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A Comparison of Methods Recovering Signals from Non-Uniform Samples

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Abstract: Band-limited signals play a vital role in signal acquisition, transmission, processing and storage for decades. However, their recovery from non-uniformly spaced samples is still subject to discussion. There are multiple authors advancing different approaches and the user has to deal with an abundance of literature. Therefore, this paper offers a brief theoretical review of the most promising reconstruction methods. We also consider the importance of regularisation if the signal samples are noisy, which, surprisingly, seems to be neglected by many authors. Subsequently, the suitability of the methods is illustrated using an example of a real-world signal. Assessment of accuracy and speed of the methods is repeated for varying noise levels in signal amplitude, and for increasing degree of non-uniformities in sampling time. Cubic spline methods and sinc function reconstruction offer the best compromise between speed and accuracy for the concrete signal tested in this paper.

Keywords: bandlimited signal, resampling, regularisation, reconstruction

1. INTRODUCTION

Band-limited signals exhibit many interesting properties which render them interesting to mathematicians and signal processing engineers. One of their most useful features is the possibility to perfectly reconstruct their amplitude for any time instant using only their discrete samples. This is the famous Whittaker-Nyquist-Kotelnikov-Shannon theorem and it is undoubtedly the main reason why these signals found such an important role in signal recording, transmission, processing and storage.

One of the less known facts, however, is that the discrete samples do not have to be acquired at uniformly spaced time instants. Indeed, signals may be sampled non-uniformly and the perfect reconstruction may still be possible; the sufficient condition is that the *average* sampling period, $\overline{T}_{\mathrm{s}}$, is smaller than half of the reciprocal value of the highest frequency, $f_{\max}$, in the signal spectrum.

$$\overline{T}_{\mathrm{s}} < \frac{1}{2 f_{\max}} \tag{1}$$

A detailed analysis of this requirement is to be found in the article by Landau (1967). The constraint is, indeed, identical to that of uniform sampling.

This paper focuses on the reconstruction problem of continuous signals from their non-uniformly spaced discrete samples. We use the term *reconstruction* because our results can be applied to extrapolation and interpolation without significant changes in the proposed methods. We also use this term since in most cases, we will avoid *interpolation*, and will allow a small deviation from the known samples instead. This is the well-known process of *regularisation* which mitigates the negative impact of noise on the quality of reconstruction, as discussed in the books by Hastie (2010); de Villiers (2017).

It seems surprising that many authors either neglect the importance of regularisation by using a constant and very small regularisation parameter, for instance Ignjatović (2018a,b); Senay (2012); or, in the worse case, ignore it altogether, e.g. Devasia (2013). This is, of course, because these articles focus on the models. Their presentation would be significantly complicated if noise was admitted. The conclusions drawn from such examples, however, may be biased and quite distinctive from the results which can be observed in practice. All our numerical experiments produced useless oscillatory reconstruction when noise was admitted and regularisation was neglected.

Therefore, this paper aims to compare the most widely used signal models and to test their performance on signals which could occur in practical digital signal processing. We include regularisation based on the Morozov discrepancy principle which requires an estimate of the noise level in the amplitude of the known samples. The reader may, however, employ other methods that suit their application the most; for instance, the L-curve method, which provides an automated means of noise-level estimation.

The paper is divided as follows. Section 2 contains the statement of the problem which we are aiming to solve. We hope that this will avoid unnecessary ambiguities which

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might otherwise arise. Section 3 provides a brief theoretical review of different signal models. Subsequently, Section 4 discusses the estimation of model coefficients and methods of their regularisation. Section 5 is the last and provides numerical experiments. The methods are compared in terms of their speed and precision of signal recovery.

2. STATEMENT OF THE PROBLEM

We will model a continuous signal using a complex-valued function $f(t)$ of time $t \in \mathbb{R}$. In this paper, we will assume that the original signal, $f(t)$, is band-limited, with band limit Ω, i.e. its spectrum has a finite support and

$$F(\omega) = 0 \quad \text{if} \quad |\omega| < \Omega. \tag{2}$$

We also assume that the signal $f(t)$ is sampled at discrete time instants, $t_1 < t_2 < t_3 < \cdots < t_K$. These samples may be stacked in a vector of signal observations

$$\mathbf{g} = [f(t_1)\ f(t_2)\ f(t_3)\ \ldots\ f(t_K)]^{\mathsf{T}} \tag{3}$$

containing K samples.

The goal of the reconstruction is to use these samples $\mathbf{g}$ in forming the approximation, $f_N(t)$, of the original signal, $f(t)$, which will be as close to the original signal as possible. As the distance, we chose the $\boldsymbol{L}_2$ distance. Its square corresponds to the well-known squared error

$$\|f - f_N\|^2 = \int_{-\infty}^{\infty} |f_N(t) - f(t)|^2 \,\mathrm{d}t, \tag{4}$$

which we aim to minimize. It is noteworthy that in a practical application, this error cannot be computed, because the continuous signal $f(t)$ is, of course, unknown.

The reader might find assumption (2) unnatural, but in such case, we refer him to the article by Slepian (1976). It contains an interesting discussion on whether the real-world signals should be considered band-limited, time-limited, or neither of these.

2.1 Features of a "good" reconstruction method

Noise immunity. Some authors completely ignore the possibility that signal samples may be corrupted by some level of noise. In fact, it is hard to think of any real situation in which we sample a physical quantity with precision up to the full 16 decimal-digit precision of the type *double*. In practice, we typically measure signals with 6-bit to 20-bit analogue-to-digital converters, so we typically count on about 2—6 decimal digits of precision.

Accuracy. The distance between the original continuous signal and its reconstructed waveform should be as small as possible. For this purpose, we will use the $\boldsymbol{L}_2$ distance defined by (4). It will never be zero, but the proper choice of the reconstruction method, its parameters and suitable regularisation decrease the distance.

Low computational complexity. If an algorithm takes too long to complete, the user may shift to a less accurate but faster method of reconstruction.

3. LINEAR SIGNAL MODELS

The literature contains a broad spectrum of reconstruction methods which solve the problem defined in Section 2. Despite their variety, it seems that most of them can be characterized according to:

(1) the signal model which they use (discussed in this section),
(2) the method by which they determine the coefficients of the model (Section 4).

Virtually all common models express the signal $f(t)$ as a superposition of N functions $\{\phi_n\}_{n=0}^{N-1}$. The model $f_N(t)$ of order N may be written as

$$f_N(t) = \sum_{n=0}^{N-1} a_n \phi_n(t). \tag{5}$$

Some of the commonly-used signal models are listed in the following table:

Table 1. Some of the most common methods for reconstruction

Type of functions	Formula	Method's name, reference
Monomials	t^n	Lagrange interpolation, e. g. Klamer (1982)
Splines	$B_{n,m}(t)$	Hastie (2010)
Sinc functions	$\mathrm{sinc}(t-n)$	Sullivan (1984)
Complex exponential functions	$e^{jn\omega_0 t}$	Gerchberg-Papoulis method, Papoulis (1977)
Spherical Bessel functions of the first kind	$j_n(t)$	'Chromatic derivatives,' Ignjatović (2018a,b)
Slepian functions (PSWF)	$\psi_n(c,t)$	Slepian (1961), applied by Senay (2012)

3.1 Monomials and polynomials

The Nth-degree interpolation polynomial

$$f_N(t) = \sum_{n=0}^{N} a_n t^n. \tag{6}$$

is a well known type of interpolation. It is a simple yet very efficient solution for problems which involve only a few samples. The coefficients of the polynomial, a_n, may be found using various methods which solve the system of linear equations

$$f(t_k) = \sum_{n=0}^{N-1} a_n t_k^n, \qquad k = 0, 1, 2, \ldots, N-1. \tag{7}$$

The left-hand side (the non-uniform signal samples) is known. The coefficients a_n have to be found.

The main disadvantage of the polynomial interpolation is its extreme susceptibility to noise when the degree of the polynomial becomes high, especially at the edges of the interval in which the known samples are located.

3.2 Splines

Splines aim to avoid the above-mentioned oscillatory problems by dividing the signal into smaller blocks and by approximating each of them by a low-degree polynomial separately, e.g., by a cubic polynomial.

Spline interpolation can be conveniently constructed using B-spline basis functions. The $B_{n,m}(t)$ stands for the nth basis function of order m. The first order **B**-spline basis is represented by the rectangular functions with knots $\{t_n\}_{n=1}^{K}$. B-splines of higher orders may be obtained using a recurrence formula, Hastie (2010). For instance, B-splines of the first-order interpolate the known samples by a piecewise-straight line with knots at the known samples. Cubic splines are doubtlessly amongst the most common tools for signal interpolation.

3.3 Sinc functions

The naïve approach to creating a band-limited signal model stems from the formula of the sampling theorem

$$f(t) = \sum_{n=-\infty}^{\infty} a_n \mathrm{sinc}(t/T_\mathrm{s} - n), \tag{8}$$

which, in theory, may be used to represent any band-limited signal. Of course, for practical purposes, we have to truncate the series at a certain number of terms, say N. The model has N coefficients, a_n, which are found by solving the linear inverse problem

$$f(t_k) = \sum_{n=-N/2}^{N/2-1} a_n \mathrm{sinc}(t_k/T_\mathrm{s} - n) \quad \text{for} \quad k = 1, 2, \ldots, K. \tag{9}$$

Typically we use a larger number of sinc functions than the number of samples, $N > K$. Therefore, we are solving an underdetermined linear system of equations with an infinite number of solutions. To resolve the non-uniqueness the solution with minimal $\boldsymbol{L}_2$ norm is sought.

It can be shown, however, that truncation involved in (9) is not needed. According to Strohmer (2000), if we seek the solution with minimal $\boldsymbol{L}_2$ norm, the original problem reduces to a finite-dimensional naturally. The model

$$f_K(t) = \sum_{k=1}^{K} a_n \frac{\sin\Omega\,(t - t_k)}{\pi\,(t - t_k)}\,. \tag{10}$$

has minimal energy from all functions that comply with band-limit (2).

3.4 Bessel functions

The sinc method models the signal spectrum as an N-term truncated harmonic Fourier series. A similar approach called the *method of chromatic derivatives* Ignjatović (2018a,b) replaces the harmonic functions with orthogonal polynomials, e.g. Chebyshev, Legendre, etc. The latter option corresponds to a series of Spherical Bessel functions in the time domain and it seems to be the most widely used, so we restrict our analysis to this variant.

The N-term model in the frequency domain is of the form

$$F_N(\omega) = \begin{cases} \displaystyle\sum_{n=0}^{N-1} a_n P_n\left(\frac{\omega}{\Omega}\right) & \text{if} \quad |\,\omega\,| \le \Omega, \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

and in the time domain

$$f_N(t) = \Omega \sum_{n=0}^{N-1} \mathrm{j}^n a_n j_n(\Omega t), \tag{12}$$

The coefficients, a_n, are again sought using the matrix inversion, as in the previous section. The models (9) and (12) are, indeed, very similar, as the Legendre polynomials $P_n(t)$ tend to harmonic functions as their order increases.

3.5 Slepian functions

Another alternative to the previous methods relies on Slepian functions, also referred to as *Prolate Spheroidal Wave Functions* (PSWF). PSWFs were recognized as a useful tool for band-limited signal modelling (extrapolation especially) by Slepian (1961).

The Slepian basis includes a further parameter (the c) which significantly affects the shapes of all its functions and may be used to tweak the signal model. More detailed information may be found either in the original series of papers starting with Slepian (1961), or in the book by Osipov (2013).

One of the remarkable properties of Slepian functions is that they are eigenfunctions of the finite Fourier transform. Hence, the model of a band-limited signal can be written in the time domain as

$$f_N(t) = \sum_{n=0}^{N-1} a_n \psi_n(c,t) \qquad t \in \mathbb{R} \tag{13}$$

and in the frequency domain as

$$F_N(\omega) = \begin{cases} \displaystyle\sum_{n=0}^{N-1} a_n\, \mathrm{j}^{-n} \sqrt{\frac{\pi T}{\Omega \lambda_n(c)}}\, \psi_n\left(c, \frac{\omega T}{2\Omega}\right) & \text{if } |\omega| \le \Omega, \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

The biggest difficulty with Slepian functions is their evaluation. There is freely available software for computing Slepian functions, Adelman (2014), but the time needed to evaluate them, e.g. in *double* precision, is considerable.

3.6 Slepian sequences

Evaluation of Slepian functions is time-consuming. The problem may be avoided if we use Slepian sequences, also referred to as *Discrete prolate spheroidal sequences* (DPSS's) Slepian (1978). Indeed, they are very similar to Slepian function but with a slightly altered definition

$$\lambda_n(c) u_n(c,t) = \sum_{k=1}^{K} \frac{\sin\Omega(t-k)}{\pi(t-k)} u_n(c,k), \quad t \in \mathbb{R},\ n \in \mathbb{N}^0. \tag{15}$$

This makes them much more suitable for digital signal processing. We only have to store K samples of each Slepian sequence. The sequences may be extended to continuous functions for any $t \in \mathbb{R}$ using (15).

4. TIKHONOV REGULARISATION

Tikhonov regularisation method for ill-posed problems, see Franklin (1974), modifies the functional of the well-known least-squares method by including a term which penalizes the energy of the reconstructed signal or its derivatives. It uses the following extended penalization functional

$$E = \sum_{k=1}^{K} |f(t_k) - f_N(t_k)|^2 + \alpha \sum_{m=0}^{p} b_m \int_{-\infty}^{\infty} \left| f_N^{(m)}(t) \right|^2 \mathrm{d}t. \tag{16}$$

The most widely used instance is obtained by setting all b_m's to zero except the $b_0 = 1$, or $b_2 = 1$. We will discuss these two options.

4.1 Penalisation of signal energy

For $b_0 = 1$ (other b_m's zero) and most signal models, the definite integral involved in (16) can be solved quite easily. We may plug the general model (5) into the functional (16), change the order of integration and summation and integrate. The resulting expression may be written in a matrix form

$$E = (\boldsymbol{f} - \mathbf{Q}\boldsymbol{a})^{\mathsf{H}} (\boldsymbol{f} - \mathbf{Q}\boldsymbol{a}) + \alpha\, \boldsymbol{a}^{\mathsf{H}}\, \mathbf{R}\, \boldsymbol{a}, \qquad (17)$$

where the elements of the symmetric matrix $\mathbf{R}$ are the scalar products of the functions used in the model

$$r_{m,n} = r_{n,m} = \int_{-\infty}^{\infty} \phi_n(t)\overline{\phi_m(t)}\, \mathrm{d}t. \qquad (18)$$

It is well-known that the vector $\boldsymbol{a}$ which minimizes (17) is

$$\boldsymbol{a} = (\mathbf{Q}^{\mathsf{H}}\mathbf{Q} + \alpha\mathbf{R})^{-1}\mathbf{Q}^{\mathsf{H}}\boldsymbol{f}. \qquad (19)$$

The matrix to be inverted is an $N \times N$ matrix. Therefore, the computational complexity increases with the model order, N.

For functions which are orthonormal over the whole real line, the matrix $\mathbf{R}$ is reduced to an identity matrix. (This is the case of Bessel functions or harmonic functions.) In the case of orthogonal functions, e.g. Slepian functions normalised so that they have unitary energy in $\langle -1; 1\rangle$, the matrix $\mathbf{R}$ becomes a diagonal matrix having eigenvalues λ_n as diagonal elements. And the non-uniformly shifted sinc functions (9), which are not orthogonal, yield even simpler expression. For these functions operation (18) represents a convolution of two sinc functions, and $\mathbf{R} = \mathbf{Q}^{\mathsf{H}} = \mathbf{Q}$. Therefore, (19) is reduced to

$$\boldsymbol{a} = (\mathbf{Q} + \alpha)^{-1}\boldsymbol{f}. \qquad (20)$$

4.2 Penalisation of signal second derivative

Compared to all the above-mentioned methods, smoothing splines represent a slightly different approach to signal modelling. Firstly, they are not band-limited, as opposed to all previous models. Secondly, the regularisation involves the energy of the model's *second derivative* instead of *energy*. All b_m's are zero except for the $b_2 = 1$.

If we use Parseval's identity, the second term may be written in the frequency domain

$$E = \sum_{k=1}^{K} |f(t_k) - f_N(t_k)|^2 + \frac{\alpha}{2\pi} \int_{-\infty}^{\infty} \omega^4 \, |F_N(\omega)|^2 \, \mathrm{d}\omega. \quad (21)$$

From this version, it becomes more obvious that the second term enforces the model to be concentrated in the frequency domain. The solution is of finite dimension K, represented by K cubic B-splines. Therefore, the method may be conveniently implemented in digital signal processing.

5. NUMERICAL EXPERIMENTS

This section concentrates on the computational complexity and quality of the reconstruction for the methods which were discussed in previous sections. There are articles which compare the quality of the reconstruction, such as Senay (2012); Ignjatović (2018b); Feichtinger (1995); Patki (2016), but these articles either compare only a subset of the methods we analyse here or were carried out in the absence of noise. The prevalent practice in band-limited signal reconstruction involves demonstrations of the methods on much simpler signals. The testing signal is often associated with a simple analytical formula or it is created as a superposition of such formulas. The works Papoulis (1977); Ignjatović (2018a); Devasia (2013); Slepian (1978) are typical examples.

5.1 Description of the signal being processed

An experiment was carried out with an audio signal, which contains sounds of birds recorded in a forest. Our main criterion for the assessment of the method performance will be the $\boldsymbol{L}_2$ error (4) of a reconstructed signal. Another sound criterion is the total running time for each of the methods. However, it is much harder to assess the latter correctly, for the method performance differs significantly with parallelization and optimization of the code. Measured running time provides only a rough comparison of the computational complexity.

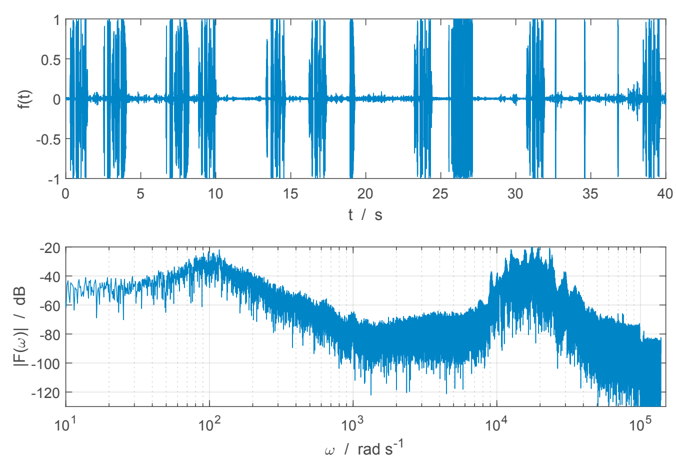

Fig. 1. The uniformly sampled test signal (top) and its spectrum (bottom).

We decided to generate the sampling non-uniformities artificially yet using a real audio recording. The original, uniform signal was 40 s long and sampled at the frequency $T_{\mathrm{s}}^{-1} = 44{,}100$ Hz. Figure 1 shows this signal in the time and frequency domain.

All tests were carried out with simulated random Gaussian noise. The non-uniform sampling instants were determined by the relation

$$t_k = kT_{\mathrm{s}} + \epsilon_k, \quad \epsilon \sim \mathcal{N}(0, \sigma_{\mathrm{s}}^2), \quad k = 1, 2, \ldots, K-1. \quad (22)$$

where the timing disturbances ϵ_k's are drawn from the Gaussian distribution $\mathcal{N}(0, \sigma_{\mathrm{s}}^2)$. The non-uniform signal is obtained by re-sampling the original signal at the instants t_k using the exact reconstruction formula of the sampling theorem. We carried out experiments for the standard deviations $\sigma_{\mathrm{s}} = T_{\mathrm{s}}/10$, $T_{\mathrm{s}}/5$ and $T_{\mathrm{s}}/2$.

The signal is very long, it cannot be processed in a single block. It has to be divided into much smaller blocks and the methods of band-limited reconstruction will be applied to each of them separately. All experiments with band-limited models were performed on $64T_{\mathrm{s}}$ long blocks with 50 %

overlap. These partial signals were then merged using cosine windows, which provided smooth transitions between consecutive blocks. Spline-based models were applied to the signal directly.

To simulate various sources of uncertainty, such as the errors induced by the A/D converters, we also added noise to the amplitude of non-uniform signal samples. Gaussian noise with standard deviation σ_a of values 10^{-6}, 10^{-4} and 10^{-2} was used. These values roughly cover the range of errors from the most widely used converters, between 6 bits and 20 bits.

5.2 Reconstruction accuracy

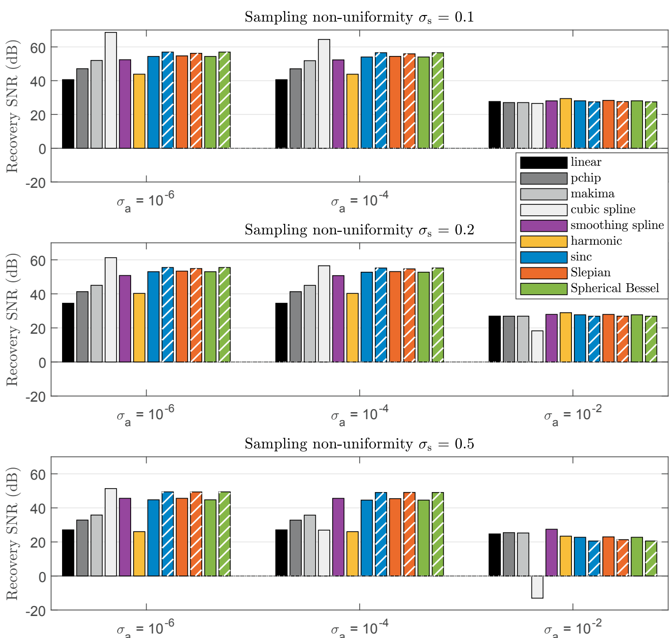

Fig. 2. Comparison of SNRs achieved by different signal reconstruction methods. Grey bars correspond to methods without regularisation. Other methods rely on Morozov's principle (full bars) or constant regularisation parameter $\alpha = 10^{-12}$ (hatched bars).

All experiments involving band-limited models were performed using the same band-limit $\Omega = 0.75\pi/T_\mathrm{s}$. Regularisation was achieved using the Tikhonov method with Morozov's discrepancy principle, as described in the previous section. The following signal models were compared:

- *linear*—piecewise-linear interpolation (Section 3.2),
- *pchip*—piecewise cubic Hermite interpolation polynomial (continuous up to the first model derivative),
- *makima*—Modified Akima cubic Hermite interpolation,
- *spline*—cubic-spline interpolation (continuous up to the second model derivative, Section 3.2),
- *SmoothinSpline*—a regularised cubic-spline model (Section 4.2),
- *harmonic*—a harmonic-function model involving sines and cosines,
- *sinc*—the sinc-function model (10),
- *Slepian*—the model consisting of Slepian sequences (Section 3.6)
- *SphericalBessel*—the model consisting of spherical Bessel functions of the first kind (12).

Figure 2 compares the quality of signal reconstruction for different methods. Each panel corresponds to a different level of sampling non-uniformities ($\sigma_\mathrm{s} = T_\mathrm{s}/10,\ T_\mathrm{s}/5$ and $T_\mathrm{s}/2$) and contains results for three different levels of noise in the sampled amplitude ($\sigma_\mathrm{a} = 10^{-6},\ 10^{-4}$ and 10^{-2}).

The first four models are not band-limited. They represent different degrees of a trade-off between the bias and variance of the model. In Figure 2, they are sorted according to this trade-off. The linear interpolation (black) has the largest bias and smallest variance. It performs more or less the same for all nine settings of the experiment. Cubic-spline model (light grey) is exactly the opposite. It has a small bias but a large variance. Hence, for noiseless signals, it yields the best results but fails for the most noisy signal (producing SNR of –13 dB).

Other methods were implemented with regularisation. Therefore we may expect that they will adapt their bias-variance trade-off according to the level of noise. Overall they produce comparable results (coloured bars), except for the harmonic model, which yielded the worst performance of the regularised models. It seems that the assumption that the modelled signal is periodic is not justified.

It seems that the adaptive regularisation-parameter selection according to Morozov hampered the reconstruction (full bars). The constant parameter produced *better* results (the hatched bars). The fact that $\alpha = 10^{-12}$ produces a nearly optimal performance for most noise levels is in accordance with the work Ignjatović (2018b).

5.3 Reconstruction speed

The models based on Slepian sequences, sinc functions and spherical Bessel functions achieved essentially the same results in terms of precision. This result is in accordance with the theory presented in the previous section. Then the only appreciable difference between them will be their computational burden, as indicated by the following figure.

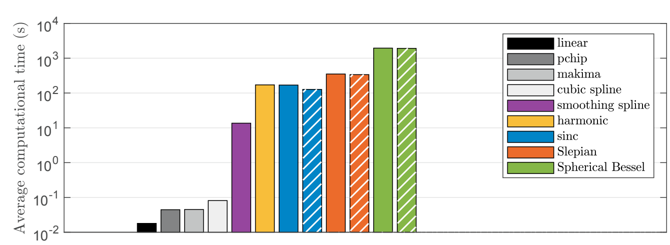

Fig. 3. Comparison of mean computational time required by different signal-reconstruction methods. Grey bars correspond to methods without regularisation. Other methods rely on Morozov's principle (full bars) or constant regularisation parameter $\alpha = 10^{-12}$ (hatched bars).

Evaluation of the spherical Bessel function model exhibited the slowest performance, mainly due to the evaluation of special functions for each signal block. Sinc functions and harmonic functions are the computationally simplest band-limited functions. Thus their use cuts the computational time roughly by another order. They still require a lot of effort though, since we have not optimized the computation of the regularised inversion. In the current implementation, the solver for the inversion is selected automatically by MATLAB. When the Morozov principle is applied, the solutions for different regularisation parameters are not reused, yet the last figure suggests that this does not play a significant role. The solution with a constant regularisation parameter saves some operations but does not seem to produce as dramatic differences as the proper selection of the model functions. The spline-based methods are the fastest. Their running time is dependent on their complexity. E.g., the piecewise-linear interpolation is the simplest, hence also the fastest.

6. CONCLUSION

This paper summarized the current state-of-the-art in band-limited signal modelling and recovery. We saw that when we have only a finite number of discrete signal samples, there is an infinite number of continuous band-limited functions which pass through the samples. The non-uniqueness is resolved by seeking the solution with the minimal $\boldsymbol{L}_2$ norm. Furthermore, a small deviation from the samples is often admitted to further reduce the $\boldsymbol{L}_2$ norm.

We have reviewed the most promising signal-recovery methods. Contemporary approaches, such as those involving Spherical Bessel functions, Slepian sequences, harmonic functions or sinc function, were compared with some of the most widely used non-band-limited signal models represented by the various versions of splines. These were verified on a practical example involving an audio signal. Methods involving cubic splines or sinc functions seem to offer the best compromise between speed and accuracy.

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