THE TWO-MASS SYSTEM PARAMETER IDENTIFICATION WITH LEVENBERG-MARQUARDT ALGORITHM

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Abstract: This paper is focused on the parameter identification of the Two-Mass mechanical flexible system for motor drive applications. The whole methodology is based on the amplitude frequency characteristic given by the Welch spectrum analysis method. Then, an initial estimate of the plant parameters is extracted from the amplitude frequency characteristic and it is used as the starting point for the Levenberg-Marquardt algorithm to enhance the parameter estimation.

Keywords: Two-mass flexible system, Welch method, Spectral analysis, Levenberg-Marquardt method

1 INTRODUCTION

Plenty of mechanical rotary systems with any flexible coupling can be modeled as the two-mass system. As the examples; the toothed belt, axial rotary flexible connection, and long torque shaft connection can be given. Work [1] can be used as a good knowledge background for the physical based modeling of the multi-mass systems. These types of systems (plants) show resonant behavior on certain frequencies. This means that there are (anti)resonant peaks on the plant frequency response. Such a behavior causes many problems in closed-loop system design and hence, it is necessary to identify this behavior as best as possible. For the satisfying parameter estimate, an appropriate identification method has to be chosen. Right beside this, an appropriate input signal has to be chosen as well. This means that the signal must be rich enough to ensure sufficient plant excitation. Widely used methods for identification of the electric drive systems are methods based on the frequency analysis or the spectral analysis. The method used in this paper is based on the spectral analysis using power spectral densities (periodograms) estimates according to:[2]. The main goal of the identification experiment presented in this paper is to obtain the plant parameters estimate, which allows successful reconstruction of the resonant and anti-resonant peaks values and their positions on the plant frequency response.

2 TWO-MASS SYSTEM MODELLING

The Two-Mass mechanical flexible system can be understood as the system with the single input and with two outputs and can be described by the following set of the differential equations.

$$\frac{d\omega_m}{dt} = -\frac{b}{J_m}\omega_m - \frac{k}{J_m}\theta_m + \frac{b}{J_m}\omega_l + \frac{k}{J_m}\theta_l + \frac{T_i}{J_m} \tag{1}$$

$$\frac{d\theta_m}{dt} = \omega_m \tag{2}$$

$$\frac{d\omega_l}{dt} = \frac{b}{J_l}\omega_m + \frac{k}{J_l}\theta_m - \frac{b}{J_l}\omega_l - \frac{k}{J_l}\theta_l \tag{3}$$

$$\frac{d\theta_l}{dt} = \omega_l \tag{4}$$

Where J_m and J_l are the inertias of the shaft and load, respectively. Parameter b is the dumping coefficient and the parameter k is the flexible coefficient. Variables ω_m and ω_l stand for the motor shaft and the load angular velocities, respectively. The variables θ_m and θ_l are shaft and load angular positions. Finally, the T_i variable stands for the input torque generated by the electric part of the machine. The typical frequency response of the plant with T_i as the input and ω_m as the output is shown in Figure 1.

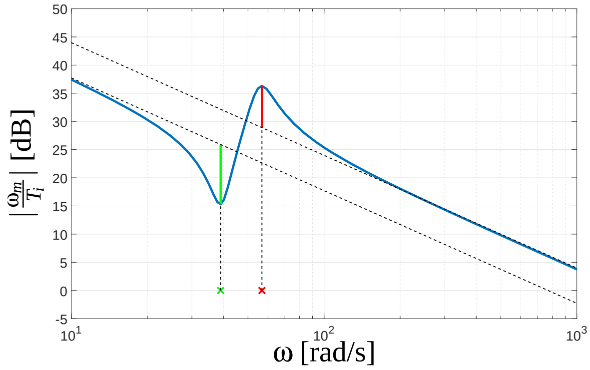

Figure 1: Flexible system frequency response (example)

The blue line represents the magnitude of the mechanical part of the drive systems with a flexible mechanical part. Red and green lines represent resonance and anti-resonance overshoots, respectively. Red and green crosses then represent their positions in the frequency. The mentioned transfer function of the examined plant is as follows:

$$G_p(s) = \frac{\omega_m(s)}{T_i(s)} = \frac{1}{J_m + J_l}\frac{1}{s}\frac{J_l s^2 + bs + k}{J_m J_l s^2 + bs + k} \tag{5}$$

where the s stands for Laplace operator. The bi-quadratic member creates the flexible element of the system. The main goal of this paper is to show the procedure of the J_m, J_l, k, and b parameters estimation. The resonant angular frequency ω_r and anti-resonant angular frequency ω_a are described by the following relations [3], [4]:

$$\omega_r = \sqrt{k\frac{J_m + J_l}{J_m J_l}} \tag{6}$$

$$\omega_a = \sqrt{\frac{k}{J_l}} \tag{7}$$

The analytical form of the amplitude frequency characteristic can be easily derived by replacing s with the $j\omega$ in (5) and evaluating the absolute value to gain the following relation:

$$|G_p(\omega)|_{dB} = 20\log_{10}\sqrt{\frac{(k - J_l\omega^2)^2 + b^2\omega^2}{(k(J_m + J_l) - J_m J_l \omega^2)^2 + b^2(J_m + J_l)^2\omega^2}} \tag{8}$$

Now, the values of the resonant and anti-resonant peaks can be easily derived by substituting (6) and (7) into (8) respectively. Then, the following relations are obtained:

$$R_{dB} = 20\log_{10}\frac{1}{J_m + J_l}\sqrt{1 + \frac{k\frac{J_l^3}{J_m}}{b^2(J_m + J_l)}} \tag{9}$$

$$A_{dB} = 20\log_{10}\sqrt{\frac{b^2}{b^2(J_m+J_l)^2+kJ_l^3}} \tag{10}$$

The model was widely discussed in the previous work: [5] Relations (6), (7), (9), and (10) are used to examine the accuracy of the identified parameters in the final part of this paper.

3 DATA PROCESSING

The used Welch method of the spectral analysis can be described by the following formula:

$$|\hat{G}_p(\omega)| = \frac{\mathscr{F}\frac{1}{N}\sum_{i=0}^{N} R_{uy_i}(\tau)w_i(\tau)}{\mathscr{F}\frac{1}{N}\sum_{i=0}^{N} R_{uu_i}(\tau)w_i(\tau)} \tag{11}$$

The method is based on the averaging of N periodograms. Where periodograms are given as the Fourier images of the cross-correlation and auto-correlation functions, respectively. For accuracy increase, there is a window function applied on each periodogram. The window function is defined as follows:

$$w(\tau) = 0.5 - 0.5\cos\left(\frac{2\pi}{L-1}\tau\right) \tag{12}$$

where L is the length of the data block (periodogram). As mentioned in the Introduction section, the spectral analysis serves only as the initial estimate for the Levenberg-Marquardt algorithm. This algorithm minimizes the nonlinear least-squares problem:

$$||\mathbf{f}(x,B)-\mathbf{y_m}||_2^2 = (\mathbf{f}(x,B)-\mathbf{y_m})^T(\mathbf{f}(x,B)-\mathbf{y_m}) = (\mathbf{r})^T(\mathbf{r}) \tag{13}$$

where $\mathbf{f}(x,B)$ is a set of nonlinear equations and B is a set of function parameters. The $\mathbf{y_m}$ is a set of measurements. For the optimum solutions, it is necessary that following partial derivatives are equal to zero:

$$\frac{\partial \mathbf{r}^T}{\partial B}\mathbf{r} = \mathbf{J}^T\mathbf{r} = \mathbf{v} \tag{14}$$

where $\mathbf{J}$ is Jacobian matrix. The vector $\mathbf{v}$ should be equal to zero for the optimal solution. Levenberg-Marquardt method presents the solution of this problem as:

$$(\mathbf{A}^{(k)}+\lambda\mathbf{D}^{(k)})B_d^{(k)} = -\mathbf{v}^{(k)} \tag{15}$$

where $\mathbf{A}^{(k)}$ is k-th estimate of Hessian defined as the $\mathbf{A} = \mathbf{J}^T\mathbf{J}$. Parameter λ is the value of a scale factor and diagonal matrix $\mathbf{D}^{(k)}$ is an additional scale matrix. Finally, $\mathbf{v}^{(k)}$ is a gradient in k-th step. According to: [6] the $\mathbf{D}$ is set as $\mathbf{D^{(k)}} = \mathbf{J}^{T(k)}\mathbf{J}^{(k)}$. The estimate of the Jacobian matrix in each step of the algorithm is calculated as follows:

$$\mathbf{J}_{*,i}^{(k)} = \frac{\mathbf{f}(x,B_i+\Delta)-\mathbf{f}(x,B_i)}{\Delta} \tag{16}$$

where Δ is a user-defined tolerance value.

3.1 PARAMETERS EXTRACTION

For the initial parameter estimate, the values of resonance frequency ω_r and anti-resonance frequency ω_{ar} are used. An additional point at low frequencies (ω_{low}) is used as well. See Figure **??** (a). The point is half of decade distant from the anti-resonance frequency. Now, from (6), (7), (9), (10) and with the additional point and with $J_{c,0} = \frac{J_{m,0}J_{l,0}}{J_{m,0}+J_{l,0}}$ one can derive:

$$\frac{1}{J_{m,0}+J_{l,0}} = J_s = |G_p(\omega_{low})| \tag{17}$$

$$J_{m,0} = \frac{\omega_{ar}^2}{\omega_r^2} J_s \tag{18}$$

$$J_{l,0} = J_s^{-1} - J_{m,0} \tag{19}$$

$$k_0 = \omega_r^2 J_{c,0} \tag{20}$$

$$b_0 = \sqrt{\frac{k_0 J_{l,0}^3}{R^2 J_{l,0}^3 J_{c,0}^{-3} - J_{l,0} J_{c,0}^{-1}}} \tag{21}$$

4 EXPERIMENT DESCRIPTION

The experimental testbench is formed by the long torque shaft connected to the Permanent magnet synchronous motor (PMSM) and the flywheel (1 kg and 10 cm diametral) placed at the other end of the shaft. The electrical part of the PMSM is considered to be controlled by the Field-oriented control strategy and its design is omitted in this paper. And the whole electrical part of the PMSM is abbreviated into a simple torque generator following: $T_i = i_q K_i$. Where i_q is the current applied on the q-axis and K_i is the torque-current constant of the PMSM. The sampling frequency for the measurement was set to 2 kHz. The q-axis current and the shaft velocity were measured and used for the spectral analysis. As the input signal, a Pseudo Random Binary Sequence (PRBS) was used. There were 65536 samples acquired and the length of the data block (length of the periodogram) for the Welch analysis was set to 8192 samples. The complete algorithm was implemented at the dSPACE DS1103 platform. In the Figure 2 (a) there are two estimates of the (5). The first one (red solid line)

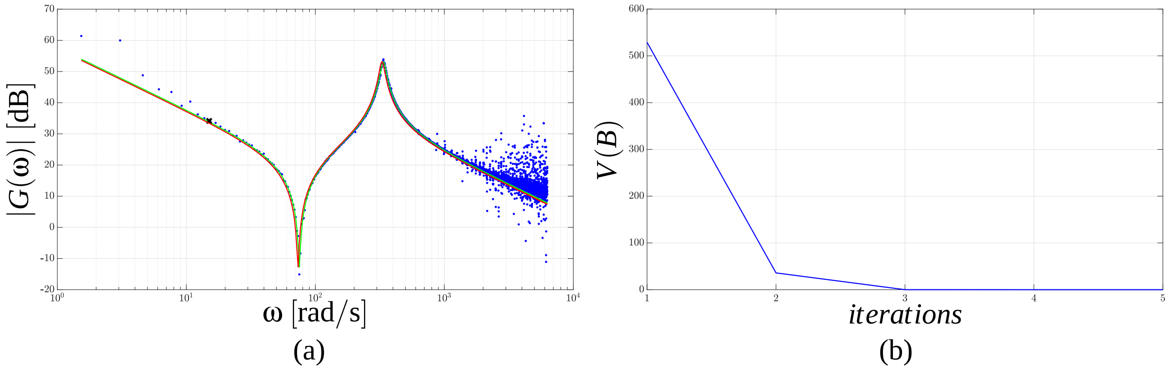

Figure 2: (a) - Blue dots represent curve obtained by the Welch method, red solid line stands for the model with the initial estimate and the green line is the model given by the Levenberg-Marquardt algorithm. The black cross is an additional point. (b) - Value of normalized cost function over the iterations

is obtained with the initial parameter estimate based on the Welch method. The second one (green) is obtained as the final value of the paramters estimate by the Levenberg-Marquardt algorithm. In Figure 2 (b), there is the normalized value of the cost function with which the algorithm is optimized. The cost function formula can be obtained from (13) as follows:

$$V(B) = \frac{1}{N}(|G_p(\omega, B)| - |\hat{G}_p(\omega)|)^T (|G_p(\omega, B)| - |\hat{G}_p(\omega)|) \tag{22}$$

where N is the number of the curve data points obtained by the Welch method (blue points in Figure 2 (a)). One can see, that the convergence of the cost function is fast. The minimum of (22) is found within the 5 iterations. The value of λ was set to $\lambda = 0.75$ and the value of Δ was set to $\Delta = 1 \cdot 10^{-1}$. The ending condition of the algorithm is given by the maximum number of iterations and the minimum change of the B parameters and minimum change of the cost function $V(B)$. Both are given by the value of Δ.

Parameter	Welch	Levenberg-Marquardt	theoretical
J_m	$6.45 \cdot 10^{-5}$ kg $\cdot$ m^2	$6.76 \cdot 10^{-5}$ kg $\cdot$ m^2	$6.5 \cdot 10^{-5}$ kg $\cdot$ m^2
l	$1.31 \cdot 10^{-3}$ kg $\cdot$ m^2	$1.33 \cdot 10^{-3}$ kg $\cdot$ m^2	$1.3 \cdot 10^{-3}$ kg $\cdot$ m^2
k	6.75 kg $\cdot$ m$^2 \cdot$ s^{-2}	6.95 kg $\cdot$ m$^2 \cdot$ s^{-2}	7 kg $\cdot$ m$^2 \cdot$ s^{-2}
b	$2 \cdot 10^{-3}$ kg $\cdot$ m$^2 \cdot$ s^{-1}	$3 \cdot 10^{-3}$ kg $\cdot$ m$^2 \cdot$ s^{-1}	$3 \cdot 10^{-3}$ kg $\cdot$ m$^2 \cdot$ s^{-1}

Table 1: Parameters comparison

In Table 1, there are results compared with the theoretical values. Theoretical values are obtained from the geometric attributes of the testbench (flywheel diameter, shaft length, shaft diameter, and shaft shear modulus). The inertia higher values are caused by the omitted additional connections in the testbench (flexible couplings etc.).

5 CONCLUSION

The main goal of this paper was to demonstrate the possible use of Welch spectrum analysis with the Levenberg-Marquardt algorithm for the motor drive plants with the two-mass mechanical load parameter estimation. The Welch method was used to create an initial estimate for the Levenberg-Marquardt algorithm. This algorithm found the minimum of (22) within the 5 iterations. Obtained results are presented and briefly discussed.

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REFERENCES

[1] Antoine Boutros, Philippe El-Jurdi, Hadi Y. Kanaan, and Kamal Al-Haddad. Modeling and simulation of a complex mechanical load using the multi-mass approach. *Proceedings of the Mediterranean Electrotechnical Conference - MELECON*, (April):373–379, 2014.

[2] Peter D. Welch. The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging Over Short, Modified Periodograms. *IEEE Transactions on Audio and Electroacoustics*, 15(2):70–73, 1967.

[3] Ming Yang, Weilong Zheng, Kaixuan Yang, and DIanguo Xu. Suppression of mechanical resonance using torque disturbance observer for two-inertia system with backlash. *9th International Conference on Power Electronics - ECCE Asia: "Green World with Power Electronics", ICPE 2015-ECCE Asia*, pages 1860–1866, 2015.

[4] George Ellis and Robert D. Lorenz. Resonant load control methods for industrial servo drives. *Conference Record - IAS Annual Meeting (IEEE Industry Applications Society)*, 3:1438–1445, 2000.

[5] Ondrej Bartik. PARAMETER IDENTIFICATION OF THE TWO-MASS MECHANICAL FLEXIBLE SYSTEM USING WELCH METHOD. *Proceedings of the 25th Conference STUDENT EEICT 2019*, (1):561–565, 2019.

[6] Roger Fletcher. A Modified Marquardt subroutine for non-linear least squares. *H. M. Stationery Office, Harwell, Berkshire*, pages 1–28, 1971.