). Amplitude spectrum is in the upper chart, Phase spectrum is in the lower chart.

Considering the size of stored data in both spectra, the Fourier Spectrum requires only half of the coefficients stored in the computer's memory, because of its well-known symmetry when the processed signal is a real-valued sequence. In this case we need only 64 coefficients, because the sampled signal consists of 127 samples. However, the Fourier Spectrum is complex-valued and storing real and complex part of the number needs twice more memory. Indeed, storage of one half of the complex-valued Fourier Spectrum requires the same amount of memory as the full Slepian Spectrum.

III. The Slepian Transformation

Slepian Transformation (ST) is operation of computing spectral coefficients (7), Inverse Slepian Transformation (IST) reconstructs the signal from the spectrum. Process of IST is illustrated in the Figure 5, where approximation order corresponds with number of used products of Sequencies and coefficients a_n. The first approximation order represents IST of only the first spectral coefficient (and others equal to zero). As our first spectral coefficient is close to zero, the first approximation shows a constant sequence where amplitude approaches 0. The higher the approximation order, the more accurate the signal approximation is. Note that roughly from 20th-order approximation, there are almost no changes in the shape of signal, which is in conformity with Figure 3: coefficients a_n with order n higher than 20 are negligible.

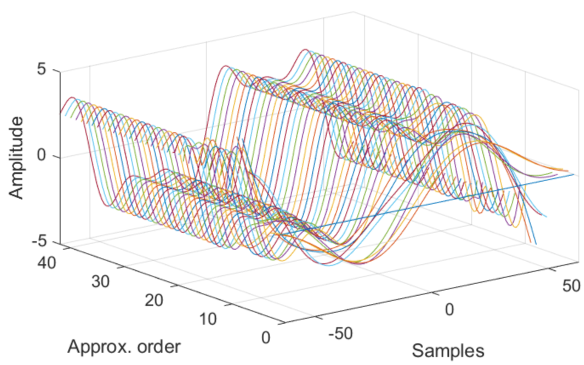

Figure 5 - Signal reconstruction using the Inverse Slepian Transform.

A. *Compressed Spectrum – Inverse Transform Example*

The hypothesis that high-order Slepian coefficients are negligible is tested by ST and IST. Example of signal employed for this task is depicted in Figure 6 and Figure 7 .

During the reconstruction, higher order coefficients are ignored, even though the full spectrum would have $K = 127$ coefficients. (The spectrum is computed from $K = 127$ signal samples). Parts of the signal are reconstructed with a truncated spectrum. Figure 8 shows the detail of signal approximation obtained from the truncated spectrum, using only ¼ of spectral coefficients. Truncated IST is compared with well-known and commonly used inverse Fast Fourier Transform (IFFT) and inverse Cosine Transform (ICT). For more information about CT see [7].

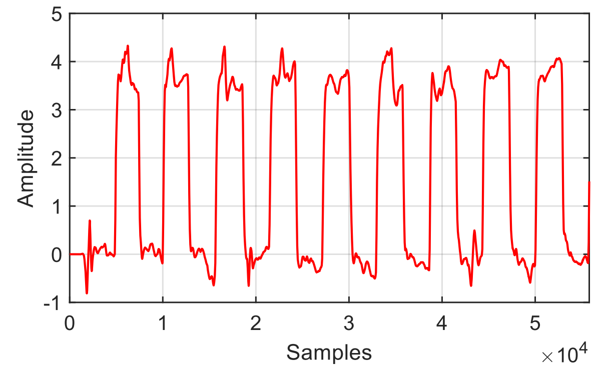

Figure 6: Example of a test signal (lateral vehicle position on a motorway).

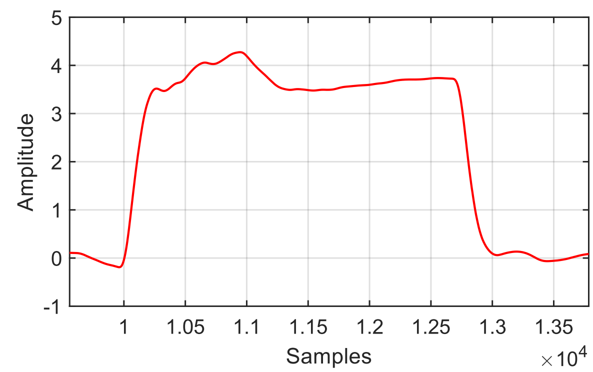

Figure 7: Example of a test signal (lateral vehicle position on a motorway), detail.

We can see that both the truncated IST and ICT can reconstruct the signal with high precision even if only a part of the spectrum is used. This might be useful to compress data in some special purposes.

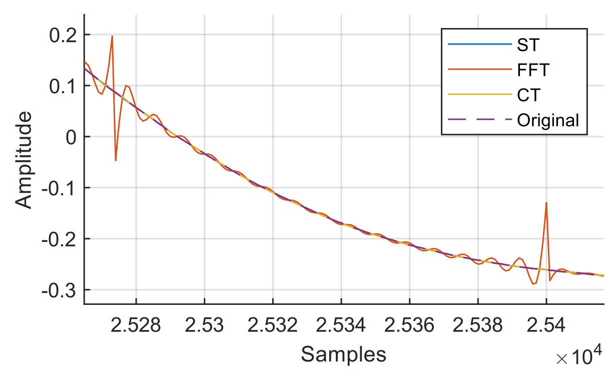

Figure 8: Detail of a compressed-spectrum signal reconstructions.

B. *Compressed Spectrum – Inverse Transform Generalization*

The process of IST with compressed truncated from Section III.A is generalized by the following algorithm:

1. Take the first spectral coefficient.
2. Execute the inverse transform (IST, ICT, or IFFT).
3. Compare it with the original signal, compute MSE (Mean Squared Error, [7]).
4. Add next spectral coefficient and go to 2.

5. If all coefficients from spectrum are used, go to 1 and continue with the next signal section of 127 samples.

6. If the whole signal is reconstructed, compute the relative global squared error

$$E_{\mathrm{R}} = \frac{\sum_{k=0}^{N-1}|f_m(k) - f(k)|^2}{\sum_{k=0}^{N-1}|f(k)|^2} \quad (9)$$

for each transform and each number of used spectral coefficients.

To compute the dataset of the following figure were used over 1,600 signal sections (from the signal in the Figure 6 or similar), each section being 127 samples long.

Figure 9 illustrates average relative global MSE of a Signal reconstruction in logarithmic axes of ST and FFT, CT to compare with. ST has the highest initial MSE because of the zero order sequency is not being constant (Slepian base has no constant sequency). In comparison, the FFT and CT both have the zero order sequency as a constant. However, after 10th coefficient used in the algorithm, ST is the most accurate with the lowest relative global MSE. The FFT MSE does not converge to zero as fast as other transforms which is caused by its characteristic "oscillation" on the edges of the interval (can be also seen in Figure 8).

It turned out that Fourier transform yields the slowest convergence of the signal approximation, as it is evident from the Figure 9. This corresponds with the Figure 8 where it is presented on a part of measured signal. On the other hand, ST yields the most accurate signal approximation even with only 10 first spectral coefficients used.

The FFT needs only ½ coefficients in comparison with ST or CT, but they are complex, as it was already discussed in Section II.C.

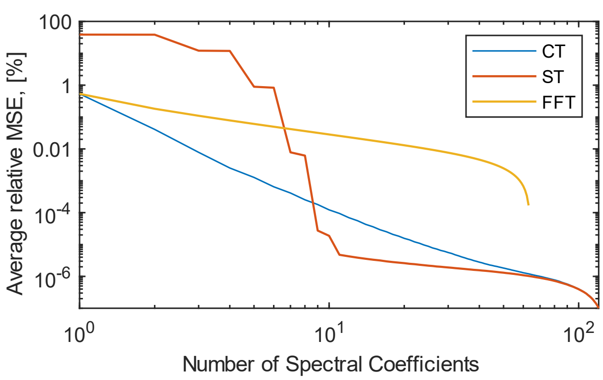

Figure 9: The relative global squared error (relative MSE) on used spectral coefficients in the signal approximation dependency.

IV. Conclusion

In this paper we briefly revisited the theory of orthogonal bases with the aim of finding bases most suitable for compression of data from a vehicle-driving simulator.

Three different signal decompositions were tested and compared, namely: FT, DCT, and ST. Our numerical experiments suggest that for the class of signals processed in this study Slepian transform affords the sparsest representation. This was illustrated by a number of mathematical tools: Plots of spectral coefficients corresponding to different bases, waveforms of signal approximations, and plots of relative MSE for increasing orders of signal approximation.

The authors are aware that there is room for further improvement in a number of ways. E.g., the free parameter of Slepian Basis (the product NW [5]) could be optimized for the given group of signals so as to yield sparsest possible representation when similar signals are processed.

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