REPORT ON DOCTORAL THESIS

Title Geometric algebras in switched systems control
Author Anna Derevianko

The main concern of this thesis is an application of the geometric algebra of conics (GAC) to the control of specific dynamical switched systems. This goal dictates the specificity of the systems analyzed, which are (two) linear switched systems whose phase trajectories (of both subsystems) are conics. The power of GAC is demonstrated in finding the switch points, which are the intersections of the conics, without solving quadratic equations. Compared to standard numerical methods, this approach eliminates an error and leads to a control with a minimum number of switches. The thesis also includes a classification of the systems in question and their controllability, regardless of whether the trajectories are conics or not.

Both geometric algebras and dynamical switched systems are large and active areas of contemporary research. As they are a priori unrelated, a successful application of one to a relevant problem of the other is an admirable achievement. In this respect, the submitted work hits the target and meets the basic requirement of providing original results. The application is interesting both conceptually and in its technical implementation. The results in the thesis include and extend those of published articles [31, 38] coauthored by the candidate.

Regarding the presentation of results, the thesis is written in an understandable but not ideal way. A certain dissatisfaction is already apparent in the introductory chapters. They should mainly serve to claim the territory and give an overview of the current state of knowledge, so that the non-expert reader can orient himself in the field without having to browse through external sources. In this respect, chapter 2 on switched systems does this job more loosely than chapter 1 on geometric algebras. Many key ideas are hidden in formal definitions without any friendly commentary, while relatively trivial notions occupy an unproportional amount of space.

Chapter 3, where the two areas meet and where the main results are presented, also contains oddities. For instance, the only statement promoted in the whole chapter is Theorem 3.5.1, which declares the controllability of switched systems of type center–saddle. An analogous but much more complicated case of center–center type does not lead to any theorem, although everything needed is covered there. To be clear, the discussion of type center–center (sections 3.1–3.3) is quite thorough, it culminates in an algorithm for finding the control, it includes illuminating examples and a comparison with numerical methods, and it should be considered the core of the work. The only doubt is about the guidelines the author followed in compiling the material.

The analysis of cases that are not controllable or controllable under certain conditions often seems to be based on particular examples or figures. This is not very convincing and can easily be based on a better foundation. As before, I do not question the conclusions, only their justification and presentation.

To conclude, the thesis is not optimally written, so the information provided is not well balanced. However, it contains original results of the candidate and meets the standards for a doctoral thesis in the field of applied mathematics. I recommend that Anna Derevianko be awarded the PhD degree.

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