# NON-TRADITIONAL GEOMETRY FOR WEDGE-SPLITTING TEST

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**Abstract.** *The wedge-splitting test is widely used in the testing of fracture mechanical parameters of concrete or concrete-like materials. This test provides a stable crack propagation, which was analysed on a cube specimen. Another interesting application for measuring fracture mechanical properties combines wedge-splitting test with a three-point bending test. In this contribution, we analyse the stress fields in such a nontraditional geometry of wedge-splitting test by employing a linear elastic fracture mechanics*

## Keywords

*Stress Intensity Factor, T-stress Linear Elastic Mechanics, WST, FEM.*

## 1. Introduction

The material's fracture mechanical properties (FMP) are obtained from various test geometries and experimental methods. Widespread and widely used test in testing concrete or concrete-like materials is the wedge-splitting test (WST) [1] (See Fig.1). This test provides a stable crack propagation throughout the test, which results in reliable values of the measured FMPs. The WST uses cube specimens with an initial notch of length $a$. This notch opens when the wedge, through which the load $P$ is applied, enters the notch. The cube geometry gives an effective use of the tested material and can be made from a standard cube used for testing the compressive strength of concrete. The WST was analysed by many studies [2][3][4], thus it is a well-acknowledged test among the researchers.

Another widely used test with reliable results of FMPs used for the evaluation of fracture mechanical properties is three-point bending test (3PBT) [5]. This test is less

effective in the case of used material during the testing as it is tested on the prismatic geometry with various span $S$ to width $W$ ratio $S/W$. Furthermore, this test setup does not need a wedge and the initial notch is placed in the midspan of the specimen and opens when the load is directly applied from the testing apparatus.
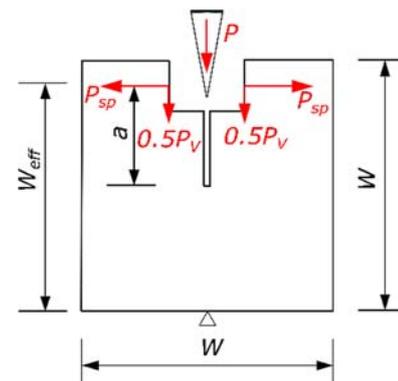


**Fig. 1:** Sketch of traditional WST geometry.

On the other hand, an interesting application for measuring fracture mechanical properties combines wedge-splitting test with a three-point bending test. This nontraditional geometry test provides more stable testing as the crack propagates from both notches simultaneously. The analysed geometry is shown in Fig.2. In this contribution, we analyse the stress fields in such a nontraditional geometry of wedge-splitting test by employing a linear elastic fracture mechanics (LEFM).
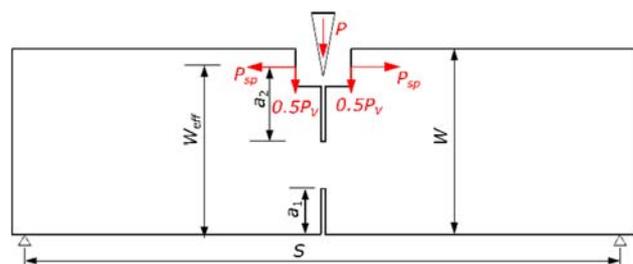


**Fig. 2:** Non-traditional 3PBT geometry with two initial notches .

The objective of this contribution is to analyse and quantify the influence of this nontraditional geometry of WST test with three-point bending test. For this, a numerical parametric study was done in a finite element method software ANSYS [8].

## 2. Theoretical Background

This contribution is based on the LEFM concept, which describes the stress fields in a crack body. For this description of stress fields a Williams expansion [6] is used, which can be expressed as:

$$\sigma_{i,j} = \sum_{n=1}^{\infty} \frac{n}{2} A_n r^{\left(\frac{n}{2}-1\right)} f_n^I(r,\theta) \qquad (1)$$

where $\sigma_{i,j}$ is the stress tensor, $A_n$ is the coefficient corresponding to mode I, $r$ and $\theta$ are the polar coordinates, $n$ is the order of the term and the $f_n^I(r,\theta)$ is the known geometry function. The stress intensity factor (SIF) for crack opening mode I can be described [7] as:

$$K_I = \sigma_{app} \sqrt{2\pi a} B_I \qquad (2)$$

where $K_I$ is the stress intensity factor for mode I, where $\sigma_{app}$ is the applied stress, $a$ is the crack length, and $B_I$ is the geometry function.

Another parameter describing the stress field is the *T-stress,* which is the second term of Williams expansion and can be calculated as:

$$T = \frac{B_2 K_I}{\sqrt{\pi a}} \qquad (3)$$

where $K_I$ is the stress intensity factor evaluated from eq. 1 and $B_2$ is the biaxiality parameter, which again depends on the tested geometry. Please note that the parameters B1 and B2 are calculated from the FEM solution using Eqs. 2-3.

## 3. Numerical Model

In this numerical study, a two-dimensional (2D) numerical model was made in commercial FEM software ANSYS [8]. The numerical model of the modified WST specimen has the dimensions mentioned in Fig. 2 and the material model is considered linear elastic. The novel WST specimen was modelled under plane symmetry conditions $u_y = 0$ in the mid span of the specimen, while the top support was considered as a rigid support ($u_x = 0$). Adequate boundary conditions were added to prevent translation of the rigid body, see Fig. 3. Please note that the span is in the direction of *Y*-axis and the *X*-axis is in the direction of crack growth.

The model was loaded with two perpendicular forces, the sizes of which depend on the angle of the wedge. It simulates the resolution decomposition of the force

applied on top of the wedge into a horizontal and vertical component. The size of the horizontal force, marked as $P_{sp}$, as a main source for crack growth/crack opening, it was established with a constant value of 1000 kN. The size of the horizontal force was calculated as: $P_v = \frac{1}{2} P_{sp} \cdot 2\tan(\alpha_w)$, in which $\alpha_w$ is the wedge angle. The load and boundary conditions are presented in Fig. 3.
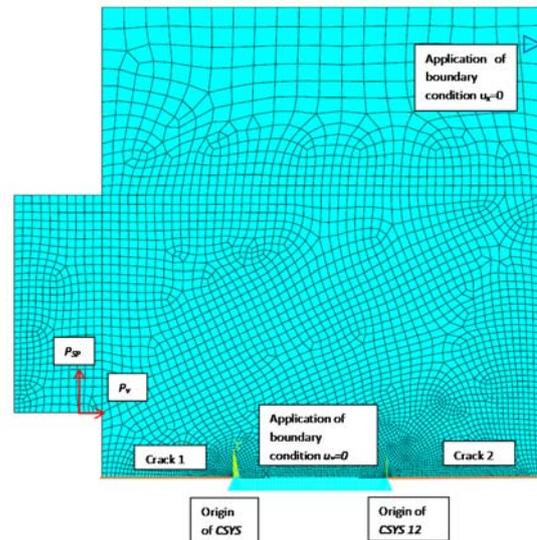


**Fig. 3:** Mesh and boundary conditions of numerical model.

The input material parameters were Young's modulus $E$ of 41 GPa and Poisson's ratio $\nu$ of 0.2. The model was meshed with quadrilateral 8-node elements (PLANE183) with plane strain boundary conditions. The crack tip was meshed using the KSCON command to take into account the crack tip singularity and to provide an angularly structured mesh. The KSCON command deforms the original quadratic elements around the crack tip into triangular ones and shifts the mid-side nodes to a distance of ¼ of the element's edge (towards the crack tip). The notch tip refinement is presented in Fig. 4.
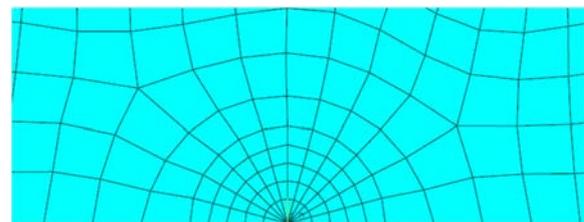


**Fig. 4:** Detail of the mesh at the crack tip.

An interaction integral [9] (which is a derivation of the J-integral [10]) is implemented in the employed FEM software. It provides a sufficient calculation of the SIF and *T*-stress. This calculation uses a path-independent integral around the notch at various radial distances. It is recommended to use of at least four different radial distances for which an interaction integral is calculated. The result is then the average value of the SIFs over these distances. The interaction integral has the following form:

$$I = \frac{2}{E^*}\left(K_1 K_1^{aux} + K_2 K_2^{aux}\right) + \frac{1}{\mu} K_3 K_3^{aux}, \qquad (4)$$

where $K_i$ is the stress intensity factor for mode I, II and III, $K_1^{aux}$ is the auxiliary stress intensity factor for modes I, II and III, $E^*$ is the Young's modulus for plane strain $E/(1-\nu^2)$, $\nu$ is the Poisson's ratio and $\mu$ is the shear modulus. For a 2D problem, the SIF for mode III $K_3$ is 0. The $T$-stress is then calculated using the following equation:

$$T = \frac{E}{(1-\nu^2)}\left\{\frac{I}{f} + \nu\varepsilon_{33}\right\}, \qquad (5)$$

where $I$ is the interaction integral from Eq. 4, $f$ is the line load applied along the crack front (typically $f = 1$) and $\varepsilon_{33}$ is the extensional strain at the crack front in the direction tangential to the crack front. Both the SIFs and the $T$-stress are calculated as an average of four contours around the crack tip.

# 4. Numerical Results

In what follows, we present the results of a preliminary study for the novel WST geometry. First, we present the values of SIF, then the results are accompanied with the values of $T$-stress as calculated for continuous crack growth from both notches. The numerical results presented here under are presented over the relative notch length $\alpha_1$ and $\alpha_2$ calculated as $\alpha_1 = a_1/W$ and $\alpha_2 = a_2/W$. The numerical results of $K_I$ values are shown in Fig. 5, while the results for $T$-stress values are shown in Fig. 6.
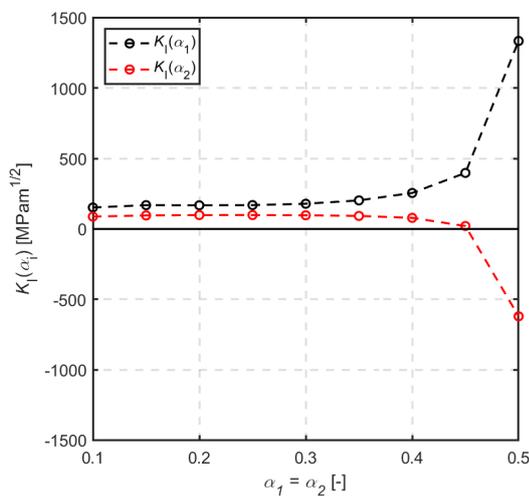


**Fig. 5:** Values of $K_I$ for a constant crack growth from both initial notches.

However, such values of $K_I$ and $T$-stress describe stress conditions for a specific specimen's size and shape. Thus, for a practical application, it is more valid to use dimensionless parameters $B_1$ and $B_2$. Both of these parameters are obtained from Eq. 2 and Eq. 3.
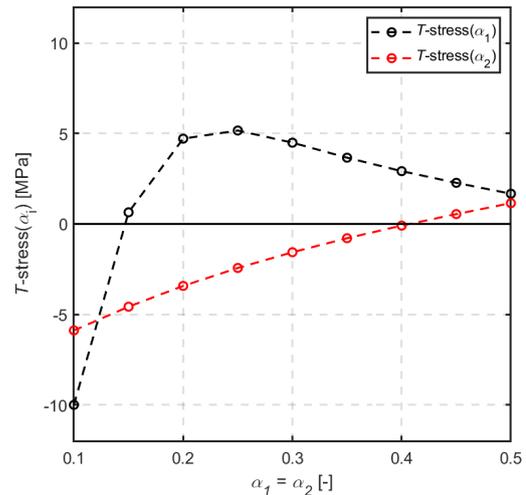


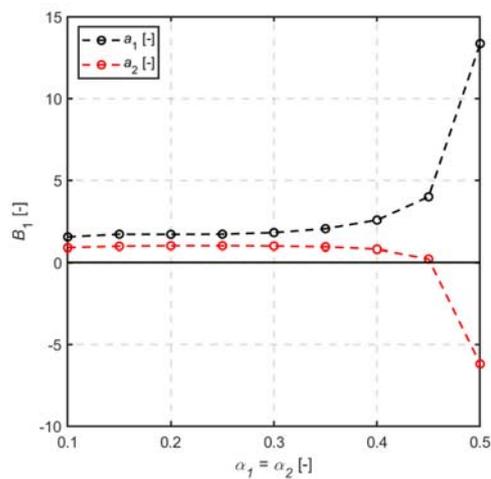**Fig. 6:** Values of $T$-stress for a constant crack growth from both initial notches.



**Fig. 7:** Dimensionless parameter $B_1$ for a constant crack growth from both initial notches.
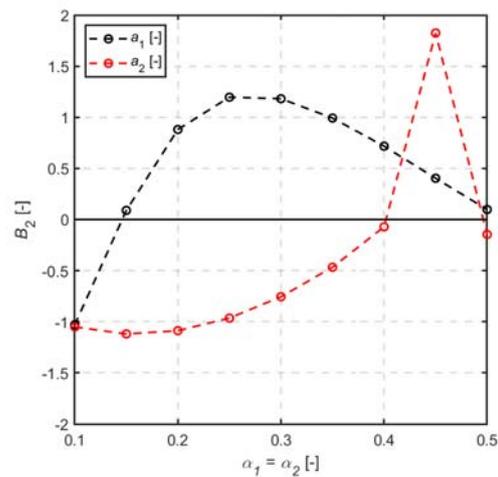


**Fig. 8:** Dimensionless parameter $B_2$ for a constant crack growth from both initial notches.

From the above presented results, it can be observed that the SIFs and parameter $B_1$ have a similar trend, i.e., both are increasing with increasing crack length. However, the trend of the $T$-stress may vary for both cracks $a_1$ and $a_2$. This applies to the parameter $B_2$ as well.

Therefore, it is interesting to see how the parameters $B_1$ and $B_2$ change when one of the cracks remain constant and the second one develops throughout the ligament. The parameter $B_1$ is presented in Fig, 9 and parameter $B_2$ is presented in Fig, 10. Please note that for the case of $B_1$ ($\alpha_2 = 0.1$) crack $a_2$ has a constant length of 0.1 and the crack $a_1$ increases in size).
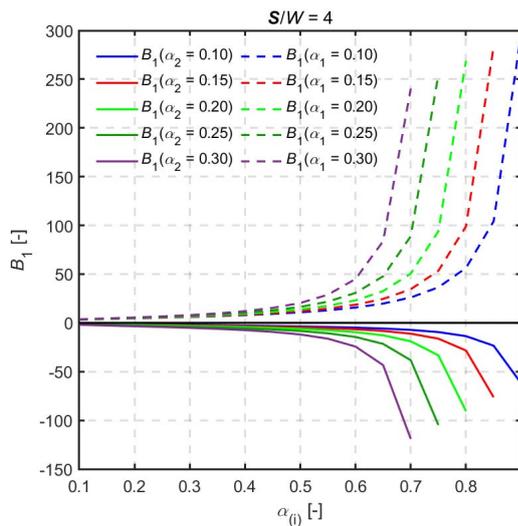


**Fig. 9:** Development of parameter $B_1$ with one crack of constant length and the second growing.
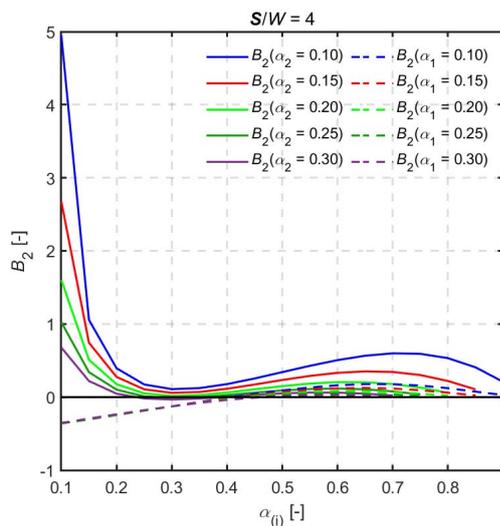


**Fig. 10:** Development of parameter $B_1$ with one crack of constant length and the second growing.

From Fig. 9, one can observe a similar trend in the development of the SIF values as for the case of

simultaneously growing cracks. However, for the case of crack $a_2$, the SIF values are less than zero and can close the crack. On the other hand, parameter $B_2$ shows a similar trend for both cracks and results are presented in Fig. 6.

## 5. Conclusion

The results presented in this study have supported the assumptions of benefits of the combination of two testing methods, commonly used to obtain the material's fracture mechanical parameters. Analogous to the traditional WST, applying force through the loading device using as wedge supports the growth of the top crack. Combination of WST and 3PBT geometry tends to help with the growth of the bottom one. Furthermore, study of the influence of span to width of the specimen has shown that the right ratio could lead to very similar values of the examined parameters at the crack tips of both cracks. Therefore, in the future experimental realization, there is an assumption of not necessarily measuring the results on both crack/notch tips, but that only one could represent both of them precisely. Furthermore, this study found a test configuration for which the crack can close. This fact can be used in further experimental research.

## Acknowledgements

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