



# Compact MOS-RC Voltage-Mode Fractional-Order Oscillator Design

KARTCI, A.; HERENCSÁR, N.; KOTON, J.; PSYCHALINOS, C.

Proceedings of the 2017 23 European Conference on Circuit Theory and Design (ECCTD 2017)

pp. 1-4

eISBN: 978-1-5386-3974-0

DOI: <http://dx.doi.org/10.1109/ECCTD.2017.8093281>

Accepted manuscript

©2017 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works. Aslihan Kartci, Norbert Herencsar, Jaroslav Koton, Costas Psychalinos, "Compact MOS-RC Voltage-Mode Fractional-Order Oscillator Design", Proceedings of the 2017 23 European Conference on Circuit Theory and Design (ECCTD 2017), pp. 1-4, 2017. DOI: 10.1109/ECCTD.2017.8093281. Final version is available at <http://ieeexplore.ieee.org/document/8093281/>

# Compact MOS-RC Voltage-Mode Fractional-Order Oscillator Design

Aslihan Kartci<sup>1,2</sup>, Norbert Herencsar<sup>2</sup>, Jaroslav Koton<sup>2</sup>

<sup>1</sup>Dept. of Radio Electronics / <sup>2</sup>Dept. of Telecommunications  
Brno University of Technology  
Technicka 3082/12, 616 00 Brno, Czech Republic  
{kartci; herencsn; koton}@feec.vutbr.cz

Costas Psychalinos

Physics Department, Electronics Laboratory  
University of Patras  
Rio Patras, GR-26504, Greece  
cpsychal@physics.upatras.gr

**Abstract**—A new voltage-mode fractional-order oscillator, employing in total 12 Metal-Oxide-Semiconductor (MOS) transistors, is introduced in this paper. The proposed circuit is composed of two operational transconductance amplifiers, two inverting voltage buffers, one resistor, and two fractional-order capacitors. Compared with the corresponding already introduced fractional-order oscillators, it offers the benefit of low transistor count and, therefore, simplicity of its structure. In addition, it offers the well-known advantages of fractional-order oscillators about the capability for achieving very low and high oscillation frequencies with reasonable component values. The behavior of the proposed oscillator has been numerically studied using the MATLAB program, while its performance has been evaluated by SPICE simulations, using TSMC 0.18  $\mu\text{m}$  Level-7 CMOS process parameters with  $\pm 1$  V supply voltages.

**Keywords**—fractional calculus; fractional-order oscillator; MOS-RC oscillator; inverting voltage buffer; operational transconductance amplifier

## I. INTRODUCTION

Although the fractional calculus has inherent complexity and lacks in providing a geometrical or physical explanation of the underlying mechanisms, it represents a natural behavior over their integer-order counterparts in many areas of engineering such as bioengineering, electronics, control theory, signal processing, etc. [1]–[8]. In the last years, the study of fractional-order oscillators started to be one of the main fundamental topics in fractional-order dynamical systems. This originated from the fact that extremely low and high frequencies of oscillation are possible through such structures [9]–[10].

Many classical fractional-order oscillators such as Wien-bridge oscillator [11], Colpitts oscillator [12], Hartley oscillator [13], and also multiphase oscillators based on all-pass filters [14], [15], were presented using conventional op-amps or their equivalent macromodels. Although the aforementioned solutions could be implemented using commercially available discrete-component ICs, from the integration point of view they suffer from the increased transistor count that they require for implementing the active cells. Therefore, in addition to the increased circuit complexity, the power dissipation of such structures could become quite high. In order to overcome this obstacle, a novel

very simple voltage-mode (VM) fractional-order oscillator topology is introduced in this paper. It is constructed from two Operational Transconductance Amplifiers (OTAs) and two Inverting Voltage Buffers (IVBs), leading into a requirement for 12 MOS transistors in total. Moreover, the fractional-order capacitor is emulated by using second-order approximation beside solid state fractional capacitors [16]. The proposed structure is presented in Section II, while the characteristic equation is numerically solved to obtain particular frequency of oscillation and start-up condition, in Section III. The performance of the oscillator is evaluated in Section IV, through SPICE simulations results.

## II. THE PROPOSED VM MOS-RC OSCILLATOR

### A. Integer-Order Oscillator Design

The proposed topology is demonstrated in Fig. 1. It is constructed from two CMOS OTAs ( $M_1 - M_4$  and  $M_9 - M_{12}$ ), two NMOS based IVBs ( $M_5 - M_8$ ), one resistor, and two capacitors. Assuming that both capacitors  $C_i$  ( $i = 1, 2$ ) conventional (i.e. integer-order) ideal elements, the characteristic equation (CE) of the oscillator is:

$$\text{CE: } s^2 + s \left( \frac{g_{m2}}{C_2} - \frac{g_{m1}}{C_1} \right) + \frac{g_{m2}}{RC_1C_2} = 0. \quad (1)$$

According to (1), the oscillation starts-up condition (CO) will be:

$$\text{CO: } \frac{g_{m1}}{C_1} = \frac{g_{m2}}{C_2}, \quad (2)$$

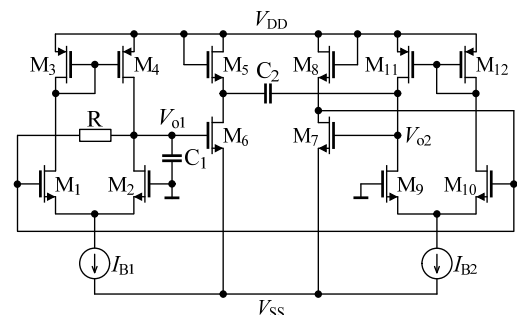


Fig. 1. The proposed compact voltage-mode oscillator.

This article is based upon work from COST Action CA15225, a network supported by COST (European Cooperation in Science and Technology). Research described in this paper was financed by the National Sustainability Program under grant LO1401 and by the Czech Science Foundation under grants no. 16-06175S, 16-11460Y. For the research, infrastructure of the SIX Center was used.

TABLE I. MAIN DESIGN PARAMETERS OF FRACTIONAL-ORDER OSCILLATOR.

No.	Case	Condition of oscillation, frequency of oscillation
1)	$0 < \alpha \leq 1$ $\beta = 1$	$\text{CO: } C_2 = \frac{\omega^{\alpha-1} g_{m2} C_{1\alpha} \sin(0.5\alpha\pi)}{g_{m1} - \omega^\alpha C_{1\alpha} \cos(0.5\alpha\pi)} = \frac{g_{m2} \left[ \omega^\alpha \cos(0.5\alpha\pi) + \frac{1}{RC_{1\alpha}} \right]}{\omega^{1+\alpha} \sin(0.5\alpha\pi)}$ $\text{FO: } \omega^{2\alpha} g_{m2} C_{1\alpha} + \omega^\alpha g_{m2} \cos(0.5\alpha\pi) \left( \frac{1}{R} - g_{m1} \right) - \frac{g_{m1} g_{m2}}{RC_{1\alpha}} = 0$
2)	$\alpha = 1$ $0 < \beta \leq 1$	$\text{CO: } C_{2\beta} = -\frac{g_{m2}}{\omega^\beta \cos(0.5\beta\pi) - \omega^{\beta-1} \frac{g_{m1}}{C_1} \sin(0.5\beta\pi)} = \frac{g_{m2}}{\omega^{1+\beta} RC_1 \sin(0.5\beta\pi) + \omega^\beta g_{m1} R \cos(0.5\beta\pi)}$ $\text{FO: } \omega^2 + \frac{\omega \cot(0.5\beta\pi)}{RC_1} (g_{m1} R + 1) - \frac{g_{m1}}{RC_1^2} = 0$
3)	$0 < \alpha \leq 1$ $0 < \beta \leq 1$	$\text{CO: } C_{2\beta} = \frac{\omega^\alpha g_{m2} C_{1\alpha} \sin(0.5\alpha\pi)}{-\omega^{\alpha+\beta} C_{1\alpha} \sin(0.5(\alpha+\beta)\pi) + \omega^\beta g_{m1} \sin(0.5\beta\pi)} = \frac{g_{m2} \left[ \omega^\alpha RC_{1\alpha} \cos(0.5\alpha\pi) + 1 \right]}{-\omega^{\alpha+\beta} RC_{1\alpha} \cos(0.5(\alpha+\beta)\pi) + \omega^\beta g_{m1} R \cos(0.5\beta\pi)}$ $\text{FO: } \omega^{2\alpha} RC_{1\alpha}^2 \sin(0.5\beta\pi) - \omega^\alpha g_{m1} RC_{1\alpha} C_{2\beta} \left[ \sin(0.5(\alpha-\beta)\pi) + \frac{1}{g_{m1} R} \sin(0.5(\alpha+\beta)\pi) \right] + g_{m1} \sin(0.5\beta\pi) = 0$

while the frequency of oscillation (FO) is:

$$\text{FO: } \omega_0 = \sqrt{\frac{g_{m2}}{RC_1 C_2}}. \quad (3)$$

Also, the outputs of the oscillator are related as follows:

$$\frac{V_{o1}}{V_{o2}} = -\frac{g_{m1} R + 1}{sRC_1 + 1}, \quad (4)$$

while their phase difference is:

$$\phi = -\pi - \tan^{-1}(\omega RC_1). \quad (5)$$

### B. Fractional-Order Oscillator Design

Depending on the selection which capacitor is fractional-order, three possible cases can be considered:

Case 1: The capacitor  $C_1$  is considered to be fractional-order with an impedance  $Z_{C1}(s) = 1/s^\alpha C_{1\alpha}$ , while the other capacitor  $C_2$  remains an integer-order capacitor with impedance  $Z_{C2}(s) = 1/sC_2$ .

Case 2: The capacitor  $C_2$  is considered as fractional-order capacitor with an impedance  $Z_{C2}(s) = 1/s^\beta C_{2\beta}$ , while  $Z_{C1}(s) = 1/sC_1$ .

Case 3: Both capacitors are considered as fractional-order capacitors with impedances  $Z_{C1}(s) = 1/s^\alpha C_{1\alpha}$  and  $Z_{C2}(s) = 1/s^\beta C_{2\beta}$ , respectively.

The characteristic equations in the aforementioned cases can be written as follows:

$$\text{CE}_1: s^{1+\alpha} + s^\alpha \frac{g_{m2}}{C_2} - s \frac{g_{m1}}{C_{1\alpha}} + \frac{g_{m2}}{RC_{1\alpha} C_2} = 0, \quad (6)$$

$$\text{CE}_2: s^{1+\beta} + s \frac{g_{m2}}{C_{2\beta}} - s^\beta \frac{g_{m1}}{C_1} + \frac{g_{m2}}{RC_1 C_{2\beta}} = 0, \quad (7)$$

$$\text{CE}_3: s^{\alpha+\beta} + s^\alpha \frac{g_{m2}}{C_{2\beta}} - s^\beta \frac{g_{m1}}{C_{1\alpha}} + \frac{g_{m2}}{RC_{1\alpha} C_{2\beta}} = 0. \quad (8)$$

Setting  $s = j\omega$  in (6)–(8), the derived FOs and COs are summarized in Table I.

In fractional-order case 1, 2, and 3, the relation between the outputs of the oscillator is:

$$\frac{V_{o1}}{V_{o2}} = -\frac{g_{m1} R + 1}{s^\alpha RC_{1\alpha} + 1}, \quad (9)$$

while the phase difference  $\phi$  between two outputs  $V_{o1}$  and  $V_{o2}$  can be calculated as:

$$\phi = -\pi - \tan^{-1} \left( \frac{\omega^\alpha \sin(0.5\alpha\pi)}{\omega^\alpha \cos(0.5\alpha\pi) + \frac{1}{RC_{1\alpha}}} \right) = \pi - \tan^{-1} \frac{\omega^\beta \sin(0.5\beta\pi)}{\omega^\beta \cos(0.5\beta\pi) - \frac{g_{m2}}{C_{1\alpha}} - \frac{1}{RC_{1\alpha}} + \frac{g_{m2}}{C_{2\beta}}}. \quad (10)$$

Note that setting  $\alpha = \beta = 1$  in the expressions in Table I as well as in (9)–(10), then the expressions in (2)–(5) are derived.

### III. NUMERICAL RESULTS

As it will be shown, the main advantage of this type of oscillator is that the fractional-order oscillators generate higher FOs than their integer-order counterparts. Considering the fractional-order oscillator of Case 3 and assuming typical active parameters of OTA and passive component values as

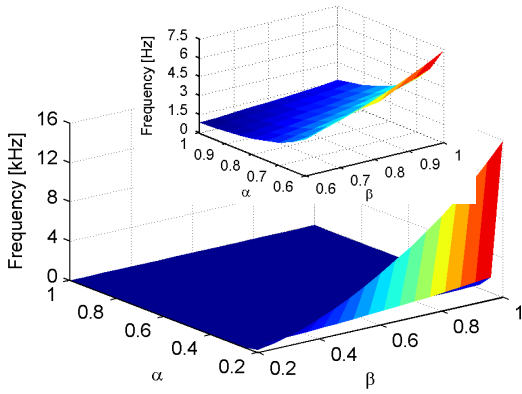


Fig. 2. Frequency of oscillation versus alpha ( $\alpha$ ) and beta ( $\beta$ ).

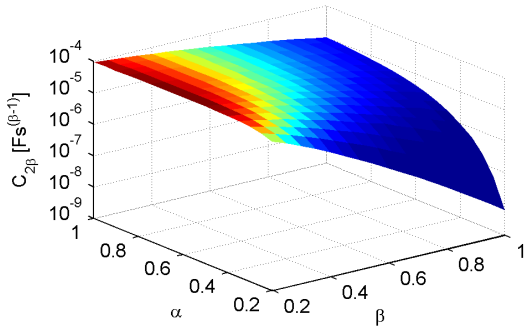


Fig. 3. Condition of oscillation versus alpha ( $\alpha$ ) and beta ( $\beta$ ).

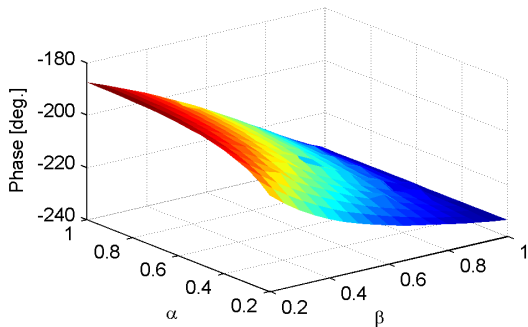


Fig. 4. Phase difference between the two outputs based on (10).

follows:  $g_{m1} = g_{m2} = 100 \mu\text{A/V}$ ,  $R = 10 \text{ k}\Omega$ ,  $C_{1\alpha} = 10 \mu\text{Fs}^{(\alpha-1)}$ , then from Table I the following frequency of oscillation and oscillation start-up condition values  $f_0 = \{1.18; 4.26; 1.25 \text{ k}\}\text{Hz}$  and  $C_{2\beta} = \{63.8 \mu; 37.3 \mu; 155 \text{ n}\}\text{Fs}^{(\beta-1)}$  for  $\alpha = \{0.65; 0.5; 0.25\}$  and  $\beta = \{0.4; 0.5; 0.9\}$ , are derived. Fig. 2 shows MATLAB plots of frequency of oscillation versus fractional-orders  $\alpha$  and  $\beta$  (see in Table I FO of Case 3). As it can be observed the FO increases while the order of  $\alpha$  decreases and  $\beta$  increases. In other words, FO is decreases while the order of  $\alpha$  increases and  $\beta$  decreases. Note that, the CO in (2) requires equality of both capacitances and transconductances. Therefore, considering an integer-order capacitor ( $\alpha = 1$ ) with the same ideal capacitance  $C_1 = 10 \mu\text{F}$ , the FO and oscillation start-up condition are respectively 1.5915 Hz and 10  $\mu\text{F}$ , as shown in Figs. 2 and 3. Furthermore, it can be observed that the start-up condition  $C_{2\beta}$  decreases

while the frequency of oscillation increases with increasing the order  $\beta$  or decreasing  $\alpha$ . Figure 4 shows the phase difference between two outputs based on (11). Considering again the orders  $\alpha = \{0.65; 0.5; 0.25\}$  and  $\beta = \{0.4; 0.5; 0.9\}$ , the phase differences were calculated as  $\phi = \{-199.4; -208.7; -231.6\}$ , respectively. Hence, by increasing the order  $\alpha$  or decreasing  $\beta$  the phase difference is increasing.

#### IV. SIMULATION RESULTS

In order to validate the numerical results of the proposed oscillator shown in Fig. 1, the simulation results were performed using SPICE program. In the design, transistors were modeled by the TSMC 0.18  $\mu\text{m}$  Level-7 CMOS process parameters ( $V_{th\_N} = 0.35 \text{ V}$ ,  $\mu_N = 327 \text{ cm}^2/(\text{V}\cdot\text{s})$ ,  $V_{th\_P} = -0.41 \text{ V}$ ,  $\mu_P = 129 \text{ cm}^2/(\text{V}\cdot\text{s})$ ,  $T_{OX} = 4.1 \text{ nm}$ ). The DC power supply voltages were set equal to  $+V_{DD} = -V_{SS} = 1 \text{ V}$ . Aspect ratios of CMOS transistors are  $W/L = 40 \mu\text{m}/1.2 \mu\text{m}$  for all PMOS and  $W/L = 0.8 \mu\text{m}/1.2 \mu\text{m}$  for NMOS in OTAs. In inverting voltage buffers both NMOS were set  $W/L = 60 \mu\text{m}/1 \mu\text{m}$ . The bias currents in OTAs were set  $I_{B1} = I_{B2} = 100 \mu\text{A}$ , which results in transconductance equal to  $100 \mu\text{A/V}$ .

First of all, the integer-order case ( $\alpha = \beta = 1$ ) with passive component values  $C_1 = 10 \mu\text{F}$  and  $R = 10 \text{ k}\Omega$  was studied. By solving the system of (2) and (3), the oscillation start-up condition and frequency of oscillation are found as  $C_2 = 10 \mu\text{F}$  and  $f_{0\_theor\_int} = 1.59 \text{ Hz}$ , which is close to simulated value  $f_{0\_sim\_int} = 1.5 \text{ Hz}$ . The simulated phase difference between the outputs was  $-203.7^\circ$ , close to  $-225^\circ$ , which is theoretically predicted by (5). The transient responses of the outputs are shown in Fig. 5(a) and simulated peak-to-peak values of oscillation amplitudes are  $V_{o1\_pp} = 798.4 \text{ mV}$ ;  $V_{o2\_pp} = 570.1 \text{ mV}$ . The theoretical ratio of amplitudes according to (4) is 1.41, very close to 1.40 ratio obtained through simulations.

Secondly, the fractional-order cases are studied for selected orders  $\alpha = \{1; 0.2; 0.2\}$  and  $\beta = \{0.2; 1; 0.8\}$ , respectively. The fractional-order capacitors were realized using the Foster I network depicted in Fig. 6. The values of passive elements have been calculated by employing the second-order Continued Fraction Expansion method, and they are given in Table II. The calculated oscillation start-up conditions are  $C_{2\beta} = \{91.21 \mu; 6.31 \text{ n}; 89.9 \text{ n}\}\text{Fs}^{(\beta-1)}$  and the FOs are  $f_{0\_theor\_frac} = \{0.25; 15.9 \text{ k}; 9.64 \text{ k}\}\text{Hz}$ , while the simulated FOs are 0.25 Hz, 15 kHz, and 10 kHz, respectively. Transient responses of the outputs for fractional-order cases are shown in Figs. 5(b)–(d). The following peak-to-peak values of oscillation amplitudes were simulated for outputs  $\{V_{o1}; V_{o2}\}$ : Case 1  $\{516.6; 253.1\}\text{mV}$ , Case 2  $\{608.9; 602.8\}\text{mV}$ , and Case 3  $\{575.1; 542.1\}\text{mV}$ , respectively. Here the theoretical ratios of amplitudes according to (9) are 1.98; 1.01; 1.06. On the other hand, the simulated ratios are 2.04; 1.01; 1.06. The simulated phase shifts were  $\{-190.05^\circ; -180.03^\circ; -180.1^\circ\}$ , while the calculated values were  $\{-225^\circ; -181.78^\circ; -181.78^\circ\}$  while. Lissajous patterns of all discussed cases showing phase shifts between outputs are depicted in Fig. 7.

TABLE II. COMPONENT VALUES USED IN FIG. 6 FOR SIMULATION OF FRACTIONAL-ORDER OSCILLATOR CASES.

Components	Component Values			
	Case 1	Case 2	Case 3	
	$\alpha = 0.2$ $C_{1\alpha} = 10 \mu\text{F}s^{-0.8}$ @ 15.9 kHz	$\beta = 0.2$ $C_{2\beta} = 91.21 \mu\text{F}s^{-0.8}$ @ 0.25 Hz	$\alpha = 0.2$ $C_{1\alpha} = 10 \mu\text{F}s^{-0.8}$ @ 9.64 kHz	$\beta = 0.8$ $C_{2\beta} = 89.9 \text{nF}s^{-0.2}$ @ 9.64 kHz
$R_a$ [ $\Omega$ ]	5.45k	5.45k	6.03k	79.1
$R_b$ [ $\Omega$ ]	4.3k	4.26k	4.7k	659.6
$C_b$ [F]	836p	52.8 $\mu$	1.25n	19.3n
$R_c$ [ $\Omega$ ]	8.6k	8.61k	9.52k	34.1k
$C_c$ [F]	5.97n	376.9 $\mu$	8.91n	376.9 $\mu$

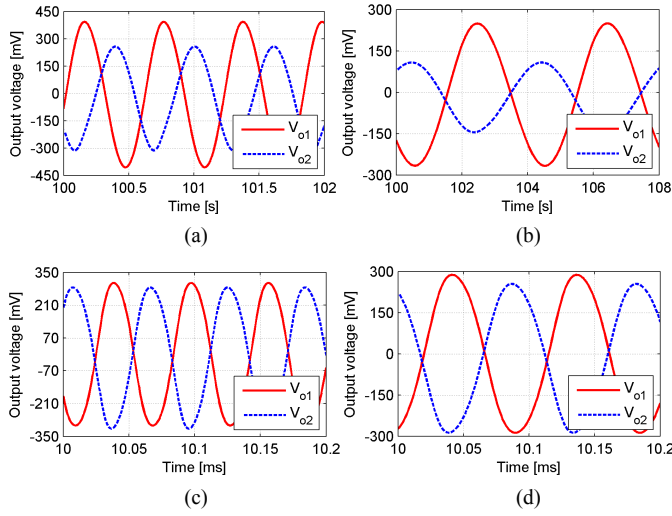


Fig. 5. Transient responses of the output voltages: (a)  $\alpha = 1$  and  $\beta = 1$ , (b)  $\alpha = 1$  and  $\beta = 0.2$ , (c)  $\alpha = 0.2$  and  $\beta = 1$ , (d)  $\alpha = 0.2$  and  $\beta = 0.8$ .

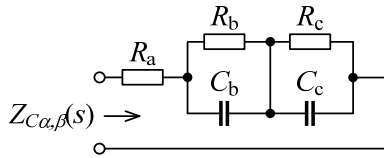


Fig. 6. RC tree for approximating fractional-order capacitors.

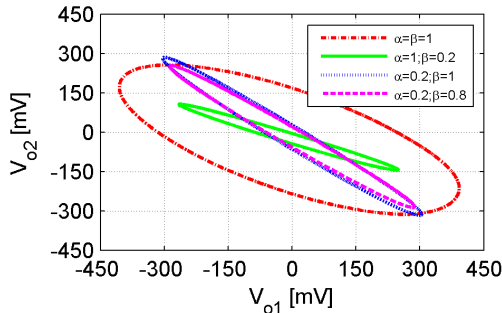


Fig. 7. Lissajous patterns for variable order capacitors showing phase shifts of  $V_{o2}$  against  $V_{o1}$ .

## V. CONCLUSION

The very first time in the literature, this paper introduced a compact voltage-mode fractional-order oscillator employing only 12 MOSFETs. The main motivation of this study by designing fractional-order oscillator was to prove that this type of oscillators offer: (i) independent tuning of the frequency and condition of oscillation, (ii) higher frequency of oscillation than its integer-order counterpart, (iii) requirement for capacitances with reasonable values, (iv) possibility for achieving different frequency of oscillation/start-up condition by only changing the order/capacitance values. The derived SPICE simulation confirmed the given theoretical analyses.

## REFERENCES

- [1] R.E. Gutiérrez, J.M. Rosário, J.T. Machado, "Fractional Order Calculus," Basic Concepts and Engineering, vol. 2010, article ID 375858, 19 pages, 2010.
- [2] N. Sebaa, Z.E.A. Fellah, W. Lauriks, C. Depollier, "Application of fractional calculus to ultrasonic wave propagation in human cancellous bone," Signal Processing, vol. 86, no. 10, pp. 2668-2677, 2006.
- [3] T.J. Freeborn, B. Maundy, A.S. Elwakil, "Field programmable analogue array implementations of fractional step filters," IET Circuits, Devices & Systems, vol.4, no.6, pp. 514-524, 2010.
- [4] I. Podlubny, I. Petras, B. Vinagre, P. O'Leary, L. Dorcak, "Analogue realizations of fractional-order controllers," Nonlinear Dynamics, vol. 29, pp. 281-296, 2002.
- [5] J.F. Gómez-Aguilar, J.J. Rosales-García, J.J. Bernal-Alvarado, T. Córdova-Fraga, R. Guzmán-Cabrera, "Fractional mechanical oscillators," Revista Mexicana de Física, vol. 58, pp. 348-352, 2012.
- [6] N. Sugimoto, "Burgers equation with a fractional derivative; hereditary effects on nonlinear acoustic waves," Journal of Fluid Mechanics, vol. 225, pp. 631-653, 1991.
- [7] S. Abbisso, R. Caponetto, O. Diamante, L. Fortuna, D. Porto, "Non-integer order integration by using neural networks," In Proceedings of IEEE International Symposium Circuits Systems, pp. 688-691, 2001.
- [8] J.F.G Aguilar, J.J.B. Alvarado, J.J.R. Garcia, T.C. Fraga, "Modeling and simulation of equivalent circuits in description of biological systems-a fractional calculus approach," Journal of Electrical Bioimpedance, vol. 3, pp. 2-11, 2012.
- [9] A. Lahiri, "Low-frequency quadrature sinusoidal oscillators using current differencing buffered amplifiers," Indian Journal of Pure & Applied Physics, vol. 49, no. 6, pp. 423-428, 2011.
- [10] A.S. Elwakil, A. Allagui, B.J. Maundy, C. Psychalinos, "A low frequency oscillator using a super-capacitor," AEU-International Journal of Electronics and Communications, vol. 70, pp. 970-973, 2016.
- [11] W. Ahmad, R. El-Khazali, A.S. Elwakil, "Fractional-order Wien-bridge oscillator," Electronics Letters, vol. 37, pp. 1110-1112, 2001.
- [12] A.G. Radwan, A.S. Elwakil, A.M. Soliman, "Fractional-order sinusoidal oscillators: design procedure and practical examples," IEEE Trans. on Circuits and Systems I: Regular Papers, vol. 55, pp. 2051-2063, 2008.
- [13] A.S. Elwakil, A. Agambayev, A. Allagui, K.N. Salama, "Experimental demonstration of fractional-order oscillators of orders 2.6 and 2.7," Chaos, Solitons & Fractals, vol. 96, pp. 160-164, 2017.
- [14] M.E. Fouda, A. Soltan, A.G. Radwan, A.M. Soliman, "Fractional-order multi-phase oscillators design and analysis suitable for higher-order PSK applications," Analog Integrated Circuits and Signal Processing, vol. 87, pp. 301-312, 2016.
- [15] B. Maundy, A. Elwakil, S. Gift, "On the realization of multiphase oscillators using fractional-order allpass filters," Circuits, Systems, and Signal Processing, vol. 31, pp. 3-17, 2012.
- [16] D.A. John, K. Biswas, "Electrical equivalent circuit modelling of solid state fractional capacitor," AEU-International Journal of Electronics and Communications, vol. 78, pp. 258-264, 2017.