COMPUTER-ASSISTED SEARCH OF THE ROOTS OF THE POLYNOMIAL

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Abstract: This article describes finding roots of polynomials, specifically aimed at finding real roots of a polynomial. Computer Support focuses on the use of generally available programs such as spreadsheets, portals to solve mathematical problems and freeware applications. Attention is devoted to a comparison of commercial and open source software.

Keywords: polynomials, Computer Support, freeware, open source.

1. INTRODUCTION

In mathematics, a polynomial is an expression consisting of variables (or indeterminates) and coefficients, that involves only the operations of addition, subtraction, multiplication, and non-negative integer exponents. An example of a polynomial of a single indeterminate (or variable),

x, is $f(x) = x^2 - 4x + 7$, which is a quadratic polynomial.

An example in three variables is

$g(x,y,z) = x^3 + 2xyz^2 + yz + 1$. [1]

Finding roots of a polynomial is a topic that is discussed in Mathematics in their first year at the military five-year study at the Faculty of Military Leadership at the University of Defense, at the civilian branch in Mathematics I. and also in learnings at other universities, like is University of technology.

Polynomials appear in a wide variety of areas of mathematics and science. For example, they are used to form polynomial equations, which encode a wide range of problems, from elementary word problems to complicated problems in the sciences; they are used to define polynomial functions, which appear in settings ranging from basic chemistry and physics to economics and social science; they are used in calculus and numerical analysis to approximate other functions. In advanced mathematics, polynomials are used to construct polynomial rings and algebraic varieties, central concepts in algebra and algebraic geometry. Generally polynomial is an expression of the form:

$$p(x) = \sum_{k=0}^{n} a_k x^k = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n ,$$

where $a_n \neq 0$. The numbers $a_0, a_1, \ldots, a_n$ are called polynomial coefficients. [2]

The number α is called a root of a polynomial p (x), if valid $p(\alpha) = 0$

Polynomial $p(x) = \sum_{k=0}^{n} a_k x^k$ can be written as

$$p(x) = (\ldots((a_n x + a_{n-1})x + a_{n-2})x + \ldots + a_1)x + a_0$$

This notation can be used to calculate polynomial in point of x in the procedure, which is known as Horner.

If you type

$c_n = a_n$,

$c_{n-1} = c_n x + a_{n-1}$,

$c_{n-2} = c_{n-1} x + a_{n-2}$,....

$c_0 = c_1 x + a_0$,

then last number c_0 is justly the value of a polynomial p (x) at x [1,4].

2. A BRIEF DESCRIPTION OF METHODS OF SOLUTION

Algebraic equation of degree n is called the equation

$$P_n(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-1} x + a_n = 0,$$

and where $a_0, a_1, \ldots, a_n \in R$, $a_0 \neq 0$, $x \in \mathbf{C}$, $n \in \mathbf{N}$.

Solution (roots) algebraic equation $P_n(x) = 0$ is any number c, which is root of the polynomial $P_n(x)$. Algebraic equation of degree n therefore has exactly n roots (in the field of complex numbers regarding multiplicity).

There is not a formula analogous to the quadratic equation for the equation of degree at least 5, so in a simplified perspective solution can be divided into methods:

- numerical,
- graphics.

Graphically, we can only determine the real roots, and only approximately. $P_n(x) = 0$ adjust the shape of f(x) = g(x). The real roots of the equation $P_n(x) = 0$ They are then equal to the x-coordinates of the intersections curve y = f (x) and y = g (x).

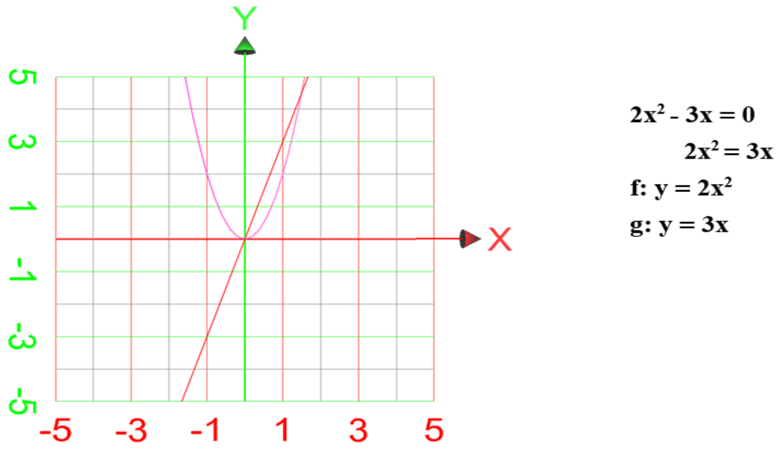

Figure 1: Example of graphical solution

With respect to iteration theory there are possibilities to represent functions using oriented functional (discrete) graphs [2,3].

Numerical Solution (Using formulas to decomposition product of root factors, using the Horner scheme, approximate numerical methods) may provide a more accurate result. We start from a few basic phrases that can be summarized as to cut a long story these basic findings:

- The degree of the polynomial P (x) with the root a1 can be reduced by dividing (x-a1).
- All the roots of the polynomial from set of all integers divides the absolute member. (With divisibility defined only for integer (generally an integral domain), hence an integral domain must be integer integer coefficients of the polynomial and the coefficient of the highest powers must be 1).

The number of positive roots is at most equal to the number of sign changes in the sequence of coefficients or differ by an even number, the number of negative roots is equal at most equal number of hits unsigned or less on an even number. The difference between the positive and negative roots is a number of complex roots- always even.

There is a whole range of terms and theorems associated with the search for roots, such as separation of roots, root approximations, interpolation, least squares methods and methods for special shapes algebraic equations. Given the scope of the article and determined focus on software support is more content devoted to the transfer of Horner's scheme into a spreadsheet, a modification example of freeware solutions and using mathematical portal.

3. A BRIEF DESCRIPTION OF SPREADSHEET SOLUTION

Unlike simple applications on the net, counting with one line objective was to gradually create a table reflecting the basic rules for the number of roots, with an emphasis on real roots. Necessity copying of rows and columns of formula led to a different absolute addressing of cells in the formula with the same result.

Original Coefficients	10	-30	-10	70	0	-40						70	The number of empty cells	4
Greatest common divisor	10	30	10	70	70	40	70	70	70	70	Greatest common divisor	10		
Roots / Coefficients	1	-3	-1	7	0	-4	0	0	0	0		Results		
-1	1	-4	3	4	-4	0	0	0	0	0		True		5
-1	1	-5	8	-4	0	0	0	0	0	0		True		6
2	1	-3	2	0	0	0	0	0	0	0		True		7

Figure 2: Image table of Horner scheme – Libre Office Calc

To simplify the equations used formula for the greatest common divisor - =GCD(B3;C3;D3;E3;F3;G3;H3;I3;J3;K3). Custom shorted coefficients provide condition =IF((COUNTBLANK(C2)=1);A1;C2/M3). The condition, which could be even better for the announcement of the results should shape: =IF(O5<(10-O2);"True";"False"). The formula within a table looks like this: =($A5*B5)+C4. Interesting option would be to automatic testing roots when applying of rules and complement the complex roots. It is clear that this possibility of referring to the fact that the set of real numbers is dense - each subset is the same size as the set of real numbers. On the other hand, for practical reasons, in the teaching are used the equations whose shape is "nice" and roots come "nice". No doubt, because math teachers are kind and welcoming people who love their students. This would allow to create effective application in narrowing down set of roots, usable in exercises and to control students. The condition of a localized version of Excel looks like this: =KDYŽ(B2=0;M2;ABS(B2)). The formula for the greatest common divisor -=GCD(B3;C3;D3;E3;F3;G3;H3;I3;J3;K3) is the same. Table between the two versions can be transferred only as numbers.

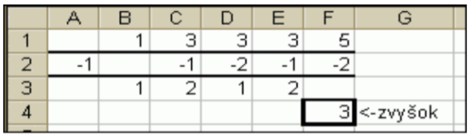

Figure 3: Example of MS Excel solution from Internet[8]

Image shows an example of the Horner scheme in Excel on the net. [8]

4. A FREEWARE SOLUTION

From the portal Slunecnice.cz simple application can be downloaded as a zip file. This application just unzip example, using freeware 7-zip. The application can also be used by flashdisc.

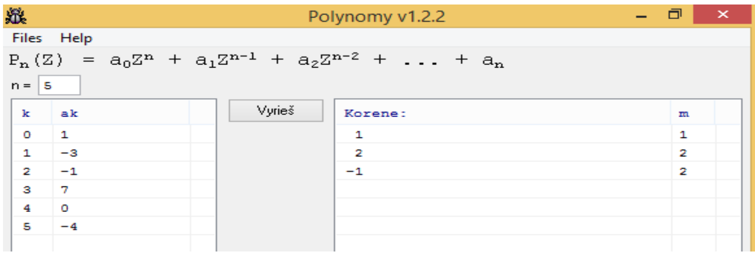

Figure 4: Freeware „Polynomy“ [5]

The polynomials is freeware for calculating the roots of the polynomial equation with real coefficients.

Main features:

- calculate the roots of a polynomial with real coefficients and determine their multiplicity
- the maximum degree of the polynomial 100
- calculation error root | Error | <1E-5 (at least five significant digits).

E.g. I need to find out the roots of the equation $2Z^2 + 8Z - 5 = 0$. This is a polynomial grade 2, so the first step is required in the box next to "n" to write degree polynomial number 2. The next steps should be gradually set values of individual coefficients with respect to the power variable. Coefficients are numbered from 0 index, so the first (A0) is the coefficient of the first term with the highest power of the polynomial (A0 Z^2), so a0 coefficient is the number 2.

The coefficients in this equation are non-zero and have values of 2, 8, -5.

To solve the equation is the click on "Solve". The roots of the equation is displayed in the window, at the same time be copied to the clipboard so that they can be simply by hitting Shift-Insert further arbitrarily used (eg. Notepad).

The application is a useful tool, in theory, can be used from a flash drive. Windows 8. in author computer blocked the use of flash drive.

5. A PORTALS SOLUTION

Embedded image demonstrates how to use the portal http://www.wolframalpha.com in solving algebraic equations. First, select the mathematics of the tabs, then Algebra - solving equations. Now you only have to insert the equation in the prescribed form, may overwrite pattern. As a result, you are not only roots, and also chart polynomial decomposition, where you can see the multiplicity of roots.

For a fee, you can obtain the gradual solution. Using the portal is nice, but it requires Internet access.

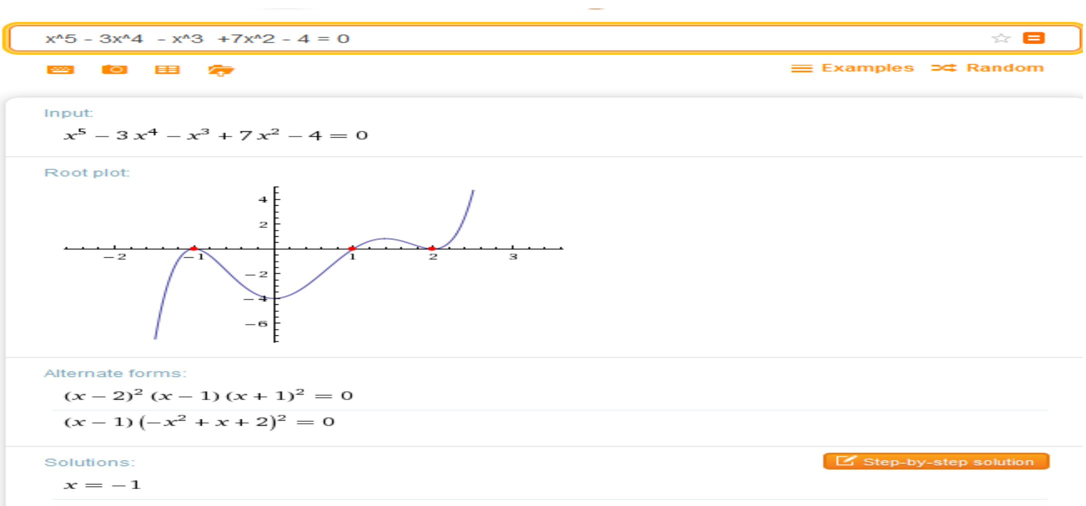

Figure 5: Portal Wolframalpha.com[6,7]

6. CONCLUSIONS

The article describes the possibilities of finding roots of polynomials using a computer. Scope of the article did not allow serious testing of the applications listed above, as well as more serious analysis of all aspects of the search for the roots of a polynomial. On the other hand, there student can find instructions for using a spreadsheet as a powerful tool for this purpose, and further inspiration for the use of various applications and Internet portals. The challenge may also be of use in the construction of mathematical logic conditional functions, allowing to solve not only other than math problems.

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