

## Research Article

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# P1-approximation for radiative transfer: application to SF<sub>6</sub> + Cu arc plasmas

**Abstract:** The objective of the paper consists of a theoretical prediction of radiative heat transfer in arc plasmas of SF<sub>6</sub> (sulphur hexafluoride) with various admixtures of copper vapours. The P1-approximation was used as a mathematical tool. Due to the very complicated frequency dependence of absorption coefficients, the Planck and Rosseland mean absorption coefficients have been derived from the calculated absorption spectrum. The main radiation quantities (radiation intensity, radiation flux density and its divergence – net emission) have been determined in cylindrical arc plasmas for several model temperature profiles. Contribution to the net emission of copper admixtures is discussed. Conclusions have been made concerning validity and utilization of various absorption means.

**Keywords:** thermal plasma radiation, mean absorption coefficients, net emission

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## 1 Introduction

The electric (switching) arc is responsible for proper disconnecting of a circuit in high power circuit breakers. In the mid and high voltage region, SF<sub>6</sub> self-blast circuit breakers are widely used. Radiative heat transfer influences

very much the physical processes occurring in the arc plasmas. Detailed information about the local arc structure can only be given by mathematical CFD modeling.

Recently, we have used an approximate method of partial characteristics to evaluate the radiation properties of SF<sub>6</sub> arc plasmas with various admixtures of Cu [1,2]. The method of partial characteristics is generally accepted as a very good mathematical tool for prediction of radiation transfer in the arc plasmas. However, the method requires beforehand tabulations, and its integration into the set of equations for CFD modelling is not straightforward.

The P1-approximation has enjoyed great popularity due to its relative simplicity and compatibility with standard methods for the solution of general CFD equations [3,4]. The P1-approximation requires the solution of one elliptic second-order partial differential equation for each frequency. Due to the very complicated frequency dependence of absorption coefficients, the efficient solution requires the proper spectral averaging. From the calculated absorption spectrum both the Planck and Rosseland mean absorption coefficients have been derived. Calculation of radiation characteristics (radiation flux, divergence of radiation flux – net emission) has been performed for cylindrical plasmas of various radii, at the pressure of 0.5 MPa and with several model temperature profiles.

## 2 Absorption properties of plasma

Plasma radiation depends on the concentrations of chemical species occurring in the plasma. In the mixture of SF<sub>6</sub> and Cu plasma, we assume the following species: SF<sub>6</sub> molecules, S, F, Cu neutral atoms, S<sup>+</sup>, S<sup>+2</sup>, S<sup>+3</sup>, F<sup>+</sup>, F<sup>+2</sup>, Cu<sup>+</sup> ions and free electrons. Molecular products of SF<sub>6</sub> dissociation were neglected due to the lack of data for their absorption coefficients. We have only taken into account SF<sub>6</sub> molecules with their experimentally measured absorption cross sections [5]. The equilibrium composition of the plasma was taken from [6]. In Fig. 1, we show the particle densities for 90% SF<sub>6</sub> + 10% Cu as a function of temperature at a pressure of 0.5 MPa.

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For atomic and ionic species, the absorption coefficients were calculated. The total coefficient of absorption is given by the linear sum:

$$\kappa_{\nu}(\nu, T, p) = \kappa_{\nu}^{ff} + \kappa_{\nu}^{bf} + \kappa_{\nu}^{bb}, \quad (1)$$

where  $\nu$  is radiation frequency while  $T$  and  $p$  are plasma temperature and pressure, respectively. Indexes *ff*, *bf*, *bb* denote, respectively, free-free transitions (bremsstrahlung), bound-free transitions (photo-recombination) and bound-bound transitions (discrete radiation).

**Bound-Free Transitions.** Because a free electron can assume non-quantized kinetic energies, its recombination with an ion will result in continuum radiation. The excess energy of an electron with velocity  $v_e$  is converted into radiation according to the relation:

$$\frac{m_e v_e^2}{2} + E_{\infty}^a - E_i^a = h\nu \quad (2)$$

$E_{\infty}^a$  denotes the ionization potential of an atom or ion,  $E_i^a$  is the energy of the  $i$ -th electronic energy state, where the electron is captured. The spectral absorption coefficient of process (2) is related to the photo-ionization cross section  $\sigma_{\nu,i}^a$  by:

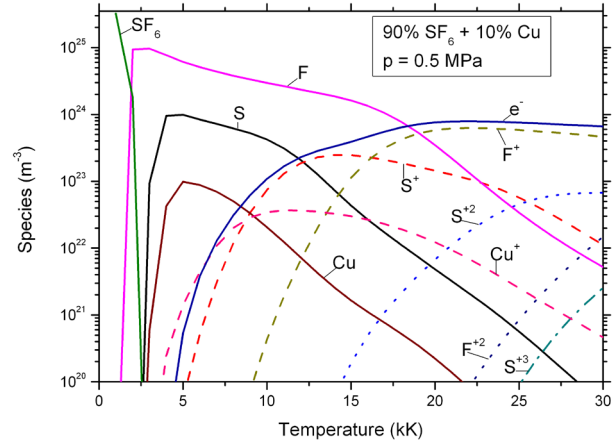
$$\kappa_{\nu,i}^{bf} = \sigma_{\nu,i}^a \cdot N_i^a \quad (3)$$

where  $N_i^a$  is the population density of the  $i$ -th electronic state of the absorbing species „a“.

The photo-ionization cross sections for neutral atoms were calculated by the quantum defect method of Burgess and Seaton [7], the cross sections of photo-ionization of ions were treated using the Coulomb approximation for hydrogen-like species [8].

**Free-free transitions.** In an ionized gas free electrons can interact with the electric field of ions resulting in a free-free transition (also known as “bremsstrahlung”). The release of a photon lowers the kinetic energy of the electron (decelerates it), the capture of a photon accelerates it. Since kinetic energy levels of free electrons are essentially not quantized, these photons may have any frequency or wavelength. The free-free transitions were calculated using the hydrogen-like approximation.

**Bound-Bound Transitions.** Due to their electronic excitation, atoms and ions emit spectra of lines such that:



**Figure 1:** Composition of 90% SF<sub>6</sub> + 10% Cu plasma as a function of temperature at a pressure of 0.5 MPa.

$$E_m - E_n = h\nu_{mn} = \frac{hc}{\lambda_{mn}} \quad (4)$$

where  $m$  and  $n$  are, respectively, the upper and lower excited levels between which the transition takes place. The radiation frequency  $\nu$  (or wavelength,  $\lambda$ ) is characteristic of both the atom and ion and the emitting level. Electrons changing their orbits remain bound to the nucleus, and this type of transition is called a “bound-bound” transition. The corresponding wavelengths extend from the infrared to the far ultraviolet. The spectral lines emitted by a gas discharge are found to be rather narrow at low gas pressure.

At higher pressures and correspondingly higher densities of atoms, ions and electrons in the plasma, there will be more frequent collisions among the radiating atoms, and the resulting effects are described as pressure broadening. When the perturbing particle is of the same type as the emitting particle, the resonance broadening occurs. When charged particles are present in significant quantities, they generate electric microfields that perturb the normal energy levels via Stark broadening of the spectral lines. The broadening effect is described by a Lorentzian profile. In addition, these effects may result in a displacement of the maximum of the lines called line shift. When the pressure is progressively reduced, collisions and Stark broadening are also reduced and the line width asymptotically approaches a temperature-dependent finite value. The remaining width of the line is due to random motion of the radiating atoms and ions and is described as the Doppler effect. This broadening effect is described by a Gaussian profile. When the Doppler width is reduced to negligible values (by reducing the thermal

velocities of the radiating atoms), the remaining width is known as the natural width and is associated with the Heisenberg's uncertainty principle. This natural width is generally much narrower than the Doppler, resonance and Stark broadening.

For each spectral line we have calculated parameters defining their half-widths and shifts. Semi-empirical formulas given in [8,9] were used. The line shapes were calculated by convolution of Gaussian and Lorentzian profiles, resulting in a Voigt profile. The fine multiplet structure and the lines overlapping were also taken into account. Examples of calculated total absorption spectra are given in Figs. 2 and 3.

In Fig. 2, total absorption coefficients for plasma of 90% SF<sub>6</sub> + 10% Cu at the pressure of 0.5 MPa for temperatures of 5 000 K and 20 000 K are plotted. The influence of copper admixture on the value of the absorption coefficient is presented in Fig. 3 for plasma at the temperature of 20 000 K. Both, strong dependence of the coefficient of absorption on the radiation frequency and dominating continuum radiation in the ultraviolet region of the spectrum, can be seen. Increasing metal admixtures lead to increasing number of spectral lines.

### Multigroup Approximation

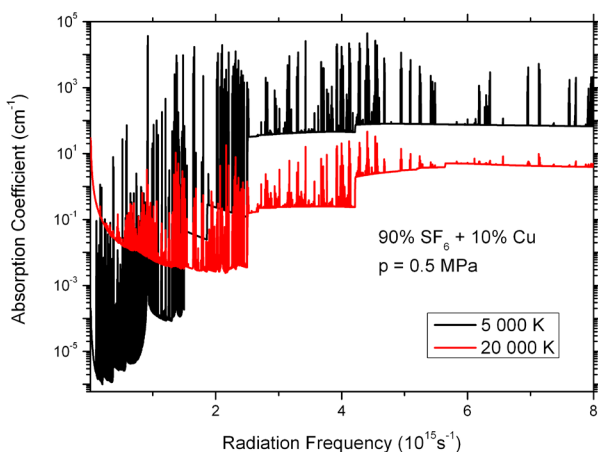
The calculated sets of absorption coefficients  $\kappa_\nu(T)$  were used for generating parameters of an approximate model – the multigroup approximation. The spectral range was split in five groups with the following cutting frequencies (in units 10<sup>15</sup> s<sup>-1</sup>)

$$\nu_k = \{0.01, 1, 2, 4.1, 6.8, 10\}, \quad (1)$$

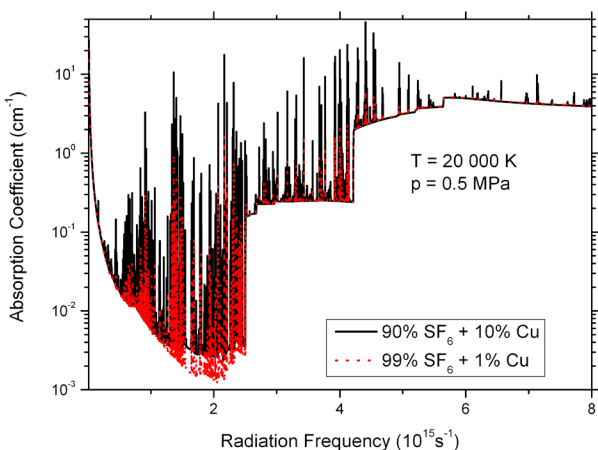
in which the absorption coefficient is assumed to be constant with certain average value. The mean values of absorption coefficients were taken as either Rosseland ( $\kappa_R$ ) or Planck ( $\kappa_p$ ) means:

$$\kappa_R^{-1} = \int_{\nu_k}^{\nu_{k+1}} \kappa_\nu^{-1} \frac{dB_\nu}{dT} d\nu \Big/ \int_{\nu_k}^{\nu_{k+1}} \frac{dB_\nu}{dT} d\nu; \quad \kappa_p = \int_{\nu_k}^{\nu_{k+1}} \kappa_\nu B_\nu d\nu \Big/ \int_{\nu_k}^{\nu_{k+1}} B_\nu d\nu, \quad (2)$$

where  $B_\nu$  designates the Planck function – the spectral intensity of equilibrium radiation. Fig. 4 shows the mean Planck and Rosseland absorption coefficients as a function of temperature for five spectral groups (1). The Rosseland and Planck mean absorption coefficients can differ by several orders of magnitude. Discrete radiation influences significantly values of Planck means mainly at lower frequencies; on the other hand, Rosseland means ignore the role of lines. The fast changing of the mean absorption



**Figure 2:** Total absorption coefficient for the SF<sub>6</sub> plasma with 10% Cu at temperatures of 5 000 K and 20 000 K.



**Figure 3:** Total absorption coefficient for the SF<sub>6</sub> plasma with various admixtures of Cu at the temperature of 20 000 K.

coefficients at about  $T = 2\,500$  K for the spectral groups  $(2.0\text{--}4.1) \times 10^{15} \text{ s}^{-1}$  and  $(4.1\text{--}6.8) \times 10^{15} \text{ s}^{-1}$  can be explained by neglecting the contribution of diatomic molecules to the absorption properties of plasma.

### 3 P1-approximation

If diffusion of light is neglected and local thermodynamic equilibrium is assumed, the radiation transfer equation can be written as:

$$\vec{\Omega} \cdot \nabla I_\nu(\vec{r}, \vec{\Omega}) = \kappa_\nu (B_\nu - I_\nu) \quad (3)$$

where  $I_\nu(r, \Omega)$  is the spectral intensity of radiation and  $\vec{\Omega}$  is a unit direction vector. In the P1-approximation, the

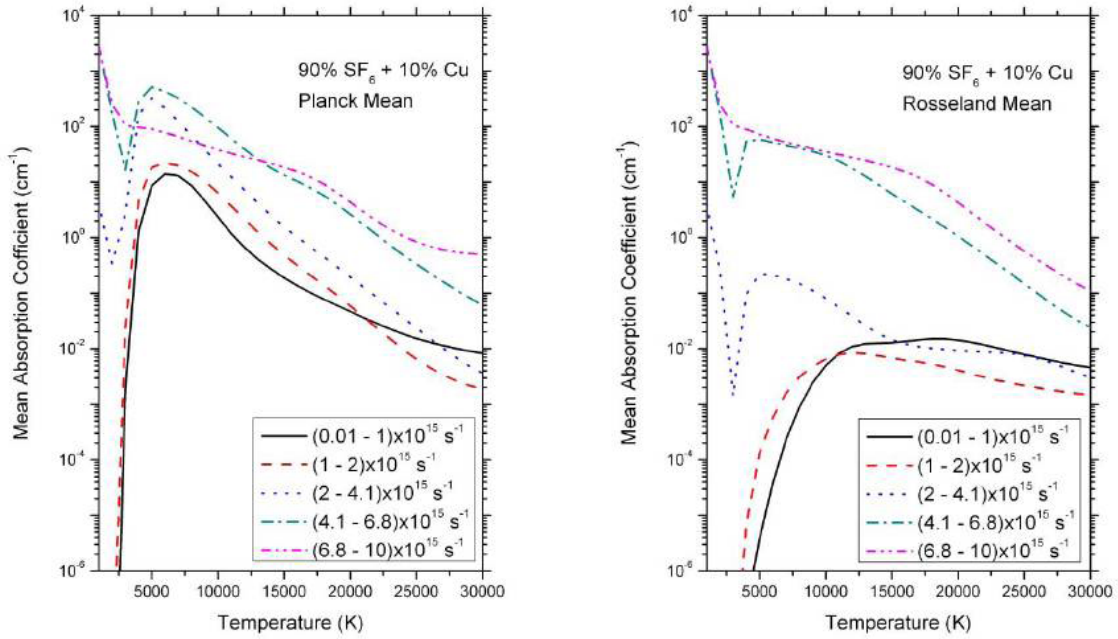


Figure 4: Planck and Rosseland absorption means as a function of temperature for plasma 90% SF<sub>6</sub> + 10% Cu at the pressure 0.5 MPa.

angular dependence of the spectral intensity is supposed to be represented by the first two terms in a spherical harmonic expansion:

$$I_\nu(\vec{r}, \vec{\Omega}) = \frac{c}{4\pi} U_\nu(\vec{r}) + \frac{3}{4\pi} \vec{F}_\nu(\vec{r}) \cdot \vec{\Omega} \quad (4)$$

where  $U_\nu$  denotes the spectral radiation field density,  $\vec{F}_\nu$  is the spectral radiation flux, and  $c$  is the velocity of light. Combining this expression with Eq. 3 one finds for radiation flux:

$$\vec{F}_\nu(\vec{r}) = -\frac{1}{3\kappa_\nu} c \text{grad} U_\nu(\vec{r}) \quad (5)$$

and a simple elliptic partial differential equation for the spectral density of radiation  $U_\nu$ :

$$\nabla \cdot \left[ -\frac{c}{3\kappa_\nu(\nu, T)} \nabla U_\nu(\nu, \vec{r}) \right] + \kappa_\nu(\nu, T) c U_\nu(\nu, \vec{r}) = 4\pi B(\nu, T) \kappa_\nu(\nu, T) \quad (6)$$

Integrating over frequency, the total density of radiation and total radiation flux are obtained:

$$U(\vec{r}) = \int_0^\infty U_\nu(\vec{r}) d\nu, \quad \vec{F}(\vec{r}) = \int_0^\infty \vec{F}_\nu(\vec{r}) d\nu. \quad (7)$$

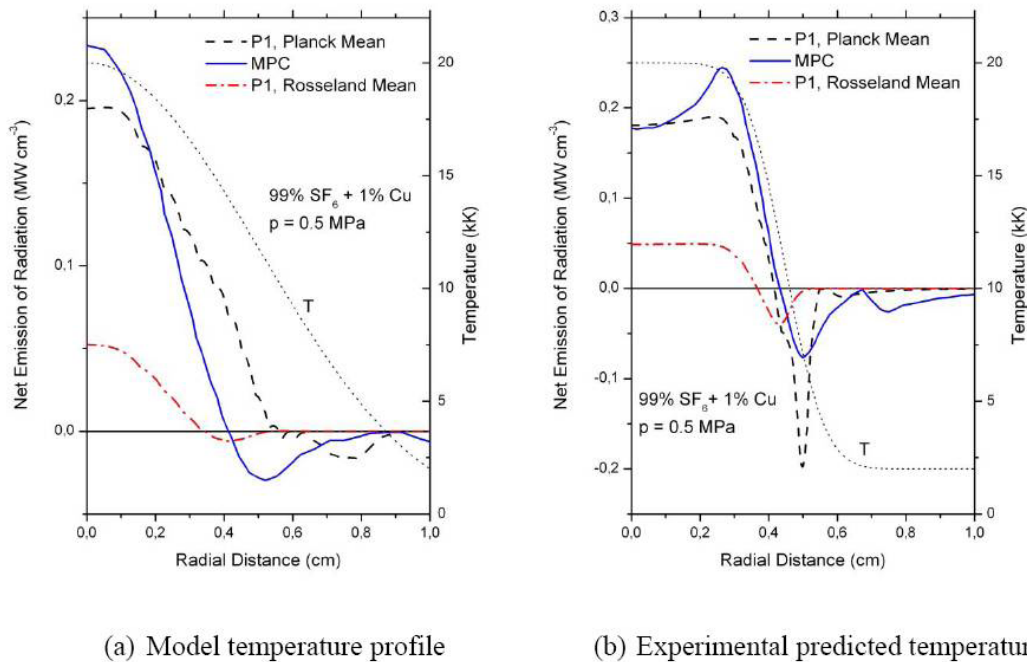
In the multigroup approximation the total density of radiation and total radiation flux are given by:

$$U(\vec{r}) = \sum_{k=1}^G U_k(\vec{r}), \quad \vec{F}(\vec{r}) = \sum_{k=1}^G \vec{F}_k(\vec{r}) \quad (8)$$

where  $U_k$  is the solution of Eq. 6 for  $k$ -th frequency group with frequency independent  $\kappa_k(T)$  and  $B_k(T) = \int_{\nu_k}^{\nu_{k+1}} B_\nu(\nu, T) d\nu$ .

## 4 Results

The P1-approximation was applied in calculations of radiation characteristics in various mixtures of SF<sub>6</sub> + Cu cylindrical plasmas using Planck and Rosseland means. The choice of mixtures (1 vol% to 10 vol% of Cu) was guided by the usual amount of evaporated electrode material in self-blast circuit breakers with copper electrodes. Two different temperature profiles were assumed. Fig. 5a, 5b shows the net emission (divergence of radiation flux) in plasma cylinder with the radius of 1.0 cm at the pressure of 0.5 MPa. Comparison of the P1-approximation and various averaging methods with results of the method of partial characteristics (MPC) [1,2] is given for plasma of 99% SF<sub>6</sub> + 1% Cu.



(a) Model temperature profile

(b) Experimental predicted temperature

**Figure 5:** Net emission of  $\text{SF}_6$  plasma with 1% Cu for two different temperature profiles (dotted line). Comparison of values calculated by P1-approximation and by the method of partial characteristics (MPC) [1], [2].

A model temperature profile:

$$T(r) = \left\{ [T_0^{1/10} - T_w^{1/10}] \times \left[ 1 - \left( \frac{r}{R} \right)^2 \right] + T_w^{1/10} \right\}^{10}, \quad (9)$$

$$T_0 = 20\,000 \text{ K}, T_w = 2\,000 \text{ K}$$

The profile is presented in Fig. 5a. This type of temperature profile is typical for a wall-stabilized plasma column.

In Fig. 5b a temperature distribution corresponding to experimental predictions for an axially blown arc is assumed:

$$T(r) = T_w + \frac{1}{2}(T_0 - T_w) \left[ \operatorname{erf} \left( \frac{0.448 + r}{0.13} \right) + \operatorname{erf} \left( \frac{0.448 - r}{0.13} \right) \right], \quad (10)$$

$$T_0 = 20\,000 \text{ K}, T_w = 2\,000 \text{ K}$$

In both cases,  $T_0$  is the temperature at the arc axes,  $T_w$  is the temperature at the plasma edge,  $R$  is the radius of the plasma cylinder, and  $r$  is the radial distance from the arc axes.

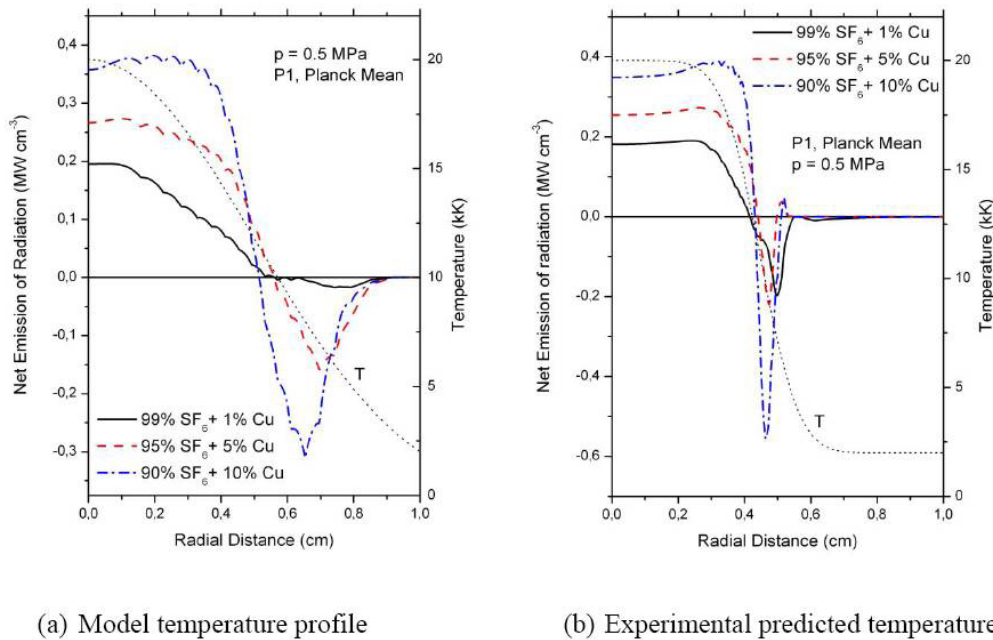
It can be seen that the Rosseland averaging underestimates both the emission of radiation in hot parts of the plasma and the absorption of radiation in cold edge of the plasma cylinder (the negative values of net emission). The Planck means give better agreement with

data calculated by the method of partial characteristics. However, in the case of the model temperature profile (Fig. 5a) the point where the net emission becomes zero (the edge of the arc) is shifted to a greater distance from the arc axes. For the experimental predicted temperature distribution (Fig. 5b), the Planck means lead to overestimation of the absorption of radiation outside the arc. This discrepancy can be explained by the overestimation of the influence of spectral lines in the Planck averaging method, which is significant particularly at lower temperatures. At high temperatures, the continuum radiation plays a more important role.

The effect of the proportion of Cu vapour to the  $\text{SF}_6$  plasma on the net emission is shown in Figs. 6a and 6b for the mentioned above temperature profiles, respectively. The P1-approximation with the Planck averaging method was used in these calculations. Admixture of Cu vapour in  $\text{SF}_6$  contributes to the increase in net emission. The contribution depends on the Cu vapour concentrations.

## 5 Conclusions

In the study we performed calculations of divergence of radiation flux (net emission) using the P1-approximation for plasma of  $\text{SF}_6$  with admixtures of Cu. The frequency



(a) Model temperature profile

(b) Experimental predicted temperature

**Figure 6:** Net emission of SF<sub>6</sub> plasma with various admixtures of Cu.

dependence of absorption coefficients has been handled by the multigroup approximation. Both Planck and Rosseland averaging methods have been applied to obtain mean values of absorption coefficients. Comparison with net emission, calculated using the method of partial characteristics, has been provided. The Rosseland means underestimate both, emission of radiation from the hot central arc and absorption of radiation in cold plasma edge. The Planck means provide better agreement with the data calculated by the method of partial characteristics. However, discrepancies occur outside the arc in the region where absorption of radiation dominates.

The use of the Rosseland and Planck mean absorption coefficients is only strictly appropriate in limited circumstances. The Rosseland mean is a suitable approach for thick plasma (absorption dominated system); the Planck means give good results for emission dominated systems (omitting self-absorption). In reality, neither mean is correct in general. Various methods have been suggested to solve this problem. The simplest procedure is to use the Planck means for frequency groups with low absorption coefficient values and the Rosseland means for groups with high absorption coefficient values. Another approach that partially solves the problem of overestimation of the role of lines in the Planck averaging method is using the escape factors for strong self-absorbing resonance lines [10]. Another correction was suggested in [3] where the frequencies are grouped not only based on frequency but also based on absorption coefficient magnitude.

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