# Examination of Newton's Method Used for Indirect Frequency Offset Estimation

*Jakub DŽUBERA*

Dept. of Radio Electronics, Brno University of Technology, Purkyňova 118, 612 00 Brno, Czech Republic

xdzube00@stud.feec.vutbr.cz

**Abstract.** *This paper deals with the topic of an indirect carrier frequency offset estimation and elimination. The main goal is to modify a conventional method as an attempt to develop a different approach and then to compare the performance of the modified method with the performance of the conventional one. The conventional approach is here represented by the gradient optimization method called the steepest descent. It is the base for the modification which utilizes Newton's method for the indirect carrier offset estimation. Both algorithms are implemented as phase-locked loops in a model of communication system. The simulation is processed in Matlab.*

## Keywords

Carrier frequency recovery, carrier frequency offset estimation, carrier frequency offset elimination, carrier synchronization, Newton's method, optimization, phase-locked loop, PLL, steepest descent.

## 1. Introduction

Coherent demodulation is widely used in many communication systems for the received signal demodulation. However such technique requires the exact knowledge of carrier frequency waveform. If the receiver has knowledge about the exact carrier frequency waveform, the received signal may be demodulated correctly and the message signal may be recovered. The process of synthesis of carrier with the exact waveform is usually done in carrier frequency synchronization subsystems. For such task the knowledge of the exact value of carrier frequency and its phase is necessary. These parameters may be estimated from the received signal waveform and then the estimates may be used to correct the waveform of the synthesized carrier waveform. PLLs (Phase-Locked Loops) are widely used in synchronization subsystems. As a base the PLL for indirect frequency offset estimation was chosen here. The default PLL implements mostly used optimization method called the steepest descent. The algorithm representing the steepest descent method is implemented as a recursive algorithm. This algorithm eliminates the carrier frequency offset of synthesized waveform indirectly via received signal phase offset estimation. The examined approach is based on different optimization method called Newton's method. It is also implemented as a PLL system and it utilizes a modified recursive algorithm.

Both methods and the representing algorithms are implemented and simulated in Matlab environment. They form subsystem parts of the communication system model. It is the core of the simulation called Simulation Engine as shown at Fig. 1 [1]. The box named Preprocessor includes the partial subsystems of a typical communication system. They are the message signal generation, the modulation, the AWGN (Additive White Gaussian Noise) transmission channel and the systems on the receiver's side such as squaring and bandpass filtration. The message signal is the form of N-level binary signal. The main task of the Preprocessor is to create a signal as it can be found at the input of the receiver. The main task of the Postprocessor is to demodulate the received signal using the newly synthesized carrier frequency waveform and recover the message signal.

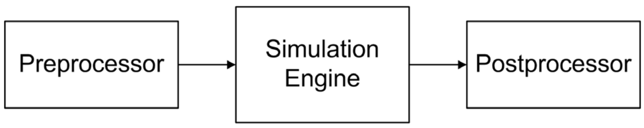


**Fig. 1.** Simulation architecture.

## 2. System Model

The system model is derived from the signal presentation in the input of the Simulation Engine which represents the synchronization subsystem. The output part of the Preprocessor actually stands for the input part of a usual receiver. This part processes squaring and bandpass filtering of the transmitted signal as it is pointed out in Fig. 2 [2].

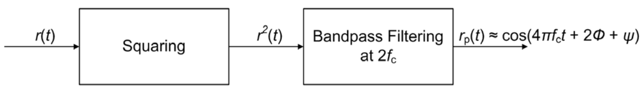


**Fig. 2.** Receiver's input blocks.

In case the input signal in the receiver's input is of the form

$$r(t) = s(t)\cos(2\pi f_c t + \phi) \tag{1}$$

where $r(t)$ is the received signal, $s(t)$ is the pulse modulated data signal, cos(.) part stands for the carrier waveform, $f_c$ is the frequency of the received signal and $\phi$ is the unknown phase offset added by the transmission channel; the output of the squaring block is

$$r^2(t) = s^2(t)\cos^2(2\pi f_c t + \phi)\,. \quad (2)$$

According to the identity $2\cos^2(x)=1+\cos(2x)$ the previous equation may be rewritten into the form

$$r^2(t) = \tfrac{1}{2}s^2(t)[1+\cos(4\pi f_c t + 2\phi)]\,. \quad (3)$$

The variable $s^2(t)$ may be considered as a constant sum of average values $s^2_{\mathrm{const}}$ and the time variation of this constant average $v(t)$

$$s^2(t) = s^2_{const} + v(t)\,. \quad (4)$$

Thus

$$\begin{aligned} r^2(t) = \tfrac{1}{2}[s^2_{const} &+ v(t) + \\ &+ s^2_{const}\cos(4\pi f_c t + 2\phi) + \\ &+ v(t)\cos(4\pi f_c t + 2\phi)] \end{aligned} \quad (5)$$

The bandpass filter with the central frequency at $2f_c$ passes only the pure cosine term. The output of the bandpass filter $r_p(t)$ (see Fig. 2) maybe approximately written as

$$\begin{aligned} r_p(t) &= \mathrm{BPF}\{r^2(t)\} \approx \\ &\approx \tfrac{1}{2}s^2_{const}\cos(4\pi f_c t + 2\phi + \psi) \end{aligned} \quad (6)$$

where $\psi$ is the phase shift added by a bandpass filter at the frequency $2f_c$. BFP stands for bandpass filtering. The variable $r_p(t)$ can be used to find the frequency and phase of the carrier, because $\psi$ is known at the receiver. The frequency of $r_p(t)$ component is twice the original carrier frequency and its phase is twice the original unknown phase $\phi$.

To develop an algorithm for frequency and phase tracking the main goal is to determine proper cost function. The base idea may be to modulate the pre-processed received signal $r_p(t)$ using a cosine wave of the known frequency $2f$ and phase $2\theta$. The cost function [2] is the form of

$$F(\theta) = \tfrac{1}{2}\mathrm{LPF}\{r_p(kT_s)\cos(4\pi f_0 kT_s + 2\theta)\} \quad (7)$$

where $r_p(kT_s)$ is a value of $r_p(t)$ sampled at the time $kT_s$, $f_0$ determines the carrier frequency generated by the receiver's local oscillator, which may differ from the received signal's carrier frequency $f_c$. The lowpass filter LPF removes the high frequency components and the down converted component can be corrected by changing $2\theta$. The value of $\theta$ that maximizes the down converted component is the same as the phase $\phi$ of $r_p(t)$. Assuming the phase shift $\psi$ of the bandpass filter is suppressed in preprocessing.

Assuming a small stepsize, the derivative of the previous cost function with respect to $\theta$ at the time instant $k$ can be approximated as

$$\begin{aligned} &\left.\frac{d\mathrm{LPF}\{r_p(kT_s)\cos(4\pi f_0 kT_s + 2\theta)\}}{d\theta}\right|_{\theta=\theta[k]} \approx \\ &\approx \mathrm{LPF}\left\{\left.\frac{dr_p(kT_s)\cos(4\pi f_0 kT_s + 2\theta)}{d\theta}\right|_{\theta=\theta[k]}\right\} = \\ &= \mathrm{LPF}\{-r_p(kT_s)\sin(4\pi f_0 kT_s + 2\theta[k])\}. \end{aligned} \quad (8)$$

Using the generic formulation of the recursive steepest descent algorithm

$$x[k+1] = x[k] - \alpha \left.\frac{dF(x)}{dx}\right|_{x=x[k]} \quad (9)$$

with respect to the fact that the synchronization system is implemented as a dual PLL system as shown at Fig. 3, the resulting algorithm is the form of

$$\begin{aligned} \theta_1[k+1] &= \theta_1[k] - \alpha_1 \mathrm{LPF}\{r(kT_s)\cdot \\ &\quad\cdot \sin(4\pi f_0 kT_s + 2\theta_1[k])\}, \\ \theta_2[k+1] &= \theta_2[k] - \alpha_2 \mathrm{LPF}\{r(kT_s)\cdot \\ &\quad\cdot \sin(4\pi f_0 kT_s + 2\theta_1[k] + 2\theta_2[k])\}. \end{aligned} \quad (10)$$

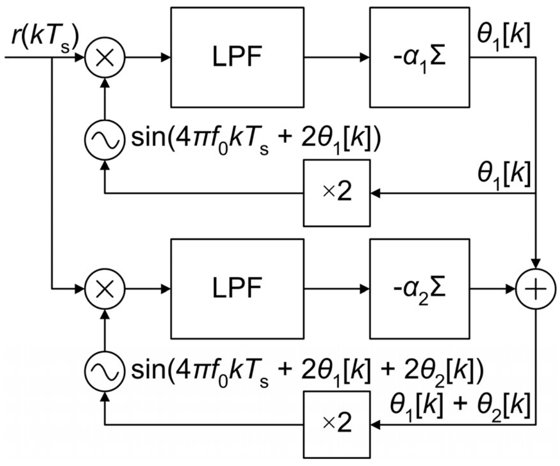


**Fig. 3.** Simulated PLL system.

The presented dual PLL system is developed from a simple PLL. The first formula in (10) corresponds to the top loop and second formula in (10) corresponds to the bottom loop. The idea is based on the fact that the PLL phase estimates converge to a line with a slope proportional to the difference between the carrier frequency of the received signal and the carrier frequency of the receiver's local oscillator; particularly to $2\pi(f_0 - f_c)$. This arises in case that the frequency of the local oscillator is $2f_0$ and the phase estimates $2\theta_1$. The top loop synthesizes the signal with the correct frequency for the bottom loop.

According to the generic formulation of the recursive Newton's algorithm

$$x[k+1] = x[k] - \alpha \left. \frac{F(x)}{\frac{dF(x)}{dx}} \right|_{x=x[k]} \quad (11)$$

the synchronization system utilizing PLL is described by the following formulas [6]

$$\begin{aligned} \theta_3[k+1] &= \theta_3[k] - \\ &- \alpha_3 \frac{\mathrm{LPF}\{r(kT_s)\sin(4\pi f_0 kT_s + 2\theta_3[k])\}}{\mathrm{LPF}\{r(kT_s)\cos(4\pi f_0 kT_s + 2\theta_3[k])\}} \\ \theta_4[k+1] &= \theta_4[k] - \\ &- \alpha_4 \frac{\mathrm{LPF}\{r(kT_s)\sin(4\pi f_0 kT_s + 2\theta_3[k] + 2\theta_4[k])\}}{\mathrm{LPF}\{r(kT_s)\cos(4\pi f_0 kT_s + 2\theta_3[k] + 2\theta_4[k])\}} \end{aligned} \quad . (12)$$

The variables $\theta_{1,2,3,4}$ stand for the phase estimates, $\alpha_{1,2,3,4}$ stand for iterative stepsizes, $f_0$ determines the carrier frequency generated by the receiver's local oscillator, $T_s$ the sampling period and LPF the low pass filtering. Fig. 4 shows the trend of the cost function.

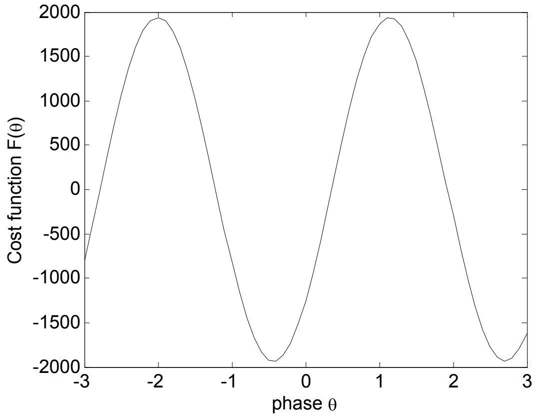


**Fig. 4.** Cost function.

## 3. Implementation and Simulation

Both methods are implemented in Matlab using the developed equations. They create the Simulation Engine. The following simulation results are the example results made for one set of parameters. The transmitted signal is modulated 4-PAM signal and it is destroyed in an AWGN channel. The noise level is set to the value of 10 dB. After transmission the modulated signal is processed in a squaring block and a bandpass filter block which simulate the input blocks of receiver. All mentioned steps are made in the Preprocessor block.

The following steps are made in the Simulation Engine. It is mainly the carrier frequency offset correction based on the phase estimation and synthesis of the new carrier frequency waveform with the corrected frequency and phase. The Postprocessor demodulates the received signal and presents the simulation results.

In Fig. 5 the modulated signal affected by AWGN is displayed. The waveform of the same signal processed by squaring block and bandpass filter is displayed as well.

The difference between the carrier frequency of the transmitted signal $f_c$ and the frequency $f_0$ of the local oscillator in the receiver is chosen to be 1 Hz. This value stands for the carrier frequency offset to be corrected. The "unknown" phase of the received signal is set to be -2 rad and the values of the stepsizes $\alpha_1$ and $\alpha_2$ are set to be $5.10^{-6}$ and $5.10^{-9}$. The initial phase values $\theta_1(0)$ and $\theta_2(0)$ are chosen to be 0.

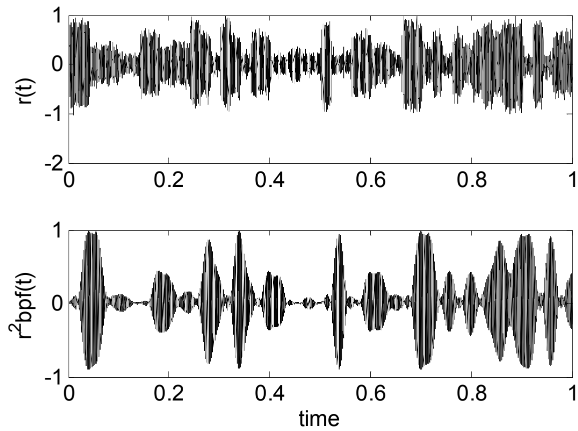


**Fig. 5.** Modulated signal affected by noise (top) and identical signal processed by receivers input blocks.

Fig. 6 and Fig. 7 show the estimated phase offsets $\theta_1$ and $\theta_2$ which result in a frequency offset elimination. These estimated phase offsets are used in the following operation for the synthesis of a carrier frequency used in the demodulation process. The stepsizes are chosen with respect to get the best results.

Fig. 8 displays the time dependency of the carrier frequency offset of the new synthesized carrier in the receiver part of the simulated communication system compared to the original carrier waveform used for modulation in the Preprocessor part.

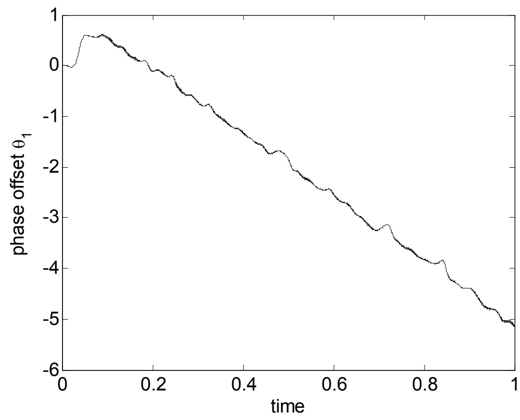


**Fig. 6.** Phase offset $\theta_1$ of the top loop.

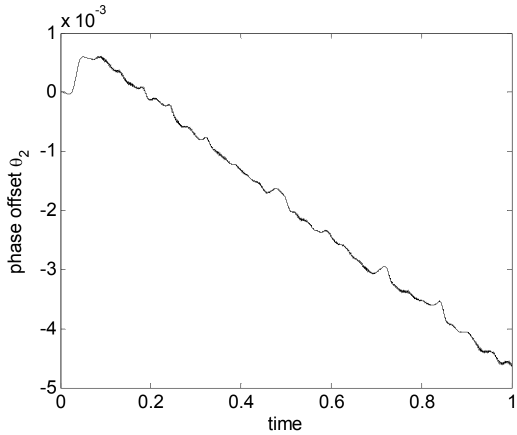


**Fig. 7.** Phase offset $\theta_2$ of the bottom loop.

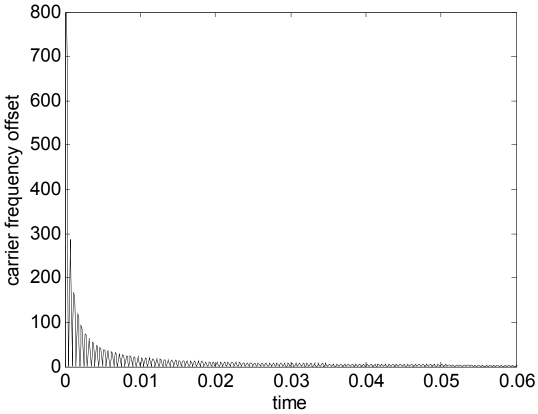


**Fig. 8.** Carrier frequency offset in time (steepest descent).

For the simulation of Newton's method the parameters are set as follows. The initial phase offsets $\theta_3(1)$ and $\theta_4(1)$ are set to 1 rad and the iterative stepsizes $\alpha_3$ and $\alpha_4$ are set to $9.10^{-6}$ and $5.10^{-6}$. The difference between the carrier frequency of the transmitted signal $f_c$ and the frequency $f_0$ of the local oscillator in the receiver is again set to 1 Hz. The "unknown" phase of the received signal remains -2 rad.

The time dependency of the estimated phase offset $\theta_3$ is presented at Fig. 9. The algorithm strictly keeps the descending trend which is proportional to the difference between the frequency of receiver's synthesized carrier and the frequency of the received signal carrier. Fig. 10 shows the time dependency of the estimated phase offset $\theta_4$.

Fig. 11 represents the time dependency of the carrier frequency offset of the carrier synthesized using Newton's method. Fig. 12 shows comparisons of the original message signal and the recovered message signal when the synthesized carrier is applied for the coherent demodulation. A slight time shift is observable. When comparing both waveforms recovered by the steepest descent method and Newton's method they appear to be very similar. The small differences can be noticed mainly in the initial time period when both of the algorithms estimate the phase offset inaccurately.

Fig. 13 shows the cross-correlation function of the original message signal and the recovered signals. The values are scaled to the maximal value. The function expresses the similarity between the original and recovered signals. Again, it can be seen that both methods give nearly identical results.

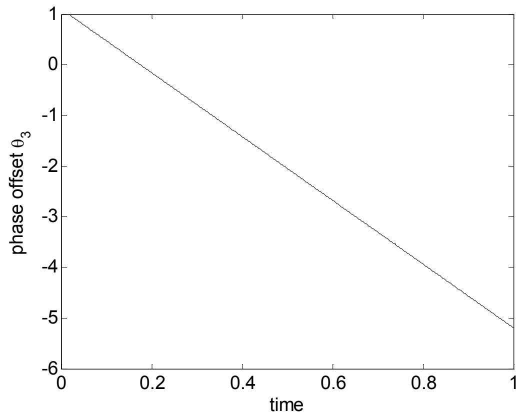


**Fig. 9.** Phase offset $\theta_3$.

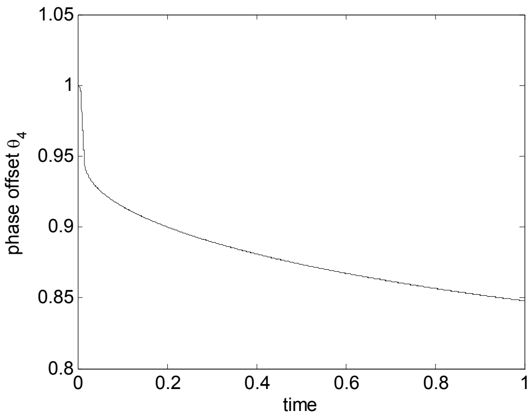


**Fig. 10.** Phase offset $\theta_4$.

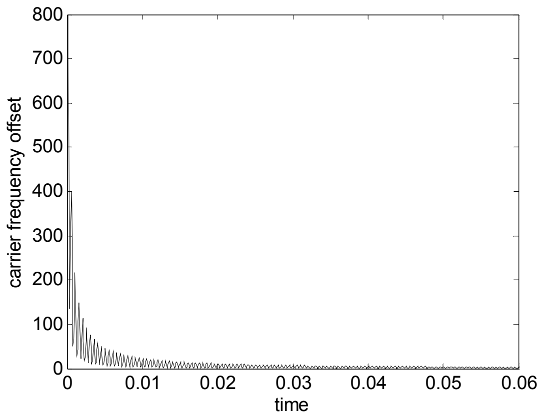


**Fig. 11.** Carrier frequency offset in time (Newton's method).

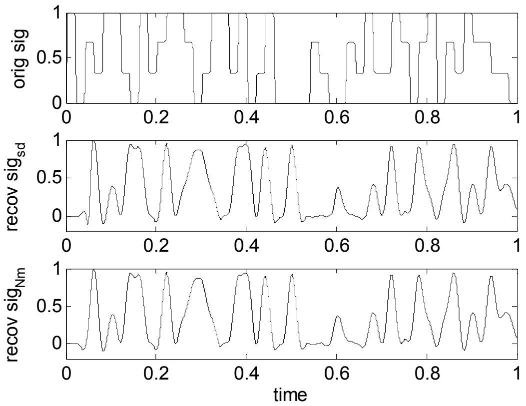


**Fig. 12.** The original signal (top), the recovered signal using the steepest descent (middle), the recovered signal using Newton's method (bottom).

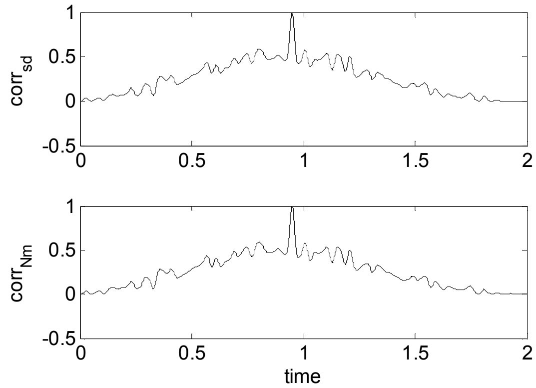


**Fig. 13.** Cross-correlation of the original message signal and the recovered signals (top – the steepest descent, bottom – Newton's method).

## 4. Conclusions

The examined PLL system is very reliable and provides accurate synthesis of both the carrier frequency and phase. The main disadvantage is the sensitivity to initial settings and the setting of stepsizes $\alpha_1$, $\alpha_2$ and $\alpha_3$, $\alpha_4$. High stepsize values lead to inaccurate phase estimation and incorrect demodulation. Reversely, extremely small values are inefficient and slow down the estimation process because more iterations are needed to reach the accurate value of the phase offset. It is found out that for the rough phase estimation the top PLL is sufficient when utilizing the steepest descent approach. Newton's method in comparison with the steepest descent method is even much more sensitive to initial setting of initial phase offset and stepsize. Reversely, the steepest descent method accepts more significant changes in both the received signal's frequency and phase. It is more flexible and thus can produce carrier frequency waveform with accurate parameters. If the initial parameters are chosen unthoughtfully for Newton's method the system is not able to estimate the phase offset accurately. The system based on the steepest descent approach performs better over the environments with significant parameter changes. However, if the initial parameters are chosen thoughtfully the Newton's method can reduce the carrier frequency offset faster in comparison with the steepest descent method. This fact can be seen when observing Fig. 8 and Fig. 11. Both methods are quite robust when operating in an AWGN channel. The performance of both methods is not significantly reduced by the noise. This feature is also supported by the proper method of signal preprocessing while squaring and bandpass filtering is utilized. The waveform of the recovered message signal can be determined by changing the lowpass filter parameters. The problem of correct symbol determination is then a problem of symbol timing synchronization. This theme is to be a future work.

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## About Authors...

**Jakub DŽUBERA** was born in Kyjov, Czech Republic, in 1980. He received his M. Sc. degree in Electronics and Communications at the Brno University of Technology in 2004. At present, he is a Ph. D. student at the Dept. of Radio Electronics at BUT.