SYMBOLIC ACTIVE RC CIRCUIT SYNTHESIS USING MATLAB

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Abstract: Implementation of the procedure for synthesis of the active RC circuit from prescribed transfer function using MATLAB® , without prior knowledge of topology or circuit architecture is presented. Method is based on the reverse process of a Gaussian pivotal elimination applied to nodal admittance matrix as proposed in [6]. This method is very useful for searching alternative circuits realizations of the same transfer function, or with modifications to find several even all realizations in some level. Given implementation has several limitations like specific input transfer function format and interaction with human for further decisions in the synthesis process.

Keywords: Nullors, Active-RC, Synthesis, Admittance matrix

1 INTRODUCTION

The concept of the circuit describing by introducing nullators and norators was first introduced by Carlin [1]. There was shown, that it is possible to approximate ideal transistor using nullators and norators in [2]. Systematic synthesis procedure of active RC circuits using resistors, capacitors and nullator - norator pairs directly was derived. Moreover there was shown, that exist unique solution for networks containing nullators - norators [3]. Using nullator - norator pair, nullor in short, it is possible to describe variety of active circuit elements e.g. ideal transistors or , ideal operational amplifiers. Method for symbolic passive RC circuit synthesis by admittance matrix expansion [4] and later active RC circuit synthesis by admittance matrix expansion [5]. Reduction of admittance matrix of the circuit to form of open-circuit voltage gain or short-circuit current is called circuit analysis. Reversing this process by expansion of network function we get synthesis. Systematic method for circuit network function expansion without topology of the circuit pre-defined beforehand using active circuits building blocks using nullors was introduced in [6] were presented. Computer aided implementation of this method using MATLAB® is proposed in this letter. The rest of this letter is organized as follows. Firstly in Section 2. brief introduction to method for matrix expansion and tools used in this method are described. In Section 3. implementation and MATLAB® functions of this method is proposed and paper is concluded in Section 4.

2 BACKGROUND

Implementation of the synthesis algorithm described in this letter is highly based on the method proposed by [6], which uses principle of port-equivalence and therefor preserves port behavior.

2.1 NULLORS ADMITTANCE MATRIX REPRESENTATION

In fig.1 is shown nullator-norator pair with corresponding node labels. Voltage and current of the nullator is constrained to zero, while voltage and current of the norator are arbitrary and are constrained by circuit. Basically nullator-norator pair create voltage controlled current source with admittance matrix (1), where for ideal active components $G_{mi} \to \infty$ and instead of using G_{mi} we will use infinity symbol threated like it is ordinary variable ∞_i, where i corresponds to $i-th$ nullor. Then $i-th$ nullors admittance matrix becomes (1) connected between nodes o,p and m,n. Symbols for given nullor-norator pair are as is in fig.1. After expansion of the port admittance matrix to admittance node matrix it is possible to replace infinity variables with corresponding admittances of real active devices.

$$\begin{array}{c} \\ m \\ n \end{array} \begin{pmatrix} G_{mi} & -G_{mi} \\ -G_{mi} & G_{mi} \end{pmatrix}^{\!\begin{smallmatrix} o & p \end{smallmatrix}} \to \begin{array}{c} \\ m \\ n \end{array} \begin{pmatrix} \infty_i & -\infty_i \\ -\infty_i & \infty_i \end{pmatrix}^{\!\begin{smallmatrix} o & p \end{smallmatrix}} \quad (1)$$

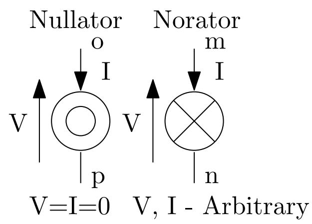

Figure 1: Nullator-Norator pair

2.2 NULLATOR REPRESENTATION OF THE ACTIVE CIRCUIT BUILDING BLOCKS

With concept of introducing the nullator - norator pair it is possible to model various types of active circuit building blocks e.g. op-amps, transistors even the current conveyors. Most used active components in real life are op-amps, transistors BJTs and FETs. Nullor models for op-amps and transistors are shown in fig.2.

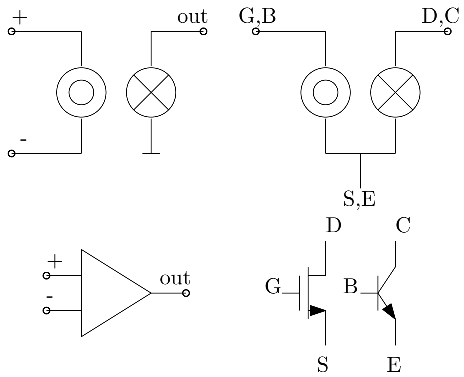

Figure 2: Nullor-Norator representation of the ideal op-amp and ideal transistor

2.3 SYNTHESIS PROCEDURE STEPS

Procedure for synthesis of the circuit starts with given current or voltage transfer function, describing behavior of the circuit, in the format of fraction $\frac{N}{D}$, where N denotes nominator and D denotes denominator and are restricted to sum of the second order powers of admittances. Performing several modifications and expansions we get one of the nodal admittance matrix after.

Steps needed to be done are as follows:

1. Choosing of appropriate admittance matrix from table 1 and table 2
2. Pivotal expansion of the N and D
3. Using theorem of arbitrary elements insertion and element shift theorem to expand admittance matrix as far as possible
4. Relaxation of the zero input admittance for voltage controlled voltage source VCVS or zero output admittance for current controlled current source CCCS.
5. Introducing additional nullators in case of existence of non-grounded or incomplete admittances in admittance matrix.

2.4 Analyzing Prescribed Transfer Functions

First step is to choose appropriate admittance matrix for prescribed transfer function. For voltage transfer function appropriate admittance matrix will be in table 1 for current function in table 2. If transfer function is positive and terms in numerator appear in denominator, then we pick Type I. If transfer function is positive and terms in denominator also appears in numerator we pick Type II. For negative transfer function and independent numerator and denominator we choose Type III. For combination of positive and negative terms in numerator denominator of the transfer function, most effective will be Type IV admittance matrix.

Type I	Type II	Type III	Type IV
$\begin{bmatrix} 0 & 0 & 0 \\ 0 & \infty_1 & -\infty_1 \\ -N & 0 & D \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ \infty_1 & 0 & -\infty_1 \\ 0 & -D & N \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \infty_1 \\ -N & -D & Q \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \infty_1 & -\infty_1 \\ -N_1 & -D_1 & P_1 & 0 \\ -N_2 & -D_2 & 0 & P_2 \end{bmatrix}$
$A_V = \frac{N}{D}$	$A_V = \frac{N}{D}$	$A_V = -\frac{N}{D}$	$A_V = \frac{N_2P_1 - N_1P_2}{D_1P_2 - D_2P_1}$

Table 1: Admitance Matrices for Prescribed Voltage Transfer Functions

Type I	Type II	Type III	Type IV
$\begin{bmatrix} \infty_1 & 0 & 0 \\ 0 & 0 & -N \\ -\infty_1 & 0 & D \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & -D \\ \infty_1 & 0 & 0 \\ -\infty_1 & 0 & N \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & -D \\ 0 & 0 & -N \\ -\infty_1 & 0 & Q \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & -D_1 & -D_2 \\ 0 & 0 & -N_1 & -N_2 \\ \infty_1 & 0 & P_1 & 0 \\ -\infty_1 & 0 & 0 & P_2 \end{bmatrix}$
$A_I = \frac{N}{D}$	$A_I = \frac{N}{D}$	$A_I = -\frac{N}{D}$	$A_I = \frac{N_2P_1 - N_1P_2}{D_1P_2 - D_2P_1}$

Table 2: Admitance Matrices for Prescribed Current Transfer Functions

Equivalence [6] of given admittance matrix for transfer function and form in terms of determinants of sub-matrices is eq.2:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & \infty_1 & -\infty_1 \\ -\frac{N}{Q_i} & 0 & \frac{D}{Q_i} \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & \infty_1 & -\infty_1 \\ -\frac{\Delta_{12,23}}{\Delta_{11,22,33}} & 0 & \frac{\Delta_{11,22}}{\Delta_{11,22,33}} \end{bmatrix} \tag{2}$$

,where Q_i is arbitrary function, consisting of pivot product composition of the transfer function, which need to be chosen manually. After selection of this arbitrary function and insertion to the admittance matrix of proper type, this should be directly fed into proposed MATLAB® script. Methods used during expansion of the admittance matrix are described in Section 3.

3 IMPLEMENTATION

Implementation of the method described in [6] is done in MATLAB® environment with help of a Symbolic Toolbox^{TM}. Input transfer function shall be modified into form described in Section 2.4 and then fed into created script. All admittances are symbolic variables and symbolic infinities representing nullors are named $Ninf_i$, where i is index of the nullor e.g. (Type IV in table.1):

$$A_V = \frac{N}{D} = \frac{bde+bdf}{abe+ace+ade+bde-bcf} \rightarrow \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & Ninf1 & -Ninf1 \\ 0 & -e & e+f & 0 \\ -\frac{b*d}{b+c+d} & -\frac{b*c}{b+c+D} & 0 & a+b-\frac{b^2}{b+c+d} \end{bmatrix} \tag{3}$$

, where $Ninf1$ represents nullor (in this form op-amp with grounded negative input) and a,b,c,d,e,f are admittances, $b+c+d$ term represents arbitrary function Q_i from (2). First step in the algorithm is analyzing of the given admittance matrix, where size of matrix and list of the symbolic variables are derived. Next step is separation of the admittances from the nullor circuits from the variable list. This is done by comparing names of variables, in the loop through all variables, with name starting with $Ninf_i$. Also positions of the nullors are derived from the admittance matrix and stored into array. After that fractions, their amount and their positions are derived. If matrix contain any fractions their denominators are checked, which have to be the same (from nature of the method for creating admittance matrix, if method is used properly, this will be satisfied always). Next step, if there are fractions in the matrix, is search for admittances common for all numerators in the fractions. There should be more then one common admittance, so here is human interaction needed, with more possible realizations of the circuit. Here comes first elements operation called matrix pivotal expansion:

$$\begin{bmatrix} a-\frac{b*c}{d} & e-\frac{b*f}{d} \end{bmatrix} \rightarrow \begin{bmatrix} a & e & -b \\ -c & -f & d \end{bmatrix} \begin{bmatrix} a-\frac{b*c}{d} \\ e-\frac{b*f}{d} \end{bmatrix} \rightarrow \begin{bmatrix} a & -c \\ e & -f \\ -b & d \end{bmatrix} \tag{4}$$

This transformation is applied to all of the fractions in the admittance matrix, which means that all of the fractions are eliminated and admittance matrix now contains only sums of the first order admittances and nullors. Final step of the matrix expansion is to loop through all admittance variables in the list and check their positions and validity. For each of the variable is firstly computed occurrence in the admittance matrix. Only 4 options are possible, there are only one, two, three or four same admittances. In case of only one occurrence, we check if its position is at the main diagonal, which means admittance is grounded.If not, it is necessary to insert arbitrary same admittances to create floating admittance, depending on the position and sign of the admittance. Rule for this operation (5) depends on the position of the nullors. If position of the nullor is not right, additional nullor insertion is necessary.

$$\begin{matrix} & o \\ m & \begin{pmatrix} \infty_i & . \\ . & . \end{pmatrix} \end{matrix} \rightarrow \begin{matrix} & o \\ m & \begin{pmatrix} \infty_i & a \\ b & . \end{pmatrix} \end{matrix} \qquad \begin{matrix} & o \\ m & \begin{pmatrix} \infty_i & \infty_i & . \\ d & . & . \end{pmatrix} \end{matrix} \rightarrow \begin{matrix} & o \\ m & \begin{pmatrix} \infty_i & -\infty_i & c \\ . & d & . \end{pmatrix} \end{matrix} \tag{5}$$

For rest of the occurrences of the admittances a strategy is the same, except mutual positions and sign of the admittances shall be investigated. This algorithm loops through all variables until correct positions and occurrences of the admittances are correct, resulting in the nodal admittance matrix describing architecture and topology of implementation of the given transfer function.

4 CONCLUSION

Provided algorithm proved, that method for symbolic synthesis described in [6], is possible to implement in the computer environment using todays tools. Given method successfully transforming voltage or current transfer function into nodal admittance matrix with little human interaction. For future improvement it is possible to extend this method for automatic pivot expansion and in some level to calculate several or most of the possible realizations of the circuit with equivalent behavior with port matrix.

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