Lumped Parameter Method for Prediction of Sound Absorption Coefficient of Loudspeaker Placed in Closed Enclosure

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***Abstract*— This paper presents a comparative analysis of a theoretical lumped parameter model used to compute the acoustic impedance of a loudspeaker placed in closed enclosure, which acts as a low-frequency absorber. The analysis aims to validate the theoretical predictions by examining the system's mechanical and acoustic parameters. The influence of diaphragm compliance, mass, and damping on acoustic impedance is explored. Additionally, an impedance tube measurement is used to experimentally verify the theoretical results. The methodology, measurement procedure, and laboratory setup are described in detail. Results indicate that the lumped parameter approach provides a reliable estimation for specific frequency ranges, with deviations attributed to non-ideal structural and material properties.**

Keywords—Sound waves, acoustic impedance, acoustics, resonator, sound absorption

I. INTRODUCTION

Low-frequency noise is one of the biggest challenges in acoustics today. Acoustic engineers dealing with noise in residential spaces near traffic or industry are faced with the problem of low-frequency noise that is not effectively attenuated by conventional building materials. Residents are thus disturbed even when complying with the regulations. Low-frequency sound also causes significant problems in small music studios and listening rooms, where standing waves and resonances significantly affect the frequency response of the room at low frequencies. Such example of standing wave problem can be seen in Fig. 1, which shows result of simulation used FDTD numerical method [1] for small shoebox-shaped room.

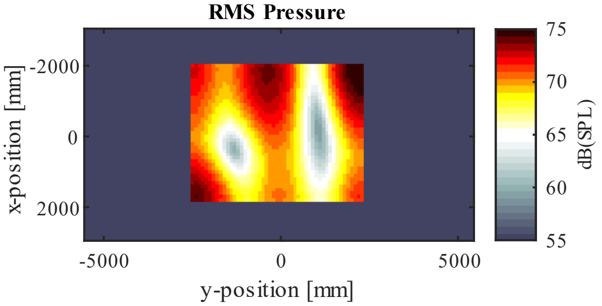

Fig. 1. FDTD simulation of spatial distribution of sound field in horizontal plane for 2nd axial mode in y direction at 68 Hz in room with dimensions of 5 x 4 x 3 m.

Conventional porous absorbers, such as foam and fiberglass panels, become ineffective at low frequencies unless they are significantly thick, often exceeding 50 cm in depth [2]. This practical limitation necessitates the use of alternative solutions such as resonant absorbers, including Helmholtz resonators and membrane absorbers, which are specifically designed to target narrow frequency bands. However, precise design of such devices is somewhat difficult as not all of physical properties are known during their design and construction. On the other hand, the loudspeaker diaphragm has all its essential properties measured and provided by the manufacturer. Using these, the lumped parameter method can be used to successfully predict its absorptive properties.

The effectiveness of these absorbers depends on their acoustic impedance, or more accurately, how well the impedance of loudspeaker diaphragm is matched to the acoustic impedance of air. Impedance matching is crucial for effective sound absorption of such device [3].

This work evaluates a theoretical lumped parameter model for predicting the acoustic impedance of a loudspeaker mounted on closed enclosure and compares it with experimental measurements conducted using an impedance tube setup. The impedance tube method allows precise measurement of the absorption characteristics of materials and structures within the low-frequency range, providing a reliable means of validation.

II. LUMPED PARAMETER MODEL

A. Mechanical properties of loudspeaker

The lumped parameter model simplifies the complex vibration behaviour of translation mechanical system by assigning its physical properties to components of electrical circuit. Mass of the system translates to inductance, compliance translates to capacity and mechanical losses to resistance. Most loudspeaker manufacturers list these parameters as M_{ms}, C_{ms} and R_{ms}.

R_{ms} is often not listed, however it can be calculated from mechanical quality factor Q_{ms} as [4]:

$$R_{\mathrm{ms}} = \frac{2\, f_{\mathrm{S}} M_{\mathrm{ms}}}{Q_{\mathrm{ms}}}, \qquad (1)$$

where f_s is resonant frequency of loudspeaker.

When loudspeaker is mounted onto enclosure, the air enclosed in the cavity acts as an additional compliance. This compliance is in this work noted as C_{enc}. Value of this compliance is calculated as [5]

$$C_{enc} = \frac{V}{\rho_0 c_0^2}, \tag{2}$$

where ρ_0 is air density ($1.29\ \mathrm{kg/m^3}$), c_0 is speed of sound in air ($344\ \mathrm{m/s}$) and V is the enclosed volume behind the loudspeaker diaphragm. When all mechanical variables are known, then acoustic impedance of entire system can be calculated by solving equivalent circuit. The equivalent electro-mechanical model of the circuit is shown in Fig. 2.

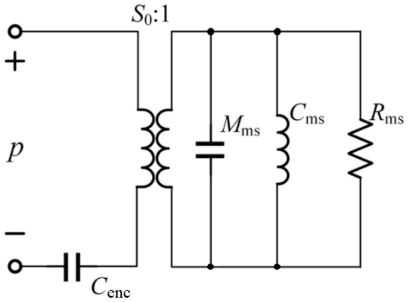

Fig. 2 Equivalent electro-mechanic circuit model of loudspeaker driven by external acoustic pressure

First, the acoustic impedance of the loudspeaker diaphragm itself must be calculated, which is done using

$$Z_d = \frac{R_{\mathrm{ms}}}{S_{\mathrm{d}}} + \frac{\mathrm{j}\,\omega\, M_{\mathrm{ms}}}{S_{\mathrm{d}}} + \frac{1}{\mathrm{j}\,\omega\, C_{\mathrm{ms}}\, S_{\mathrm{d}}}, \tag{3}$$

where $\omega = 2\pi f$ is the angular frequency, j is imaginary unit and S_{d} is the effective area of loudspeaker diaphragm.

After that, compliance of enclosure can be added using:

$$Z_{\mathrm{lspk}} = Z_{\mathrm{d}} + \left(\frac{S_{\mathrm{d}}}{j\,\omega\, C_{\mathrm{enc}}}\right). \tag{4}$$

Because, the whole resonant structure is placed and measured in impedance tube the of known cross-section area, as described in next part of this paper, it is important to take into account the transformation of acoustic impedance between the diaphragm surface and the cross-section area of the impedance tube. Acoustic impedance of the entire system is then calculated using [6]:

$$Z_{\mathrm{x}} = Z_{\mathrm{lspk}} \left(\frac{S_0}{S_{\mathrm{d}}}\right), \tag{5}$$

where S_0 is the cross-section area of the impedance tube. Acoustic impedance of the entire system can be converted into absorption coefficient values using [7]:

$$\alpha = \frac{4\rho_0\ \mathrm{c}_0\ Re(Z_{\mathrm{x}})}{(\rho_0\ \mathrm{c}_0\ \mathrm{Re}(Z_{\mathrm{x}}))^2 + (\mathrm{Im}(Z_{\mathrm{x}}))^2}. \tag{6}$$

B. Prediction of absorption coefficient of real structure

Using the above equations, the absorption coefficient values were calculated for a specific loudspeaker type. The Dayton Audio PC105-8 was selected. Its mechanical parameters are listed in the table below.

TABLE I Mechanical Properties of loudspeaker selected for verification

Dayton PC105-8			
Table column subhead	***Symbol***	***Value***	***Unit***
Moving mass	M_{ms}	4.2	g
Mechanical compliance	C_{ms}	0.84	mm/N
Mechanical Quality factor	Q_{ms}	2.29	-
Resonant frequency	f_{s}	84.8	Hz
Mechanical losses	R_{ms}	0.97	kg/s
Diaphragm area	S_{d}	52.8	cm^2

The resulting absorption coefficient values for enclosure volume of 4 liters are shown in Fig. 3.

Lumped parameter model allows quick investigation of influence of changes in the structure to final sound absorption coefficient values. For example, if an additional mass is added to the mass of the moving diaphragm, the effect on the resulting sound absorption coefficient can be determined. The calculation was performed for three different variants, each with an increasing mass of 2.5 g. (See in Fig. 4.)

The values of the sound absorption coefficient do not reach the value of 1, because this is a calculation of acoustic impedance of the entire system. Only the area of the loudspeaker diaphragm is acoustically active, which is approximately 6% of the total cross-section area of the impedance tube.

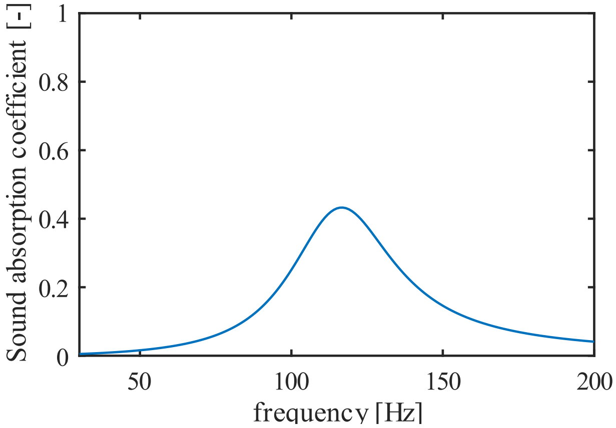

Fig. 3 Calculated sound absorption coefficient of complete system with Dayton PC105-8 placed in enclosure with volume of 4 liters

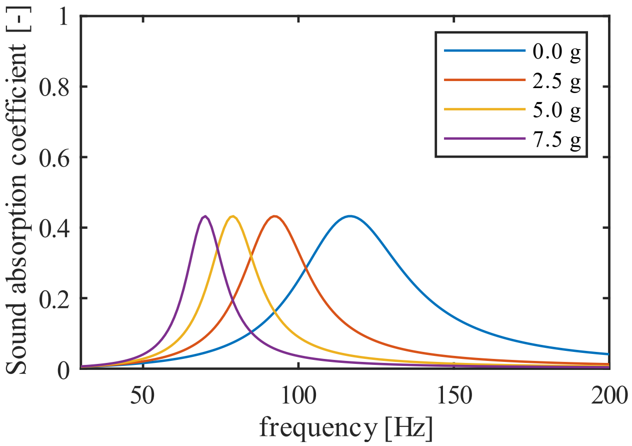

Fig. 4 Calculated sound absorption coefficients for different values of added mass

III. VERIFICATION MEASUREMENTS

A. *Measureement technique*

Measurement using impedance tube method was conducted. The impedance tube used in this study has a square cross-section of 30 cm x 30 cm, that allows testing of larger samples. The impedance tube is made of MDF board with thickness of 18 mm plated with 3 mm thick metal sheets. This ensures high reflection and sound insulation even for low frequencies. Driving loudspeaker is located on one side of the tube whereas a highly reflective surface, where specimen shall be placed, on the other. As illustrated in Fig.6.

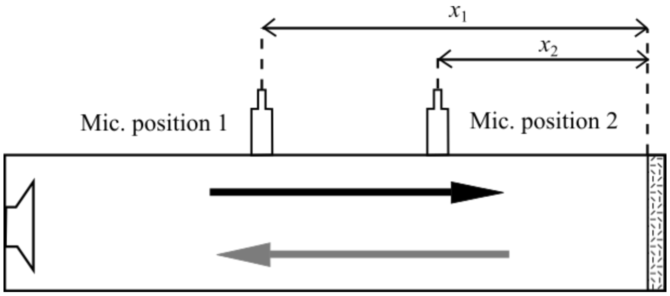

Fig. 5 Diagram of impedance tube that uses transfer function method of measurement of acoustic impedance

The impedance tube has two positions for microphones. The microphones pick up the acoustic pressure at their respective positions and then the transfer function between those two positions is calculated using

$$H_{12} = \frac{P_2 P_1^*}{P_1 P_1^*} \, , \tag{7}$$

where P_1 and P_2 are Fourier transforms of acoustic pressures acting on the microphones. The * symbol stands for complex conjugate [8]. H_{12} is then used to compute the reflection coefficient using

$$r = \frac{H_{12} - \mathrm{e}^{-\mathrm{j}k_0(x1-x2)}}{\mathrm{e}^{\mathrm{j}k_0(x1-x2)} - H_{12}} \mathrm{e}^{2\mathrm{j}k_0 x1} \tag{8}$$

where x_1, x_2 are distances of microphones from the measured sample, k_0 is the wavenumber. The reflection coefficient is then used to compute absorption coefficient using [8]

$$\alpha = 1 - |r|^2 . \tag{9}$$

B. *Measurement of sound absorption coefficients and comparsion to theoretical model*

A prototype was created for comparative measurement of the absorption coefficient. MDF board with thickness of 16 mm was used for the enclosure. The internal dimensions are 20 cm x 20 cm x 10 cm and thus the volume is 4 liters. There are four openings for future experiments with coupled Helmholtz resonators, which were sealed with 3D printed plugs during the measurement. Its photograph is shown in Fig. 6.

Fig. 6 Photograph of the absorber prototype.

If we compare the calculated values of the absorption coefficient with the measured ones, a slight deviation can be observed, mainly in the shift of the resonant frequency towards higher values, see Fig. 7. This is probably due to the fact that the loudspeaker unit partially fills the internal volume of the enclosure. Therefore, the volume is actually slightly smaller than 4 liters, which leads to a decrease in compliance and therefore the resonant frequency increases.

Furthermore, the maximum absorption value is slightly lower, which is probably due to additional losses inside the enclosure, since the volume of enclosure was represented only by ideal compliance without resistance which would include losses caused by leaks and other imperfections. Another reason for inaccuracies may be deviations in real TS parameters, which may be slightly different from the data provided by the manufacturer.

However, these are relatively small deviations and this is a very promising initial step in the case of the design of a specific acoustic element.

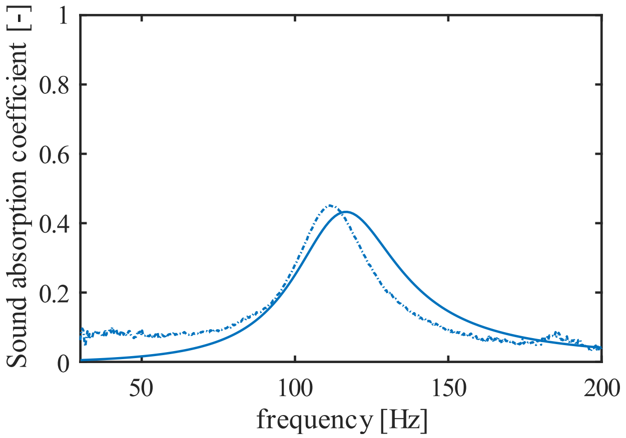

Fig. 7 Comparison of measured and calculated absorption coefficients. Dotted line shows measured values.

When additional weights are added to the theoretical model and to the prototype, then high accuracy of the prediction be observed when comparing measured and predicted values. It can even be observed that the deviation in both resonance frequency and maximum value of the absorption coefficient decreases in comparison with the initial state. (see Fig. 8)

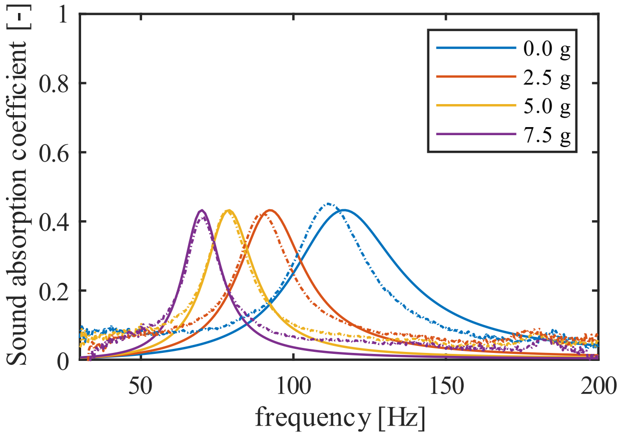

Fig. 8 Comparison of measured and calculated absorption coefficients with added mass. Dotted line shows measured values.

IV. CONCLUSION

The lumped parameter model provides a useful approximation of the acoustic impedance of a loudspeaker diaphragm acting as a low-frequency absorber. This approach effectively captures the fundamental impedance characteristics at low frequencies, where mass, compliance, and resistance play a crucial role in defining the acoustic behaviour of the system. Acoustic impedance values are then used for prediction of absorption coefficients where theoretical predictions align well with experimental results obtained from the impedance tube setup, demonstrating the model's reliability for practical applications in low-frequency sound absorption.

One of the key advantages of the lumped parameter model is its ability to provide insights into the behaviour of absorbing structures with minimal computational complexity. By tuning parameters, such as diaphragm mass, compliance, and damping, it is possible to design absorbers optimized for specific frequency ranges. This makes the model particularly useful for applications in room acoustics, and architectural noise control, where effective low-frequency absorption is required.

Future work should focus on extending the model to incorporate more complex structures and configurations. One potential area of exploration is the application of the lumped parameter approach to bass-reflex loudspeaker enclosures, which exhibit additional resonant behaviours due to the interaction between the port and the internal air volume. Another promising direction is the investigation of shunt resistance applied to loudspeaker terminals, which can provide additional control over the system’s impedance characteristics and further enhance absorption performance. [6]

One promising direction for further research is the application of the lumped parameter model in the design and analysis of acoustic metamaterials. [9] Acoustic metamaterials are engineered structures with unique wave-manipulating properties, often achieving negative effective mass or bulk modulus, leading to unconventional sound absorption and wave propagation characteristics. By incorporating the lumped parameter approach, researchers can model the interaction of resonant elements within metamaterials, predicting their impedance characteristics and optimizing them for targeted frequency ranges. For instance, arrays of membrane-based absorbers with precisely tuned mass and compliance could be designed to achieve extreme low-frequency absorption in compact structures. [10] Additionally, combining lumped parameter modelling with metamaterial concepts such as locally resonant units or tuned Helmholtz-like inclusions can lead to advanced noise reduction solutions for architectural acoustics, aerospace applications, and underwater acoustics, where traditional absorbers are impractical due to size constraints [1].

REFERENCES

[1] *k-Wave: A MATLAB toolbox for the time domain simulation of acoustic wave fields [online]*. Copyright ©. Available from: http://www.k-wave.org/manual/kwave_user_manual_1.1.pdf

[2] COX, T.; D'ANTONIO, P. *Acoustic absorbers and diffusers: theory, design and application. Third edition*. Boca Raton: CRC Press, Taylor & Francis Group, [2017]. ISBN 978-1-4987-4099-9.

[3] KIM, Y. *Sound propagation: an impedance based approach. Hoboken*, N.J.: Wiley, c2010. ISBN 978-0-470-82583-9

[4] DICKASON, V. *The loudspeaker design cookbook. 6th ed., new, expanded*. Peterborough: Audio Amateur Press, 2000. ISBN 1-882580-33-8.

[5] ZHANG P., CONG, C., TAO, J. *et al. Dual frequency sound absorption with an array of shunt loudspeakers*. Sci Rep 10, 10806 (2020). https://doi.org/10.1038/s41598-020-67810-z

[6] LI, Xin; CAO, Zhigang; LI, Zihao a LIU, Bilong. *Sound absorption of a shunt loudspeaker on a perforated plate*. Online. Applied acoustics. 2022, roč. 193, s. 108776. ISSN 0003-682X.

[7] MORSE, Philip M. a INGARD, K. Uno. *Theoretical acoustics*. Princeton: Princeton University Press, 1986. ISBN 0-691-02401-4.

[8] ISO 10534:1996, *Acoustics – Determination of sound absorption coefficiend in impedance in impedance tubes*

[9] NORRIS, A.; HABERMAN, M. *ACOUSTIC METAMATERIALS. Online*. The Journal of the Acoustical Society of America. 2012, roč. 132, č. 4. ISSN 0001-4966. [cit. 2024-04-05].

[10] LISSEK, H; BOULANDET, R and RENÉ, P.. *Shunt loudspeakers for modal control in rooms, 2009*