Beampattern Synthesis using Reweighted ℓ_1-Norm Minimization and Array Orientation Diversity

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Abstract. *The pattern synthesis of sparse antenna arrays has many practical applications in situations where the weights, size, and cost of antennas are limited. In this work the antenna array synthesis problem, with minimum number of elements, is studied from the new perspective of sparseness constrained optimization. The number of antenna elements in the array can be efficiently reduced by casting the pattern synthesis problem into the compressive sensing (CS) framework of sparseness constrained optimization and solving with the reweighted ℓ_1-norm minimization algorithm. Besides, the proposed method allows exploitation of the array orientation diversity in the CS framework to address left-right radiation pattern ambiguity problem. Numerical examples are presented to show the high efficiency of achieving the desired radiation pattern with the minimum number of antenna elements.*

Keywords

Sparse array pattern synthesis, reweighted ℓ_1-norm minimization, orientation diversity, convex optimization.

1. Introduction

The goal of array pattern synthesis (APS) is to calculate the excitations and positions for all antenna elements that produce a radiation pattern as closely as possible to the desired one. Reducing of the number of antenna elements in the array is particularly useful in many applications where the weight, size and cost of the antennas are limited, such as phased array radar, satellite communication and MIMO radar system [1], [2], [3], [4]. Up to present many analytical formulations have been derived for the antenna array synthesis problem, such as the Dolph-Chebyshev and Taylor methods for uniformly spaced antenna arrays [5]. These methods are generally based on the assumption that the array elements are equally spaced with uniform distribution which results in a large number of antenna elements to synthesis the desired radiation pattern. To reduce the number of elements in the array, An alternative strategy is to use unequally spaced and non-uniform excitation for APS. Actually, the unequally spaced non-uniform antenna array synthesis problem is complex and generally could not be efficiently solved with analytical methods. Fortunately, some existing global optimization methods, such as genetic algorithm (GA), particle swarm optimization (PSO) method and simulated annealing (SA), can been successfully used for the synthesis of non-uniform linear, planar and circular arrays [3], [6], [7], [8], [9].

Convex optimization technique has also been proposed to solve the APS problem, which can be formulated as a second-order cone programming (SOCP) problem or, more generally, a semi-definite programming (SDP) problem. Though SOCP and SDP can be readily solved by the SOCP solver and SDP solver, respectively, a general non-uniform array design problem cannot be directly formulated as a convex problem [10]. Wang *et al.* [10] proposed an iterative procedure to optimize the array pattern at each iteration by solving an SDP problem. To design the optimal non-uniform array, all the above mentioned approaches try to construct an objective function to minimize the peaks of sidelobes, or more generally the synthesis error. When the number and positions of elements are known, nonuniformly spaced arrays can be optimized using convex programming in essentially the same way as that for uniformly spaced arrays. The required changes are rather trivial. It is impossible to solve the APS problem by complex programming if the positions of the non-uniformly spaced array elements are unknown. In addition, more elements are usually required to obtain the desired array performance. Recently, a novel non-iterative synthesis algorithm based on the matrix pencil method has been proposed that efficiently reduces the number of elements in a linear antenna array with very short computation time [2],[4], and Zhang *et al.* [11] cast the array synthesis problem into the framework of sparseness constrained optimization and solved the problem by the Bayesian compressive sensing (BCS) inversion algorithm.

In this paper, we extend our method [12] to a new version by using reweighted ℓ_1-norm minimization [13], array orientation diversity and convex optimization [14]. Merits of the algorithm include 1) it does not need a thorough search in the multidimensional parameter space, and 2) it can achieve the same array performance with fewer antenna elements, and thus reduces the array cost significantly.

2. Array Pattern Synthesis with Reweighted ℓ_1-Norm Minimization and Orientation Diversity

2.1 Problem Formulation

We assume that transmit signals and the array are coplanar, so the antenna array synthesis problem can be described as follows:

$$\min(DM) \quad \text{s.t.} \left\{ \min_{\{R_{\alpha i}, d_{\alpha i}\} \substack{\alpha=1...D \\ i=1...M}} \|F_d(\theta) - F(\theta)\|_{\ell_2} \right\} \le \varepsilon \tag{1}$$

where $F(\theta) = \sum_{\alpha=1}^{D} \sum_{i}^{M} R_{\alpha i} e^{jkd_{\alpha i}\cos(\theta-\theta_\alpha)}$, $F_d(\theta)$ is the desired radiation pattern, M is the number of identical antenna elements in each linear array, $R_{\alpha i}$ is the excitation coefficient of the ith element located at $d_{\alpha i}$ in the αth array, k is the wavenumber in the freespace, and D array orientations θ_α (α = 1,...,D). The objective of the problem is to synthesize the desired radiation pattern $F_d(\theta)$ with the minimum number of elements under a small tolerance error ε. For one linear array at orientation θ_α to the incident plane wave from the bearing θ, the array factor is given by

$$F_\alpha(\theta) = \sum_{i}^{M} R_{\alpha i} e^{jkd_{\alpha i}\cos(\theta-\theta_\alpha)}. \tag{2}$$

Suppose that all the antenna elements in each array orientation θ_α ($\alpha \in 1,\ldots,D$) are symmetrically distributed within a range of $-d_s$ to d_s along the array orientation θ_α, respectively, the combination pattern of all the linear orientation arrays can be written as

$$F(\theta) = \sum_{\alpha=1}^{D} F_\alpha(\theta). \tag{3}$$

In order to solve the equations (2) and (3), we can assume that all the antenna elements are equally spaced from $-d_s$ to d_s with a small interelement spacing Δd. Although it is supposed that there is one element at each position, not each antenna element is necessarily radiating waves or excited with current. All the antenna elements can be in two states: "on" states (when the element is in the supposed position or has an excitation) or "off" state (when there is no element in the supposed position or without an excitation). Through discretization, (3) can be written in a matrix form

$$[F(\theta)]_{h\times 1} = [H]_{h\times n}[r]_{n\times 1} \tag{4}$$

where h is the number of sampled antenna radiation pattern, $n{=}D\left\lceil \frac{2d_s}{\Delta d} \right\rceil$, the vector $\mathbf{F}[F(\theta_1),F(\theta_1),...,F(\theta_h)]^T$ contains the sampled radiation pattern at different angles, $\mathbf{H}$ is an $h\times n$ matrix with the (i, l)-th element $\mathbf{H}_{il}{=}e^{jkd_{\alpha l}\cos(\theta_i-\theta_\alpha)}$, $l \in \left((\alpha-1)\frac{n}{D}+1 \sim \frac{n}{D}\alpha\right)$, and $\alpha \in [1,2,...,D]$. Choosing $h < n$, matrix $\mathbf{H}$ forms an overcomplete dictionary. $\mathbf{r}$ is the excitation vector, $R_{\alpha l}$=0 means the antenna in the lth position of the αth array is not excited or is absent from the supposed position, and the antenna location space in practice is sparse, thus we can exploit this priori information for APS, which can be casted as the following convex optimization problem,

$$\min \|\mathbf{r}\|_{\ell_1} \text{ subject to } \|\mathbf{F}-\mathbf{H}\mathbf{r}\|_{\ell_\infty} < \varepsilon. \tag{5}$$

In (5) we seek to find the smallest number of non-zero elements in the excitation vector $\mathbf{r}$. Regarding the notation of this paper, $\|\cdot\|_{\ell_1}$, $\|\cdot\|_{\ell_2}$, $\|\cdot\|_{\ell_\infty}$ indicate ℓ_1 norm, ℓ_2 norm, ℓ_∞ norm, respectively. And $\lceil x \rceil$ denotes the smallest integer not less than x.

2.2 Compressive Sensing

Notations and some main results of the compressive sensing (CS) theory [15], [16], [17], [18], [19], [20] are summarized in this section. The CS theory addresses the following underdetermined and noisy problem:

$$\mathbf{y}{=}\mathbf{A}\mathbf{x}{+}\mathbf{v} \tag{6}$$

where $\mathbf{A}$ is a known sensing matrix of the size $M\times N$ with $M < N$. The main goal of the CS theory is to recover the signal $\mathbf{x}{=}\,[x[1],\ldots,x[N]]^T$ of length N from measurements $\mathbf{y}{=}\,[y[1],\ldots,y[M]]^T$ of length M, contaminated by a white zero-mean Gaussian noise $\mathbf{v}$, with covariance matrix $\mathbf{G}_v{=}\mathbf{I}\sigma_v^2$. The solution to this ill-posed problem is possible only if some of the properties of signal $\mathbf{x}$ can be considered. The CS theory assumes the signal $\mathbf{x}$ to be "sparse" or "compressible" in some sparsity basis $\{b_n\}_{n=1}^{N}$, providing the following representation:

$$\mathbf{x}{=}\mathbf{B}\mathbf{d}{+}\mathbf{w} \tag{7}$$

where columns of sparsity basis matrix $\mathbf{B}$ are the vectors from the sparsity bases, vector of sparsity coefficients $\mathbf{d}$ of size $N\times 1$ contains only $J \ll N$ significantly large elements, and

$$\mathbf{w} = \mathbf{x} - \mathbf{x}_J \tag{8}$$

contains a nonsparse part of compressive signal $\mathbf{x}$, where $\mathbf{x}_J$ is a pure J-sparse signal.

In [19], it was shown that in the noiseless scenario, when the sensing matrix $\mathbf{B}$ obeys the restricted isometry property (RIP) [16], [19] or the uniform uncertainty principle (UUP) [20], the sparse signal $\mathbf{x}$ can be recovered exactly from the measurement $\mathbf{y}$ via the following linear programming optimization:

$$\min_{\hat{\mathbf{x}}\in\Re^N} \|\hat{\mathbf{x}}\|_{\ell_1} \quad \text{subject to} \quad \mathbf{A}\mathbf{x}{=}\mathbf{y}. \tag{9}$$

The matrix $\mathbf{A}$ is considered to obey the RIP with the J-restricted isometry constant δ_J, which is the smallest value that satisfies

$$(1-\delta_J)\|\mathbf{x}\|_{\ell_2}^2 \le \|\Phi\mathbf{x}\|_{\ell_2}^2 \le (1{+}\delta_J)\|\mathbf{x}\|_{\ell_2}^2. \tag{10}$$

For the noisy scenario in (6), the general estimator that holds for any signal $\mathbf{x}$, not necessarily sparse, was presented in [17] as the following convex optimization:

$$\min_{\hat{\mathbf{x}}\in\Re^N} \|\hat{\mathbf{x}}\|_{\ell_1} \quad \text{subject to} \quad \|\mathbf{v}\|_{\ell_2} \le \varsigma \tag{11}$$

where the residual power $\|\mathbf{v}\|_{\ell_2}$ is upperbounded by ς, which is small compared with the power of the strong sources. It was shown that using the assumption that $\delta_{2J}<\sqrt{2}-1$, the performance of this estimator is bounded by

$$\|\hat{\mathbf{x}}-\mathbf{x}\|_{\ell_2} \leq c_0 J^{-1/2} \|\mathbf{x}_J-\mathbf{x}\|_{\ell_1} + c_1\varsigma \tag{12}$$

where c_0 and c_1 are well behaved and small constants. Note that this results suggests that when the signal $\mathbf{x}$ is a pure J-sparse, the estimation error is bounded only by energy of the measurement noise $\mathbf{v}$.

Just as it is represented in [13], the regular ℓ_1 minimization can not obtain exact recovery with substantially fewer measurements. To deal with this problem, Candes *et al.* [13] designed an algorithm which consists of solving a sequence of weighted ℓ_1-minimization problems where the weights used for the next iteration are computed from the value of the current solution. And some existing convex optimization algorithms enforce sparsity by the reweighted ℓ_1 minimization.

The iterative ℓ_1 reweighting is presented in [13]:

$$w_l^{(q+1)} = \left[\left|x_l^{(q+1)}\right| + \delta\right]^{-1} \tag{13}$$

where x_l denotes the lth entry of the recovered signal and w_l is the corresponding weighted value, $\delta>0$ is an application-dependent parameter and it must be carefully designed, q is the iteration count number.

2.3 The Proposed Algorithm

In this paper, we outline the new solution of (1) as follows:

- Creating a Virtual Array and Initializing a Weight Matrix

  To obtain more elements than those of a conventional array with the same array aperture, we first create D virtual uniformly spaced linear orientation arrays with spacing $\lambda/16$ ($\lambda/2$ inter-element spacing of the conventional uniformly linear array (ULA)), and initialize a weight matrix $DM \times DM$ $\mathbf{Q}$ as an identity matrix.

- Finding the Sparse Weight Vector

  A weight vector is chosen to produce a beampattern specified by $F(\theta)$. According to the general problem of minimizing the peak value of the error between the synthesized pattern and the desired pattern, the weight vector can be obtained by solving the following weighted ℓ_1-norm minimization convex problem:

$$\min\ ||\mathbf{Q}\boldsymbol{w}||_{\ell_1}\ \text{s.t.}\ ||F(\theta)-F_d(\theta)||_{\ell_1} \leq \xi, \quad \forall\theta \in [-180°, 180°] \tag{14}$$

  where ξ is the fitting error between the synthesized pattern and the desired one. Minimizing $||\mathbf{Q}\boldsymbol{w}||_{\ell_1}$ makes the vector $\mathbf{Q}\boldsymbol{w}$ sparse, which is useful to create D non-uniformly spaced linear orientation arrays. Here, let the weight vector $\boldsymbol{w} = [w_1, w_2, ...]^T$ from (14) be the original weight vector. Weights of the original weight vector that are so small that they can be ignored without significantly changing the array performance.

  If the absolute value of an element from the original weight vector is bigger than a threshold which is determined according to the anticipated purpose, the element will be retained as a nonzero value element of the original weight vector; otherwise, it is assigned zero. The sparse weight vector $\boldsymbol{w_s}$ is thus obtained.

- Updating the Weight Matrix

  After obtaining the original weight vector $\boldsymbol{w} = [w_1, w_2, ...]^T$, the weight matrix $\mathbf{Q}$ is updated according to $\mathbf{Q} = diag\left(\left[(|w_1|+\delta)^{-p}, (|w_2|+\delta)^{-p}, ...\right]\right)$ (usually, p is an integer greater than 1), where $diag(\boldsymbol{x})$ indicates the diagonal matrix with main diagonal elements equaled to the vector $\boldsymbol{x}$. We introduce the parameter $\delta>0$ in order to ensure that a zero-valued component in $\boldsymbol{w}$ does not strictly prohibit a nonzero estimate at the next step. As empirically demonstrated in the next section, δ should be set slightly smaller than the expected nonzero magnitudes of $\boldsymbol{w}$. According to reference [13], reweighted ℓ_1 minimization can improve the signal reconstruction. Here, it was demonstrated experimentally that p=2 is a better choice.

- Forming the Non-uniform Arrays

  After having determined the sparse weight vector $\boldsymbol{w_s}$, the antenna elements corresponding to nonzero valued positions of the sparse weight vector are retained to form D non-uniform linear arrays with fewer elements as well as different orientations.

  The above steps $(2,3,4)$ are repeated until the final synthesized pattern performance is satisfactory or the specified maximum number of iterations is attained.

- Optimizing the Sparse Weight Vector

  To further improve the performance of the array beampattern formed by the sparse weight vector, convex optimization is further conducted to obtain the optimal weight vector, that is,

$$\text{Find } \boldsymbol{w_{opt}} \text{ that minimizes } ||F(\theta)-F_d(\theta)||_{\ell_\infty}, \quad \forall\,\theta \in [-180°, 180°]. \tag{15}$$

  The optimal sparse weight vector $\boldsymbol{w_{opt}}$ can be obtained from (15) readily.

3. Simulation Results and Discussion

Given the array physical size, the objective is to design an array with the desired array pattern as shown in Fig. 1, where the region $|\theta| \leq \theta_s$ corresponds to the main beam and the region $|\theta| \geq \theta_s$ corresponds to the sidelobe. We set $\theta_s = 2.3°$, and take a "dense set" of the interval

$[-180°, 180°]$ with the angles sampled at 2° from $-180°$ to 180°.

To show the performance of our beampattern synthesis, we will consider two cases: same element number array and approximate beampattern performance, since all formulated problems in (1) (5) (14) and (15) are convex, so we adopt the optimization toolbox [21] to solve the formulated problems.

3.1 Same Element Number Array with Array Orientation Diversity

The influence of the array orientation diversity on the beampattern synthesis is analyzed in this section. We choose four-orientation virtual ULAs (named Array a, Array b, Array c, Array d, with array orientation -10°, 0°, 10°, and 20° respectively) with each subarray aperture length of 25 λ having a uniform spacing of $\lambda/16$ between neighboring elements. In addition, we initialize $\mathbf{Q}$ as the identity matrix, $\delta = 1 \times 10^{-4}$, and $p = 2$ in our simulations. Fig. 2 shows a 23-element beampattern synthesis performance in four cases with 1, 2, 3, and 4 array orientations, and Fig. 2 shows the beampattern synthesis performance of our proposed method and BCS approach improve with increasing array orientation diversity (from 1 to 3), but too high array orientation diversity (from 3 to 4) can reduce beampattern synthesis performance, which will also be demonstrated in Fig. 3. The optimal antenna positions and the corresponding excitation amplitudes of the four cases are displayed in Tab. 1, 2, 3 and 4, respectively. Note that for the two cases of Tab. 1 and 2, the required normalized radiated energy of BCS approach [11] and our proposed method are almost the same to obtain the desired beampattern, while for cases of Tab. 3 and 4, the BCS approach requires normalized radiated energy 64.7370 and 67.0517 respectively, which is correspondingly bigger than 48.6094 and 48.7165 of our proposed method. Besides, from Tab. 2, we can see that the BCS approach needs a 24-element array to obtain the performance of our 23-element array.

3.2 Approximate Beampattern Performance with Array Orientation Diversity

To show another advantage of our approach, we exemplify the synthesis of an 18-element array with one orientation, 11-element array with two orientations, 9-element array with three orientations, and 10-element array with four orientations using our method, respectively. The optimal beampatterns exhibit maximal sidelobes of -7.78 dB, -8.10 dB, -7.77 dB and -7.89 dB respectively, which are shown in Fig. 3. Tables 5, 6, 7, and 8 display all the corresponding antenna positions and excitation amplitudes. Obviously, when the array physical size is given, using orientation diversity can offer economization of 8 (or 9) elements without reducing the array performance. Besides, the normalized radiated energy data can also provide that our proposed method needs less radiation energy for two cases of Tab. 7 and 8.

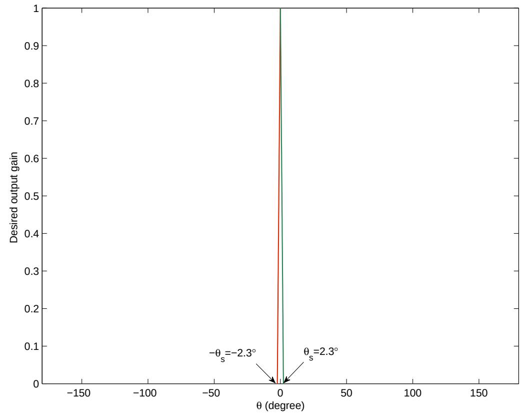

Fig. 1. Desired beampattern.

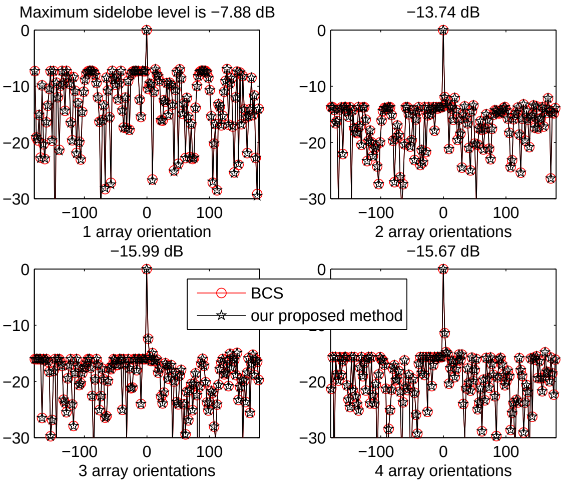

Fig. 2. "Reference beampattern of a 23(24)-element array by using BCS inversion algorithm[11]" vs. "Our beampattern of a 23-element array".

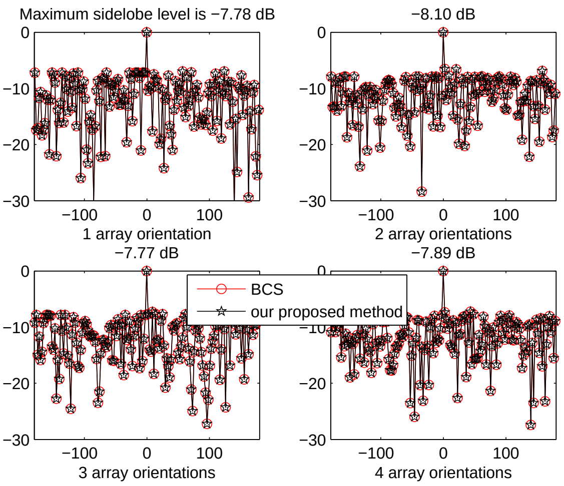

Fig. 3. Optimal beampattern of different element number array by using "BCS inversion algorithm [11]" vs. "Our method" .

Element Indices	Positions (λ)	Excitation amplitudes (w)	
Array b	Array b	Array b (BCS)	Array b (Our)
1, 23	±12.5000	51.3198, 47.9195	12.2921, 11.4768
2, 22	±12.4375	62.5594, 56.5721	14.9799, 13.5478
3, 21	±12.1875	19.9681, 17.3741	4.7825, 4.1609
4, 20	±11.8125	7.3736, 7.7327	1.7659, 1.8519
5, 19	±11.4375	4.5632, 4.8394	1.0928, 1.1590
6, 18	±11.0000	2.3530, 2.4080	0.5635, 0.5767
7, 17	±10.5625	1.3405, 1.3437	0.3210, 0.3218
8, 16	±9.6875	1.7990, 0.8586	0.4309, 0.2056
9, 15	-4.875, 8.75	1.5282, 1.4926	0.3660, 0.3575
10, 14	-2.9375, 4.875	0.7863, 0.4661	0.1883, 0.1116
11, 13	-1.0, 2.9375	1.5120, 1,1712	0.3621, 0.2805
12	3.9375	1.0407	0.2492

Tab. 1. Element positions and excitation amplitudes in a 23-element one-orientation antenna array obtained by BCS inversion algorithm [11] vs. our method.

Element Indices	Positions (λ)	Excitation amplitudes (w)	
Array (a, b)	Array (a, b)	Array (a, b) (BCS)	Array (a, b) (Our)
1, 1	-8.0625, -10.2500	1.2357, 0.8602	0.2047, 0.1488
2, 2	-6.4375, -9.5625	1.3310, 1.3180	0.2204, 0.2280
3, 3	-5.1875, -7.6875	1.2404, 0.7458	0.2054, 0.1290
4, 4	-3.6250, -6.4375	1.5706, 1.5549	0.2601, 0.2690
5, 5	-1.4375, -3.6250	0.7106, 1.7101	0.1177, 0.2958
6, 6	-0.6250, -2.4375	2.2701, 0.9225	0.3760, 0.1596
7, 7	0, -1.8750	0.3516, 0.9005	0.1165, 0.1558
8, 8	3.6250, -0.6250	1.2664, 1.5396	0.2097, 0.2663
9, 9	6.9375, 6.4375	0.6381, 0.6793	0.1057, 0.1175
10, 10	9.4375, 7.6875	0.7758, 0.8363	0.1285, 0.1447
11, 11	10.125, 9.5625	1.0089, 0.9005	0.1671, 0.1558
12	10.25	0.8021	0.1388
13(BCS)	0(BCS)	0.3516(BCS)	

Tab. 2. Element positions and excitation amplitudes in a 24-element two-orientation antenna array obtained by BCS inversion algorithm [11] vs. a 23-element two-orientation antenna array obtained by our method.

Element Indices	Positions (λ)	Excitation amplitudes (w)	
Array (a, b, c)	Array (a, b, c)	Array (a, b, c) (BCS)	Array (a, b, c)(Our)
1, 1, 1	-9.3750, -10.625, -6.75	0.6986, 0.9990, 0.8895	0.1103, 0.1116, 0.1256
2, 2, 2	-6.8125, -7.7500, -5.125	0.5428, 0.7880, 1.1561	0.0857, 0.0880, 0.1633
3, 3, 3	-5.1250, 1.3750, -3.625	0.9754, 0.9085, 0.8375	0.1540, 0.1014, 0.1183
4, 4, 4	-1.6250, 7.7500, -0.75	0.5837, 1.0895, 0.8879	0.0922, 0.1217, 0.1254
5, 5, 5	-0.8125, 10.625, 2.375	0.7196, 1.1484, 0.9246	0.1136, 0.1282, 0.1306
6, 6	2.3750, 3.625	1.0262, 1.1047	0.1621, 0.1560
7, 7	3.6250, 5.125	1.3672, 0.8900	0.2159, 0.1257
8, 8	5.1250, 9.4375	0.9062, 0.7077	0.1431, 0.1000
9,	6.8125,	0.9340,	0.1475,
10,	9.3750,	0.7333,	0.1158,

Tab. 3. Element positions and excitation amplitudes in a 23-element three-orientation antenna array obtained by BCS inversion algorithm [11] vs. our method.

Element Indices				Positions (λ)				Excitation amplitudes (w)							
Array (a, b, c, d)				Array (a, b, c, d)				Array (a, b, c, d) (BCS)				Array (a, b, c, d) (Our)			
1,	1,	1,	1	-12.1875,	-10.625,	-10.4375,	-9.5625	0.7358,	0.6999,	0.6413,	0.8625	0.1039,	0.0605,	0.0716,	0.1140
2,	2,	2,	2	-6.7500,	-1.3125,	-8.3750,	-7.6875	0.9060,	0.9059,	0.7063,	0.7072	0.1280,	0.0784,	0.0789,	0.0934
3,	3,	3,	3	-4.9375,	1.3125,	-5.1250,	-3.5625	1.2890,	0.8468,	1.0876,	1.1947	0.1821,	0.0732,	0.1215,	0.1578
4,		4,	4	-3.6875,		5.1250,	-2.3750	0.9672,		1.2757,	0.9623	0.1366,		0.1424,	0.1271
5,		5,	5	0.5000,		8.3750,	0.5625	0.8397,		1.0613,	0.8544	0.1699,		0.1185,	0.1129
6,			6	3.6875,			6.5625	1.0262,			0.6671	0.1186,			0.0881
7,			7	4.9375,			7.6875	1.0175,			0.7799	0.1437,			0.1030
8,				6.7500,				0.8331,				0.1177,			

Tab. 4. Element positions and excitation amplitudes in a 23-element four-orientation antenna array obtained by BCS inversion algorithm [11] vs. our method.

Element Indices	Positions (λ)	Excitation amplitudes (w)	
Array b	Array b	Array b (BCS)	Array b (Our)
1, 18	±12.5000	20.6823, 14.8158	4.3819, 3.1390
2, 17	±12.4375	20.1761, 14.1658	4.2747, 3.0012
3, 16	±12.1875	3.5416, 3.5069	0.7504, 0.7430
4, 15	±11.4375	0.9698, 0.2283	0.2055, 0.0484
5, 12	±8.7500	2.3088, 0.8985	0.4892, 0.1904
6, 11	±3.9375	1.3389, 1.0416	0.2837, 0.2207
7, 10	±2.9375	0.7729, 1.5217	0.1638, 0.3224
8, 9	±1.9375	0.9379, 0.4899	0.1987, 0.1038
13, 14	9.6875, 10.5625	1.3023, 1.0206	0.2759, 0.2162

Tab. 5. Element positions and excitation amplitudes in a 18-element one-orientation antenna array obtained by BCS inversion algorithm [11] vs. our method.

Element Indices		Positions (λ)		Excitation amplitudes (w)			
Array (a, b)		Array (a, b)		Array (a, b) (BCS)		Array (a, b) (Our)	
1,	1	-4.2500,	-10.750	2.5745,	2.0409	0.1818,	0.3057
2,	2	-1.4375,	-10.250	1.4217,	1.7936	0.1004,	0.2687
	3		-5.4375		2.4323		0.3644
	4		-4.3125		1.1876		0.1779
	5		-2.3750		1.8990		0.2845
	6		-1.4375		1.8245		0.2733
	7		2.3750		1.6808		0.2518
	8		4.3125		1.7988		0.2695
	9		10.250		2.0026		0.3000

Tab. 6. Element positions and excitation amplitudes in a 11-element two-orientation antenna array obtained by BCS inversion algorithm [11] vs. our method.

Element Indices			Positions (λ)			Excitation amplitudes (w)					
Array (a, b, c)			Array (a, b, c)			Array (a, b, c) (BCS)			Array (a, b, c) (Our)		
1,	1,	1	-3.875,	1.6250,	-2.8125	2.4616,	2.0282,	3.2427	0.2459,	0.1754,	0.2290
2,	2,	2	1.0000,	4.6250,	8.7500	1.4886,	2.0513,	2.5409	0.1487,	0.1774,	0.1794
3,	3		3.8750,	10.250		2.5571,	2.4902		0.2554,	0.2154	
4			6.7500			1.5011			0.1499		

Tab. 7. Element positions and excitation amplitudes in a 9-element three-orientation antenna array obtained by BCS inversion algorithm [11] vs. our method.

Element Indices	Positions (λ)	Excitation amplitudes (w)	
Array (a, b, c, d)	Array (a, b, c, d)	Array (a, b, c, d) (BCS)	Array (a, b, c, d) (Our)
1, 1, 1, 1	-8.9375, -10.1250, -5.5625, -9.6250	1.3440, 1.9919, 2.0703, 2.7822	0.1162, 0.1989, 0.1034, 0.1965
2, 2 2	-6.8125, -4.4375 -2.1875	1.4221, 2.4865 2.6819	0.1230, 0.2483 0.1894
3, 3	-3.6875, 4.4375	1.6143, 2.9534	0.1396, 0.2950
4	7.3750	1.4390	0.1437

Tab. 8. Element positions and excitation amplitudes in a 10-element four-orientation antenna array obtained by BCS inversion algorithm [11] vs. our method.

4. Conclusion

The proposed APS algorithm based on reweighted ℓ_1-norm minimization and array orientation diversity is demonstrated to be effective in reducing array elements, suppressing the sidelobe, and reducing the energy consumption to some extent. The sensitivity and robustness of the proposed design tool in real-life application will also be considered in our further work.

Acknowledgements

This work was supported in part by the National Natural Science Foundation of China under grant 60772146, the Key Project of Chinese Ministry of Education under grant 109139. The first author is specially supported in part by the Special Fund of Central Colleges Basic Scientific Research Operating Expenses under grant E022050205 as well as the CSC under grant No. 2011607043. We would like to thank Z. L. Zhang for the permission to use the code of T-SBL, which is effective when the measurement matrix is highly coherent.

References

[1] KUMAR, B., BRANNER, G. Design of unequally spaced arrays for performance improvement. *IEEE Transaction on Antennas and Propagation*, 1999, vol. 47, no. 3, p. 511 - 523.

[2] LIU, Y., NIE, Z., LIU, Q. Reducing the number of elements in a linear antenna array by the matrix pencil method. *IEEE Transaction on Antennas and Propagation*, 2008, vol. 56, no. 9, p. 2955 - 2962.

[3] YE J., PANG, W. Immune algorithm in array-pattern synthesis with side lobe reduction. In *Proceedings of the 6th International Conference on Microwave and Millimeter Wave Technology.* Nanjing (China), 2008, vol. 3, p. 1127-1130.

[4] LIU, Y., LIU, Q., NIE, Z. Reducing the number of elements in the synthesis of shaped-beam patterns by the forward-backward matrix pencil method. *IEEE Transaction on Antennas and Propagation*, 2010, vol. 58, no. 2, p. 604 - 608.

[5] BALANIS, C. *Antenna Theory: Analysis and Design*. 3rd ed. New York: Wiley, 2005.

[6] CHEN, K., YUN, X., HE, Z., HAN, C. Synthesis of sparse planar arrays using modified real genetic algorithm. *IEEE Transaction on Antennas and Propagation*, 2007, vol. 55, no. 4, p. 1067 - 1073.

[7] CHEN, Y., YANG, S., NIE, Z. The application of a modified differential evolution strategy to some array pattern synthesis problems. *IEEE Transaction on Antennas and Propagation*, 2008, vol. 56, no. 7, p. 1919 - 1927.

[8] KURUP, D., HIMDI, M., RYDBERG, A. Synthesis of uniform amplitude unequally spaced antenna arrays using the differential evolution algorithm. *IEEE Transaction on Antennas and Propagation*, 1999, vol. 47, no. 3, p. 511 - 513.

[9] KHODIER, M. M., CHRISTODOULOU, C. G. Linear array geometry synthesis with minimum side lobe level and null control using particle swarm optimization. *IEEE Transaction on Antennas and Propagation*, 2005, vol. 53, no. 8, p. 2674 - 2679.

[10] WANG, F., BALAKRISHNAN, V., ZHOU, P. Y., CHEN, J., YANG, R., FRANK, C. Optimal array pattern synthesis using semidefinite programming. *IEEE Transaction on Signal Processing*, 2003, vol. 51, no. 5. p. 1172 - 1183.

[11] ZHANG, W. J., LI, L., LI, F. Reducing the number of elements in linear and planar antenna arrays with sparseness constrained optimization. *IEEE Transaction on Antennas and Propagation*, 2011, vol. 59, no. 8, p. 3106 - 3111.

[12] CHEN, H., WAN, Q., HUANG, A. M., WANG, J. Array pattern synthesis via sparse weight vector finding and convex programming. In *Proceedings of the 5th International Conference on Wireless Communications, Networking and Mobile Computing.* Beijing (China), 2009, p. 1 - 4.

[13] CANDES, E. J., WAKIN, M. B., BOYD, S. Enhancing sparsity by reweighted ℓ_1 minimization. *Journal of Fourier Analysis and Applications*, 2008, vol. 14, no. 5 - 6, p. 877 - 905.

[14] BOYD, S., VANDENBERGHE, L. *Convex Optimization*. Cambridge (UK): Cambridge University Press, 2003.

[15] CANDES, E., WAKIN, M. An introduction to compressive sampling. *IEEE Signal Processing Magazine*, 2008, vol. 25, no. 2, p. 21 - 30.

[16] CANDES, E., ROMBERG, J., TAO, T. Robust uncertainty principles: Exact signal reconstruction form highly incomplete frequency information. *IEEE Transactions on Information Theory*, 2006, vol. 52, no. 2, p. 489 - 509.

[17] CANDES, E., ROMBERG, J., TAO, T. Stable signal recovery from incomplete and inaccurate measurements. *Communications on Pure and Applied Mathematics*, 2006, vol. 59, no. 8, p. 1207 - 1223.

[18] DONOHO, D. Compressed sensing. *IEEE Transactions on Information Theory*, 2006, vol. 52, no. 4, p. 5406 - 5425.

[19] CANDES, E., TAO, T. Decoding by linear programming. *IEEE Transactions on Information Theory*, 2005, vol. 51, p. 4203 - 4215.

[20] CANDES, E., TAO, T. Near-optimal signal recovery from random projections: Universal encoding strategies? *IEEE Transactions on Information Theory*, 2006, vol. 52, no. 12, p. 5406 - 5425.

[21] STURM, J. F. *Using SeDuMi 1.02, a Matlab Toolbox for Optimization over Symmetric Cones*. [Online] Cited 2001-10-31. Available at: http://www.optimizationonline.org/DB HTML/2001/10/395.html.

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