Comments and Remarks over Classic Linear Loop-Gain Method for Oscillator Design and Analysis. New Proposed Method Based on NDF/RR$_T$

José Luis JIMÉNEZ-MARTÍN[1], *Vicente GONZÁLEZ-POSADAS*[1], *Ángel PARRA-CERRADA*[1], *Luis Enrique GARCÍA-MUÑOZ*[2], *Daniel SEGOVIA-VARGAS*[2]

[1]DIAC, Universidad Politécnica de Madrid, Ctra valencia km 7, Madrid 28031, Spain
[2]Grupo RADIO, Universidad Carlos III, Avda. Universidad 30, Leganés 28911, Spain

vgonzalz@diac.upm.es, dani@tsc.uc3m.es

Abstract. *This paper describes a new and original method for designing oscillators based on the Normalized Determinant Function (NDF) and Return Relations (RR_T). Firstly, a review of the loop-gain method will be performed. The loop-gain method pros, cons and some examples for exploring wrong solutions provided by this method will be shown. This method produces in some cases wrong solutions because some necessary conditions have not been fulfilled. The required necessary conditions to assure a right solution will be described. The necessity of using the NDF or the Transpose Return Relations (RR_T), which are related with the True Loop-Gain, to test the additional conditions will be demonstrated. To conclude this paper, the steps for oscillator design and analysis, using the proposed NDF/RR_T method, will be presented. The loop-gain wrong solutions will be compared with the NDF/RR_T and the accuracy of this method to estimate the oscillation frequency and Q_L will be demonstrated. Some additional examples of plane reference oscillators (Z/Y/Γ), will be added and they will be analyzed with the new NDF/RR_T proposed method, even these oscillators cannot be analyzed using the classic loop gain method.*

Keywords

Oscillator, Normalized Determinant Function, *NDF*, Stability, Loop-Gain.

1. Introduction

Oscillators are one of the most important elements in all radiofrequency and microwave systems as for example the Radar systems [1]. Unfortunately, they have a great inconvenience, which is being one of the hardest circuits to be designed because of their inherent non-linear behavior.

Nowadays, linear simulation and its approximation to the first harmonic are widely used for simplifying the initial design [2], [3] and for other useful and interesting reasons as estimating some important parameters. For example, linear simulation requires less computational capacity than non-linear simulation and the non-linear models are not always available or accurate enough. Before starting a non-linear analysis it is usually required to have a good oscillation frequency and Q_L approximation, being also advisable to have, at least, some knowledge of non-linear simulation and approximations [4] and, in some hard cases, of non linear solutions stability analysis [5], [6], [7], [8].

As linear simulation requires less computational burden, linear simulation is inherently quicker and easier, more suitable for circuit parameters tuning and very useful to development of new topologies. Linear simulation only requires active device *S* parameters or a linear model, which is easier to obtain than the non-linear one. On the other hand linear simulation, for oscillators design, is only suitable for oscillation frequency and Q_L estimation, but it is not suitable for output power, phase noise (although it can be estimated from Q_L value), harmonics level and time domain signal estimation. Common methodology consists of a first linear design and coarse tuning with a final fine optimization using harmonic balance and transient simulation [9].

Classic linear analysis methods can be classified into two groups. The first one is called loop-gain [3], [10] and the second one is referred to as Reference Plane [2], [11], [12], [13]. This last group includes negative resistance, negative conductance and reflection coefficient methods. Each one has its own pros and cons, but the main advantage of reference plane methods is that they can be directly used for designing *RF* and microwave circuits. This advantage becomes clearer when circuits including distributed elements are used. The distributed elements make it really difficult or even impossible to use the Alechno virtual ground method [3]. On the other hand, important circuit parameters, such as gain margin and loaded *Q*, (directly related to phase noise) can be obtained directly using Loop-Gain method. The start-up time can be obtained using loaded *Q* and gain margin [9], [10], [14]. It is important to point out that these parameters are difficult to extract using plane reference methods.

This paper shows a novel and general application method based on a “true loop-gain”, term deeply described in the next section. At the same time, the Loop Gain method is explained and its problems are analyzed and the conditions for proper use are also defined. Throughout the following four sections, the new proposed method is described and design examples are provided.

2. Previous and Necessary Conditions for the Proper Use of Loop-Gain Method.

Any oscillator can be analyzed by means of $Z/Y/\Gamma$ network functions. They are more accurate to use than general transfer functions or loop-gain function because they include all the system poles [15]. These two different ways of view of an oscillator are drawn in Fig. 1.

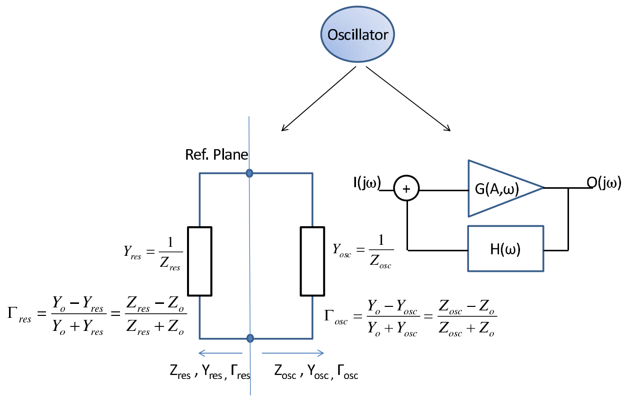


Fig. 1. Oscillator analysis: Reference plane method(left bottom) and Feedback scheme(right bottom).

The use of a plane reference methods depend on being able to identify the resonator circuit or not. They have been widely used when the feedback path and the resonator, as a quadrupole, are difficult to identify. When the feedback path is identified, the Loop-Gain is commonly used [9]. Many times, choosing between both only depends on the designer’s preferred design topology or even on his experience using each method.

The start point for the loop-gain analysis is the general function of a loopback system (Fig. 1), defined by (1)

$$X_0(s) = \frac{G(s)}{1 - G(s)\cdot H(s)} \cdot X_i(s). \tag{1}$$

In a real circuit, specially in RF and MW, it is difficult, or even impossible, to define (1) as an analytical function in Laplace domain. Designers usually tend to use Nyquist analysis for oscillator start-up conditions verification [16], and sometimes, they even use simplifications of the Nyquist criteria as Barkhausen criteria, although it has much more restrictions. Nyquist criteria fixes the oscillation condition by means of $(1 - G(s)\cdot H(s))$ zeros location, which in fact, are loop-back system function poles. The Nyquist criteria uses the argument principle, so what is really being calculated is the difference between zeros and poles of $(1 - G(s)\cdot H(s))$ lying in the Right Half Plane (RHP). For a well-conditioned start-up condition and achieving a single frequency periodic solution, the necessary condition is to have only a pair of complex conjugated poles in the Right Half Plane (RHP).

Loop-gain [10], is based on previous works, mainly in a feed-back structure stated by Randall and Hock (Fig. 2). Their work determined the Z parameters network function, which can be rewritten as an S parameters expression (2), which became a great success in linear oscillator design

$$\begin{aligned} \frac{I_0}{I} &= \frac{Z_{21} - Z_{12}}{Z_{11} - Z_{12} - Z_{21} + Z_{22}} \\ &= \frac{S_{22} - S_{11} + 2S_{21} - S_{12}S_{21} + S_{11}S_{22} - 1}{1 + (S_{12} + S_{21} - S_{12}S_{21} + S_{11}S_{22})}. \end{aligned} \tag{2}$$

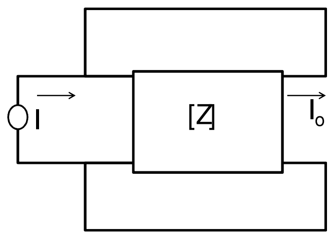


Fig. 2. Randall Feed-Back proposed structure.

For the network function (2), poles in the RHP can be calculated by the frequency response of the denominator, which is widely known as Characteristic Function (CF) (3). This equation is useful to show and explain some problems on the loop-gain analysis.

$$CF = 1 + (S_{12} + S_{21} - S_{12}S_{21} + S_{11}S_{22}). \tag{3}$$

For a proper use of the Nyquist analysis (frequency response) it is imperative to assure that the RHP poles of the network function (2) only come from CF zeros (3). When the network function is analysed, it can only be guaranteed if none of S parameters has poles (visible or hidden [17], [18], [19]) in the RHP. This lack of poles can only be demonstrated by using the Normalized Determinant Function (NDF) as Platzer [17], [18] or Jackson [19] defined to determinate the stability of a Z_0 loaded quadrupole. But, this specific condition was not envisaged by Randall and Hock in their work [10].

NDF test is conceptually equivalent to the Rollet proviso in amplifiers design [19], but, in our case, it is used to assure the open-loop gain stability of the oscillator. Open-loop stability is necessary before starting a Nyquist analysis of the CF. This condition is equivalent to K [20] or μ [21] analysis for amplifiers design.

Related to the *CF* use, it is necessary to point out that the use of Nyquist analysis for *CF* only determines the existence of poles in the *RHP*. Even if it is supposed that the $\Im(CF)$ crossover zero is the oscillation frequency (which is more accurate for high Q poles), as the *CF* is not a loop gain it cannot provide information about the gain margin and the Q_L. They are necessary parameters to obtain the start-up time and phase noise estimations of oscillators. Even if *CF* is properly used and it predicts rightly the oscillator start-up condition, this method has an inherent variance of the solution depending on the opening loop point for the analysis. The obtained S parameters (that comply the *NDF* test function criteria) are functions of the point where the loop is opened. Even if they have the information about the poles of the system, the Nyquist analysis of the different *CF* provide different traces and different zero crossovers.

This *CF* problem made Randall and Hock to obtain a more suitable equation, which is invariant with the opening point of the loop. It is the main advantage compared with Rhea and Alechno's work [22], [3]. At a first attempt, Alechno only used S_{21} value, subsequently, Alechno redefined and modified the equation [23] to include S_{11} and S_{22} influence. The final equation (4), defined by Randall and Hock work, takes into account all S-parameters influence. It was obtained by calculating the eigenvalues of an infinite chain of Z_0 loaded quadrupoles [10]

$$G_L = \frac{Z_{21}-Z_{12}}{Z_{11}+Z_{22}-2Z_{12}} = \frac{S_{21}-S_{12}}{1-S_{11}S_{22}+S_{12}S_{21}-2S_{12}}. \quad (4)$$

Anyway this new open-loop gain also requires, like the *CF* expression, to verify some conditions for proper *RHP* poles prediction. Even being independent from the opening point of the loop, it still depends, like the *CF* expression, on the position of the virtual ground. This variance will be clearly shown with examples in Section 4. This variance modifies oscillation frequency, quality factor and gain margin results where, obviously, a unique solution should be obtained. The virtual ground position can even cause a false negative oscillation condition.

In order to define the required additional conditions for proper loop-gain method use the authors of this paper have rewritten network function (2) as it is shown in (5)

$$\frac{I_0}{I} = \frac{\left(\dfrac{1-S_{11}S_{22}+S_{12}S_{21}-2S_{12}-S_{22}+S_{11}}{1-S_{11}S_{22}+S_{12}S_{21}-2S_{12}}\right)}{\left(1-\dfrac{S_{21}-S_{12}}{1-S_{11}S_{22}+S_{12}S_{21}-2S_{12}}\right)}. \quad (5)$$

The loop-gain (4) can be identified on the denominator of (5), which is one minus Randall loop gain. But in our case it will be possible to obtain a "true loop-gain" as network function with similarities with a general feedback system function

Once the general feed-back system function (2) has been rewritten using G_L expression (6), the necessary conditions for guaranteeing the use of G_L for oscillator design can be analysed.

$$\frac{I_0}{I} = \frac{Z_{21}-Z_{12}}{Z_{11}-Z_{12}-Z_{21}+Z_{22}} = \frac{\left(-\dfrac{G_L(S_{22}-S_{11})}{S_{21}-S_{12}}+1\right)}{(1-G_L)}. \quad (6)$$

Nyquist analysis of the loop-gain must search for $+1$ encircling. To assure oscillation stability, it is necessary that the poles of the system only come from the zeros of $(1-G_L)$. To guarantee that the poles of the system only come from the zeros of $1-G_L$ it is necessary that:

- G_L does not have any poles in the *RHP*. This condition is fulfilled if "test function" $TF = 1-S_{11}S_{22}+S_{12}S_{21}-2S_{12}$ does not have any zero in the *RHP*; and S_{21} and S_{12} do not have any poles in the *RHP*. This test function can be properly analyzed with Nyquist criteria if none of the S parameters have any poles, hidden or not, in the *RHP*. If defined conditions are not satisfied, the zeros of $1-G_L$ function could be hidden to Nyquist analysis.
- The numerator in (5) must not have any visible or hidden poles in the *RHP*. It is assured if none of S parameters of the circuit have any poles in the *RHP* and the expression $1-S_{11}S_{22}+S_{12}S_{21}-2S_{12}$ does not have any zeros in the *RHP*.

In order to verify the previous conditions it is necessary to apply the Normalized Determinant Function (*NDF*) criteria to the Z_0 loaded quadrupole to guarantee that none of the S parameters have any pole in the *RHP*. The next step is to analyze the Nyquist trace of the G_L denominator, to assure that it does not have any zeros in the *RHP*. After checking these requisites the analysis of G_L can be used to determine the existence of poles of the I_0/I function. The oscillation frequency predicted by the Nyquist analysis of G_L is only accuracy for poles with high Q. But it is important to remember that the multiple virtual ground possibilities of the circuit provides different solutions of frequency, Q_L and gain margin, as it will be shown in Section 4. The common used approximation of $S_{11} \approx S_{22} \approx S_{12} \approx 0$ presents similar issues as the described ones.

The necessary conditions to guarantee the proper analysis of the oscillation condition have been described. It is possible to extend these considerations to the first harmonic approximation. The conditions of the characteristic equation for oscillation stability and minimum phase noise are summarized in Tab. 1, where $G_T = 1-G_L = 1-G_{osc}\cdot G_{res}$. These conditions have been obtained directly from those described for oscillators analyzed by the reference plane [15], [11] as their characteristic equations are formally the same one.

In Tab. 1, G_L is divided into two terms: active part G_{osc}, and resonator term G_{res}. The A variable is the amplitude, usually the incident wave for S parameters,ω is the frequency; A_0 and ω_0 are the amplitude and the frequency at the oscillation condition. But even it has an attractive formal aspect, it is important to remember that this is only an approximation to the first harmonic. The approximation is more accurate when A_0 is purer, in other words, when the resonator has a higher Q and the compression is lower.

Equations in Tab. 1, also suppose that G_{osc} has a small variation with the frequency. This condition is easy to fulfill if active and passive elements are properly isolated, but in fact, it is difficult to achieve in a real circuit.

Parameter	Definition
Characteristic Equation	$G_T(A,\omega) = -\dfrac{1}{G_{osc}(A)} + G_{res}(\omega) = 0$
Oscillation Condition	$G_T(A_0,\omega_0) = -\dfrac{1}{G_{osc}(A_0)} + G_{res}(\omega_0) = 0$
Stability	$\frac{1}{G_{osc}(A)}$ with $G_{res}(\omega)$ cross into a clockwise angle from 0 to π
Minimum noise	$\frac{1}{G_{osc}(A)}$ with $G_{res}(\omega)$ cross into a $\frac{\pi}{2}$ clockwise angle

Tab. 1. Loop Gain Oscillation Conditions.

3. Proposed Method Based on NDF/RR$_T$

One of the most important conclusions that were obtained in Section 2 is that it is required to use the *NDF* before performing the G_L analysis. On the same way, this conclusion can be applied to plane reference methods, like negative conductance, negative resistance and refection coefficient, as authors stated in [15]. There is only one way to assure that Y_{osc}, Z_{osc}, and Γ_{osc} do not have any hidden or visible poles in the *RHP*. This is to use *NDF* to analyse the active subcircuit network loaded with a short-circuit for Y_{osc}, with an open circuit for Z_{osc} or with Z_0 for Γ_{osc}. This guaranteeing method was explained by the authors in [15] taking [17], [18], [19], [25] as references. At this point, the proper question would be "*Why not use the NDF for oscillator design?*". This question is the starting point for this section, where the *NDF*/RR_T method is described pointing out its advantages over the reference plane and loop-gain methods.

NDF (7) was proposed by Platzer [17], [18] to verify the Rollet proviso. They proposed a practical and rigorous method for an N-port network stability analysis. *NDF* is defined as the quotient between the network determinant and the normalized network determinant. This last one is obtained by disabling active devices,

$$NDF = \frac{\triangle(s)}{\triangle_0(s)}. \quad (7)$$

Properties of this function have been deeply described by Platzer [18], pointing that it is suitable to assure if a network function has any poles in the *RHP*. A clockwise Nyquist *NDF* trace encircling of the origin indicates the existence of a pole (a complex pair of poles) in the *RHP*. The *NDF* has unity as upper boundary so it is easy to determine the maximum frequency to be analysed.

Return Relations (RR_i) were defined by Bode [26]. Platzer [17] describes its use to calculate the *NDF* as

$$NDF = \prod_{i=0}^{n} (RR_i + 1). \quad (8)$$

The RR_i term is the Return Relation for the *i*-depending generator while previous $i-1$ have been disabled. The active devices must be replaced with linear models as the one shown in Fig. 3. So, as it was explained before, it is necessary to have a linear model of the transistor, if it is not available, it can be extracted by simulation from the non-linear [19] or *S* parameters one.

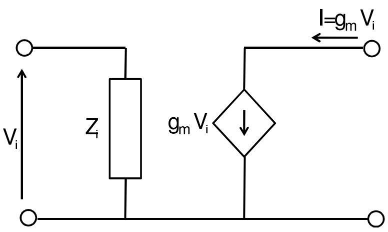


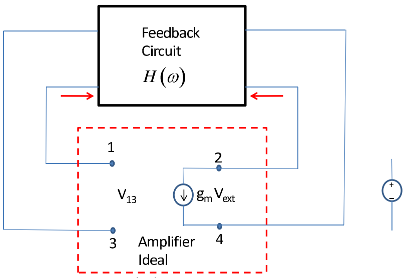
Fig. 3. Lineal BJT Model.

Redrawing linear equivalent circuit of the oscillator to have access to the terminal of the depending generator, and all other components including transistor parasites in the passive feedback sub-circuit, it is possible to calculate *RR* and so, *NDF* (Fig. 4).

NDF used as design function does not need any additional calculus or supposition for determining the oscillation frequency of the first harmonic approximation (Kurokawa approximation). It provides direct information about the *RHP* poles of the system, which should only be a pair of complex poles to achieve a proper oscillating start-up and stability condition. The oscillation frequency is obtained without requiring the compression of the transistors, the Q_L and gain margin of the oscillator is obtained without any ambiguity. This unambiguous result is achieved thanks to using the *RR*, which is the only proper way to consider an oscillator as a feedback system by isolating the active element from the rest of the circuit. It should be pointed out that, as it is presented in Fig. 4, the loop gain can be related to *RR* as

$$\begin{aligned} True\ Loop\ Gain &= RR_T = -RR = RR_{osc} \cdot RR_{res} \\ &= g_m \cdot H(\omega). \end{aligned} \quad (9)$$

Using (9) and Tab. 1 it is possible to redefine the conditions for oscillation start-up, oscillation stability and minimum phase noise. These conditions for RR_T have been defined in Tab. 2 following an analogous process as it was done for G_L. The RR_T is divided into its active device contribution $RR_{osc} = g_m$, which is function of the depending generator, and its passive contribution $RR_{res} = H(\omega)$, which includes all passive elements and transistor parasitics. In Tab. 2, V is the control variable of the depending generator, ω is the frequency; and V_0 and ω_0 are respectively the voltage at oscillation condition and the oscillation frequency. It is important to point out that this solution is the first harmonic approximation, so it will be more accurate when V becomes a purer tone. The tone is purer when the resonator has a higher Q and the gain margin is smaller. Also, it is supposed that RR_{osc} is invariant with the frequency invariant. It is absolutely true because RR_{osc} only depends on the transconductance of the transistor.



$$RR = \frac{-V_{13}}{V_{ext}} = \frac{-H(\omega)\cdot g_m \cdot V_{ext}}{V_{ext}} = -g_m \cdot H(\omega)$$

Fig. 4. Oscillator Model. As feedback system for RR extraction.

Parameter	Definition
Characteristic Equation	$1 - RR_T(V,\omega) = -\frac{1}{RR_{osc}(V)} + RR_{res}(\omega) = \ldots$ $\ldots = -\frac{1}{g_m(V)} + H(\omega) = 0$
Oscillation Condition	$1 - RR_T(V_0,\omega_0) = -\frac{1}{RR_{osc}(V_0)} + RR_{res}(\omega_0) = \ldots$ $\ldots = -\frac{1}{g_m(V_0)} + H(\omega_0) = 0$
Stability	$^1/_{RR_{osc}(V)}$ with $RR_{res}(\omega)$ cross into a clockwise angle from 0 to π
Minimum noise	$^1/_{RR_{osc}(V)}$ with $RR_{res}(\omega)$ cross into a $\pi/2$ clockwise angle

Tab. 2. NDF/RR_T Oscillation Conditions.

4. Practical Examples

An oscillator circuit without specific ground reference is the base circuit used in this section. Different virtual ground points can be defined for this basic circuit using Alechno [24]. Some resulting possibilities of this example are the well-known classic topologies: common collector (also named Colpitts), common emitter (also named Pierce) and common base. Using virtual ground concept, it is demonstrated that there is a unique oscillator topology and, as it will be explained throughout this example, NDF/RR_T is the best tool to analyze it.

The circuit in Fig. 5 is an oscillator without ground reference. It uses as active device a BFR360F transistor biased with $I_C = 10$ mA and $V_{CE} = 3$ V. AWR software has been used for all the simulations shown in this paper. The circuits in Figs. 6, 7 and 8 include the parasitics of the package of all devices. These figures show classic topologies obtained by using virtual ground theorem. It should be pointed out that these figures have been already drawn with open-loop point to be used for G_L and CF analysis.

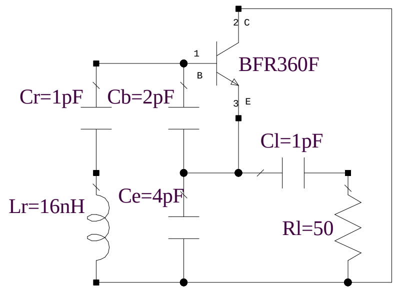


Fig. 5. General oscillator model without ground reference.

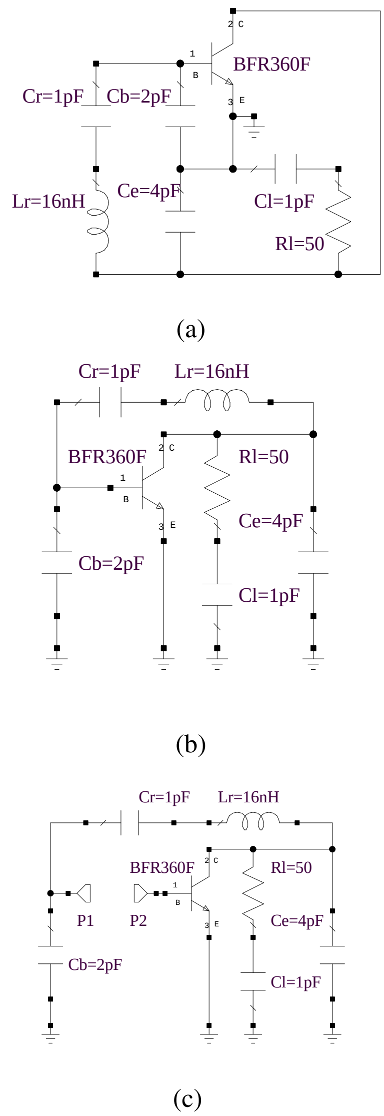


Fig. 6. a) Common emitter oscillator. b) Pierce classic topology. c) Common emitter oscillator opened for G_L analysis.

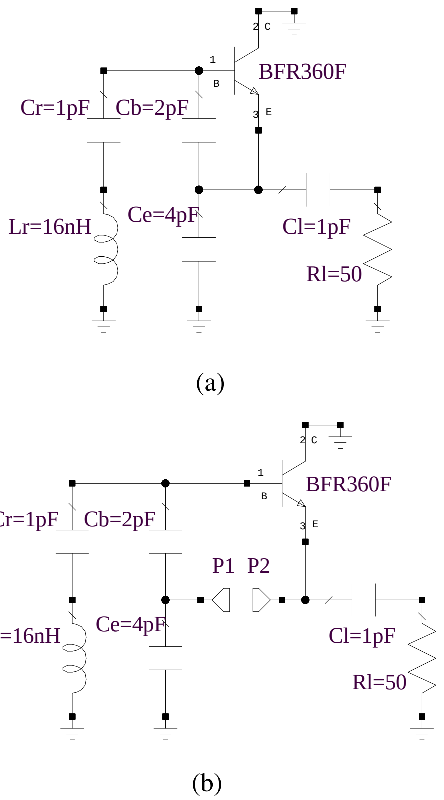


Fig. 7. Common collector oscillator. a) Colpitts classic topology. b) Opened for G_L analysis.

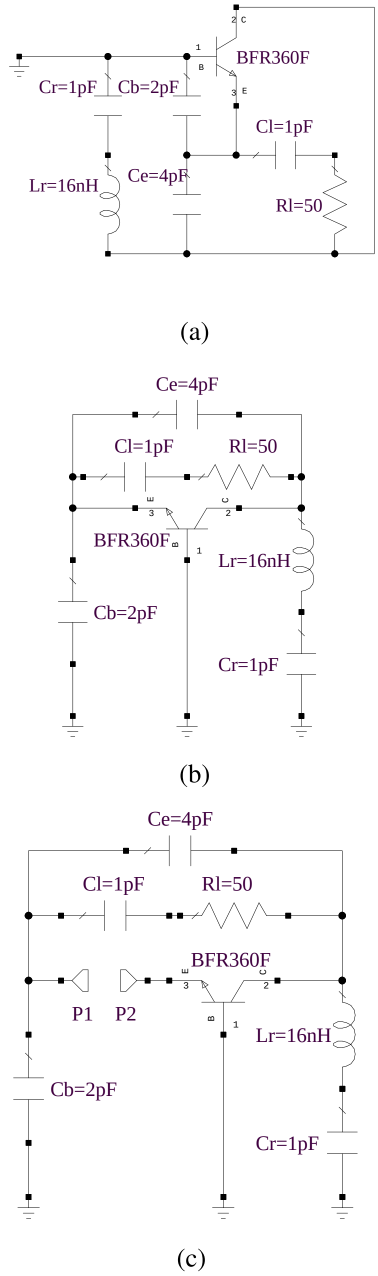


Fig. 8. a) Common base oscillator. b) Classic topology. c) Common base oscillator opened for G_L analysis.

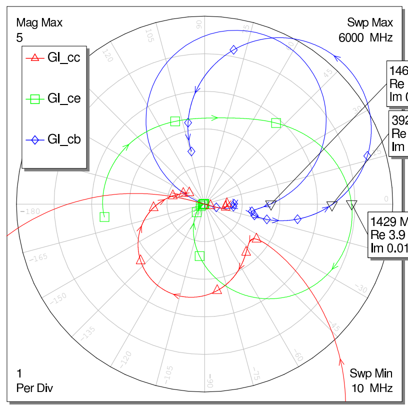


Fig. 9. G_L Nyquist plot for common emitter, common collector and common base.

Obviously the three "topologies" must have the same oscillation frequency, gain margin and quality factor (Q_L). As first step, Randall and Hock G_L is going to be used to analyze all the obtained circuits topologies, specifically with the circuits on Fig. 6(c), Fig. 7(b) and Fig. 8(c).

The open-loop analysis, Fig. 9, predicts that only the common emitter topology will oscillate, but "*How can it be possible if the three schematics represent the same circuit?*". The problem appears due to the Nyquist analysis of G_L expression is not valid for common collector and common base topologies. In these two cases the denominators of G_L have two hidden zeros which make Nyquist analysis not to encircle $+1$.

It is interesting to point out that the G_L analysis of the common collector circuit does not cross the positive real axis with a real part greater than the unit. So, it does not comply with the Barkhausen criteria, and from a traditional point of view, it will not oscillate. Meanwhile, common base circuit complies the Barkhausen criteria at two different frequencies, 1467 MHz and 3927 MHz. The first one crosses from a positive to a negative phase, but the second one crosses from a negative to a positive phase. This example can be considered as complementary to Nguyen examples [16], but $+1$ is not encircled.

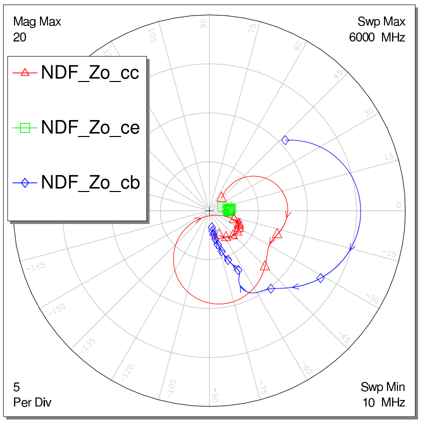


Fig. 10. NDF Nyquist plot of Z_0 loaded circuits.

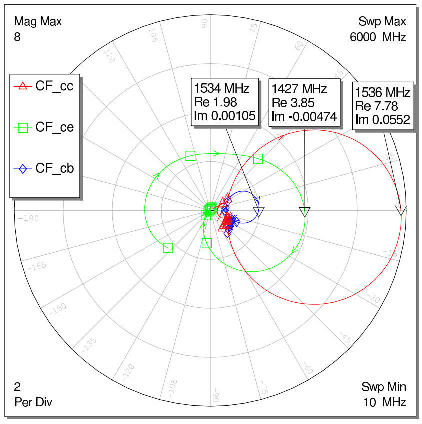


Fig. 12. CF Nyquist plot of Z_0 loaded circuits.

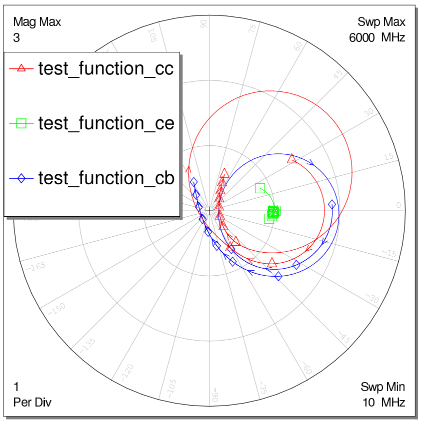


Fig. 11. "Test Function" Nyquist plots.

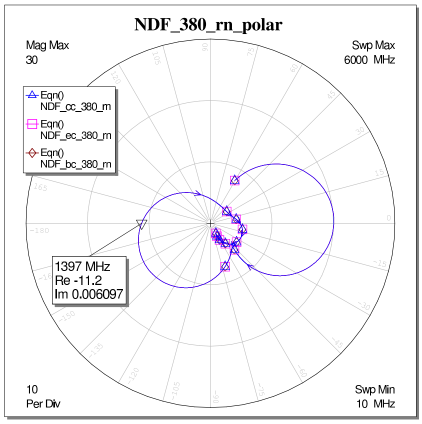


Fig. 13. NDF Nyquist plot for the three circuit topologies.

In Section 2, the required conditions for a proper G_L oscillator analysis were defined. The conditions for an S parameter analysis were the Nyquist *NDF* analysis of the open-loop quadrupole loaded with Z_0 and the Nyquist verification of "test function" $(1 - S_{11}S_{22} + S_{12}S_{21} - 2S_{12})$. None of these function analyses can have any zeros in the *RHP*.

Fig. 10 shows the Nyquist *NDF* plots of Fig. 6(c), Fig. 7(b) and Fig. 8(c) quadrupoles loaded with the characteristic impedance Z_0. The nyquist plots confirm that all the circuits are stable when they are loaded with Z_0 because none of the *NDF* plots encircle the origin.

At the same time, Fig. 11 shows the Nyquist plots of the three "test functions" showing that the common collector and the common base plots encircle the origin. These two "test functions" have a pair of zeros in the *RHP*, so it is not possible to use G_L expression to analyze them as oscillators.

Another possibility is to use the *CF* (3) to analyze the oscillator instead of using the G_L. In this case, only the first condition of the G_L case (*NDF*) is required. It has been previously verified that none of the proposed topologies have any zeros in the *RHP*, so the *CF* analysis should provide correct solutions. Fig. 12 shows the *CF* Nyquist plots of the three circuits. The obvious advantage of requiring only the first condition (*NDF*) is not such a big advantage because *CF* function has a strong dependence with the selected open-loop point and with the virtual ground position. Meanwhile, although G_L has no dependence with the open-loop point, it has dependence with the virtual ground position.

On the other hand, the *NDF*, or the RR_T, analysis has a unique solution for the three schematics, Fig. 13. As all the *NDF* analyses are identical and they predict a unique complex pair of poles in the *RHP*, so the required condition for proper oscillation is satisfied. Although the *AWR* commercial simulation software is used and it already has the *NDF* function, it is possible to calculate *RR* with any simulator. The RR_T is usually calculated, it is $-RR$ and "the true loop gain". All the simulations presented in this paper do not use *AWR NDF* function, they have been solved using Platzer definition, Fig. 4.

The second example is a similar oscillator scheme that has additional resistors for circuit biasing. Now that the bias circuit is included, it is possible to use the transistor non linear model (Fig. 14). As in the previous example, the base circuit has no ground reference point and it is redrawn as the three classic topologies using the virtual ground concept.

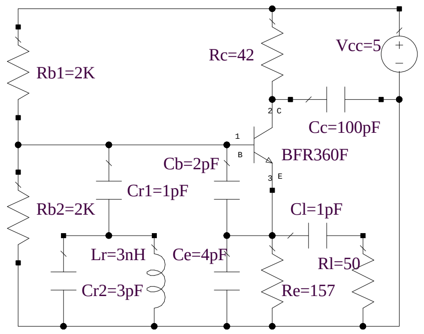


Fig. 14. Biased general oscillator without ground reference.

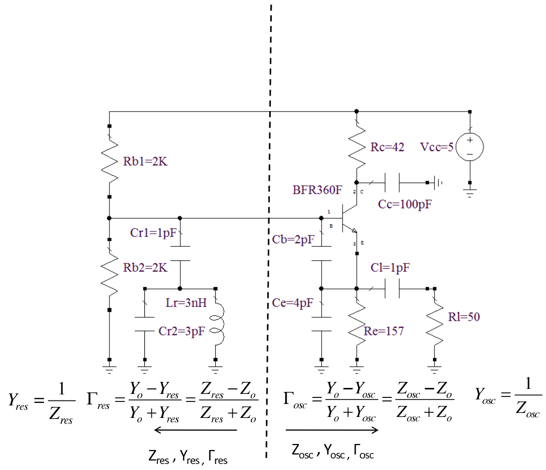


Fig. 15. Biased common collector oscillator.

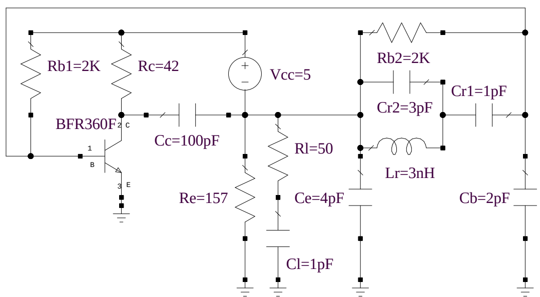


Fig. 16. Biased oscillator as common emitter.

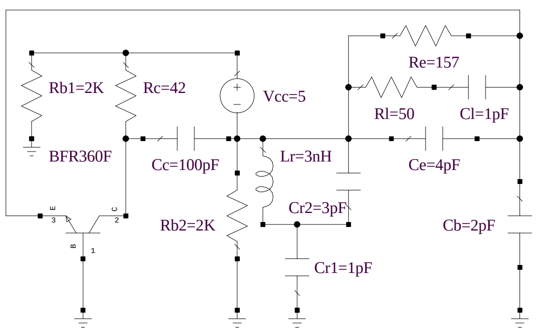
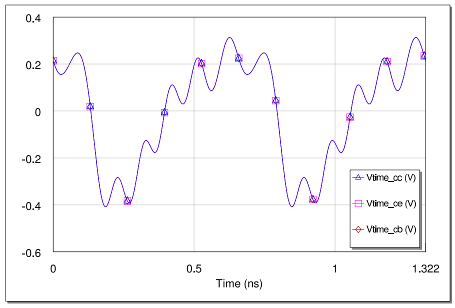


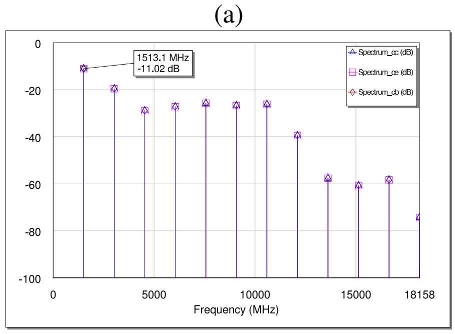
Fig. 17. Biased oscillator as common base.

Figs. 15, 16 and 17 respectively show the three common topologies: common collector, common base and common emitter. Common collector oscillator, also named Colpitts or Clapp, is usually analyzed with the negative resistance method or the reflection coefficient method, as it is shown in Fig. 15. For an effective demonstration of the three schematics are the same circuit, the Harmonic Balance analysis (HB) and Time Domain analysis(TD) are used to obtain the signal spectrum and the phase noise.

Fig. 18 shows the time domain responses and spectra of the three different oscillator configurations. As it can be checked, the time domain responses and spectra are the same for the three circuits.



(a)



(b)

Fig. 18. Time(HB) and Spectrum of the CC, CE and CB models.

The phase noise of the common emitter and the common base circuits are identical, Fig. 19. The phase noise of the common collector is nearly identical,but it differs at noise floor. This small difference must be caused by numerical errors, maybe because of Jacobian matrix conditioning [27].

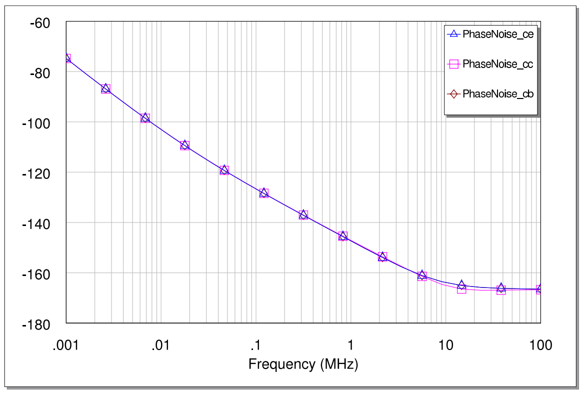
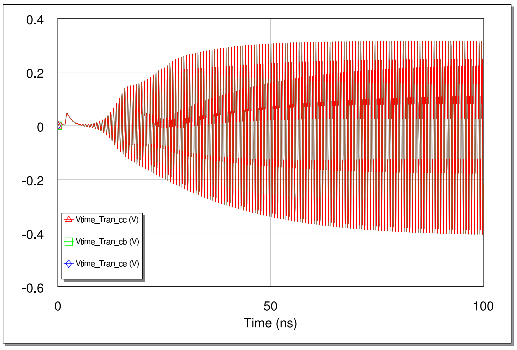
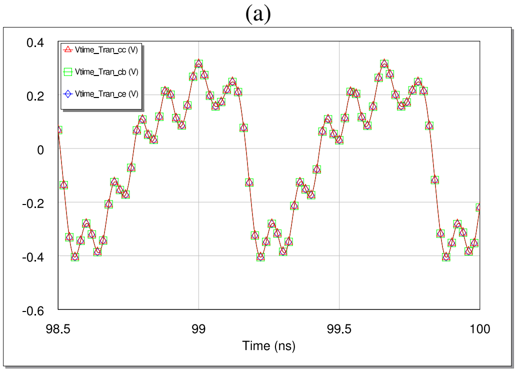


Fig. 19. Phase noise of the CC, CE and CB models.

A transient time simulation has been performed using a *DC* linear ramp source from 0 V to 5 V on 5 ns as transient generator signal. Start-up transients and steady state signals are shown in Fig. 20. They are identical for the three circuits. With all these simulations and results it can be assured that the three circuits are the same.



(a)



(b)

Fig. 20. Transient time (a) and steady state signal (b) of the CC, CE and CB models.

Now, the three circuits are analyzed using *CF*, the G_L and the RR_T expressions. The open-loop circuits from Figs. 15, 16 and 17 used for *CF* and G_L analysis are shown in Figs. 21, 22 and 23 respectively. It has been added a high value inductance in series, for biasing the circuit, and two high value capacitors for DC-block. In this way, the *S* parameters, needed for *CF* and G_L calculus, have been extracted by simulation.

The first step is to check the necessary conditions before starting a *CF* or G_L analysis. The *NDF* Nyquist plots of the quadrupoles loaded with $Z_0 = 50$ must not clockwise encircle the origin. Fig. 24(a) shows that none of the *NDF* encircle the origin, so the three schematics are suitable for *CF* analysis. But for the G_L, it is necessary a second condition to check, it is the "test function" Nyquist plot. Plots of these three "test functions" are shown in Fig. 24(b). The "test functions" of the common emitter and common collector circuits do not encircle the origin, so these two circuits are suitable for G_L analysis. But, the common base one is not suitable to be analysed by G_L because the "test function" Nyquist plot encircles the origin, so G_L analysis of this circuit will provide wrong results.

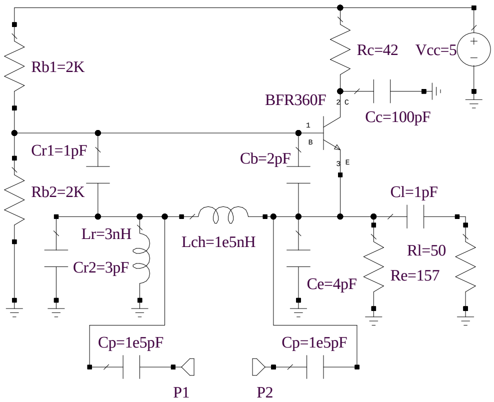


Fig. 21. Circuits for CF and G_L analysis common collector.

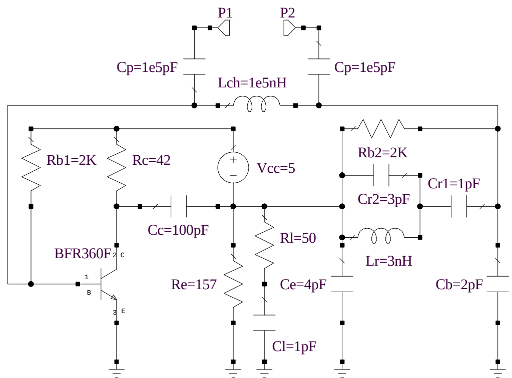


Fig. 22. Circuits for CF and G_L analysis common emitter.

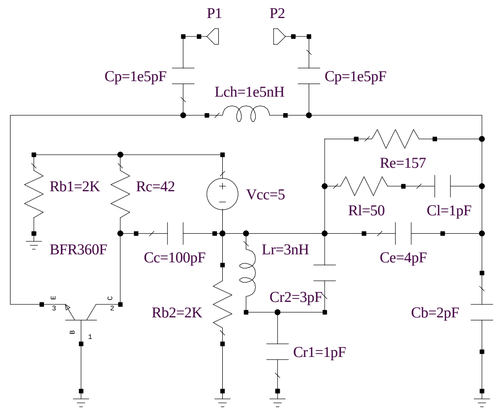


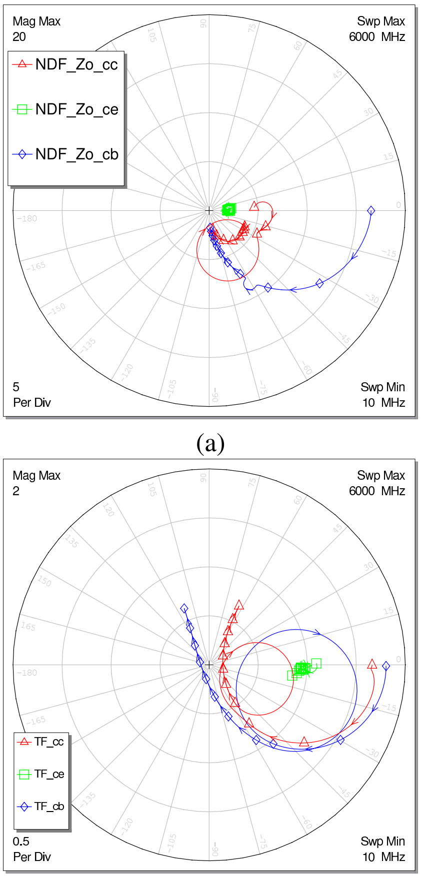
Fig. 23. Circuits for CF and G_L analysis common base.

CF expressions of all three circuits have been simulated and they are presented in Fig. 25. The *CF* plots predict proper start-up condition for all circuits, but each circuit present a different gain margin. Common base gain margin has an important difference with the other two ones. This fact demonstrates that *CF* solution is a function of the virtual ground location.

It is also possible to define the open loop loaded quality factor (Q_L) as

$$Q_L = -\frac{\omega}{2} \cdot \frac{d}{d\omega} Arg\left(F\left(\omega\right)\right) = -\frac{f}{2} \cdot \frac{d}{df} Arg\left(F\left(f\right)\right). \quad (10)$$

Q_L is a function of the speed of the phase variation with the frequency [9] and it also has dependence of the selected function F(f) to be analysed. In our examples, this definition can be applied to CF, G_L, RR_T or NDF, although it is only suitable for being used with RR_T. Fig. 26 shows the Q_L results using the CF function. Significant differences between circuits can be observed, but these differences cannot exist because they are all really the same circuit.



(a)

(b)

Fig. 24. Three circuits: a) Zo = 50 loaded NDF Nyquist plot. b) Test function Nyquist plot.

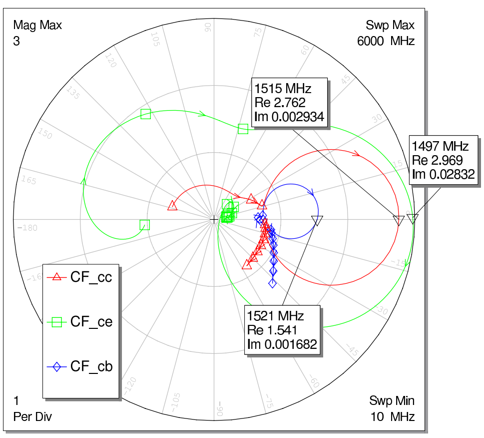


Fig. 25. CF of the three circuit models.

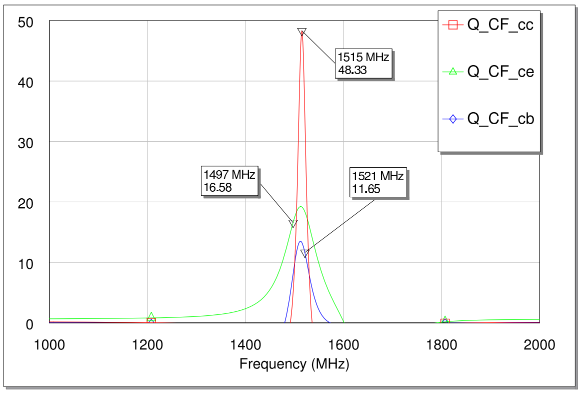


Fig. 26. Oscillator models Q_L obtained from its CF.

To sum up, CF properly predicts oscillation condition for three topologies of the oscillator of this example, but the obtained gain margin and Q_L are not unique. All of these point us that this method is not a suitable option for designing.

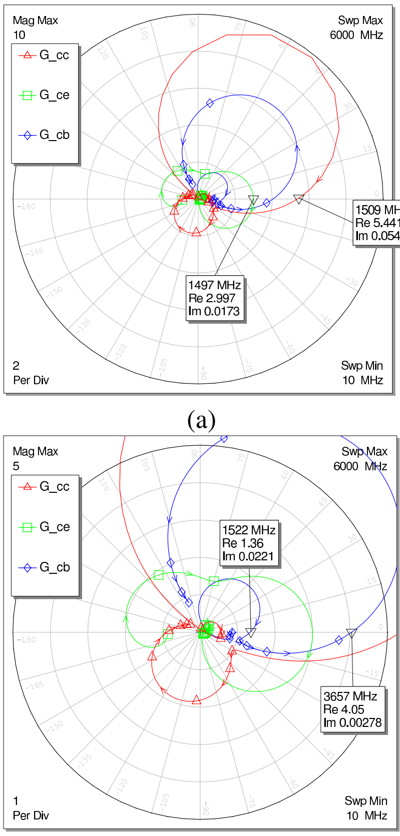
It seems to be that G_L is a better option than CF, because G_L has no dependence with the open-loop point. But, it is important to point that G_L has dependence with the virtual ground position. The G_L Nyquist plots of the three topologies are shown in Fig. 27. These plots predict oscillation for the common collector and common emitter circuits, but not for the common base one.

The G_L Nyquist plot of the common base circuit has two crosses with the real positive axis, but it does not clockwise encircle the $+1$, then it does not comply with Randall and Hock oscillation condition. Although common emitter and common collector predict similar oscillation frequencies, the gain margin and Q_L values have a significant difference, Fig. 28. When Randall start-up time approximation (11) is used with the data from Fig. 28 and 29 different results will be obtained. The same difference appears when Q_L is used to estimate the phase noise using Lesson's model,

$$t_r \approx \frac{38.2 \cdot Q_L}{\omega_0 \cdot G_{0dB}}. \tag{11}$$

But, NDF and RR_T Nyquist plots are identical for the three circuits, as it is shown in Fig. 29. This way the Q_L factors are also identical, Fig. 30, and then the start-up time and phase noise estimation are also identical.

All the CF and G_L previous analysis are small signal analysis. g_m must be compressed to apply the first harmonic analysis approximation. When g_m value is compressed to make the complex zeros (poles of the network function) to locate over the imaginary axis, the Nyquist plots of G_L and CF will cross over $+1$, as shown in Fig. 31. In this case CF and G_L crossing frequencies match the NDF and RR_T one. But, even with the compressed g_m the Q_L factors, Fig 32, are still different.



(a)

(b)

Fig. 27. G_L of the three circuit models. a) full view. b) detailed view.

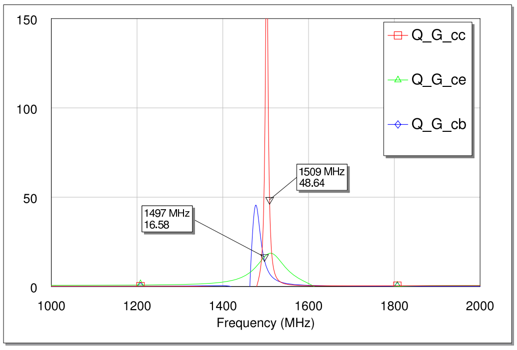


Fig. 28. Oscillator models Q_L obtained from its G_L.

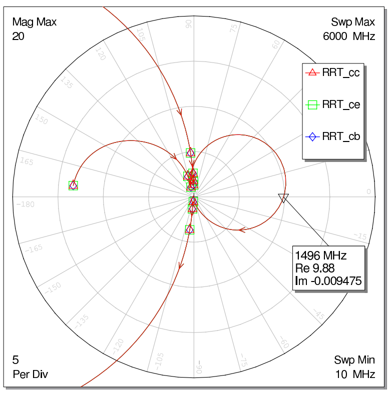


Fig. 29. RR_T of the three circuit models.

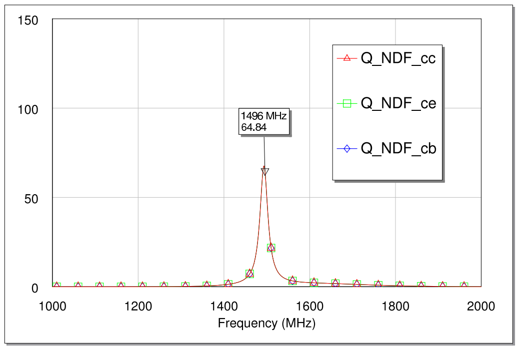
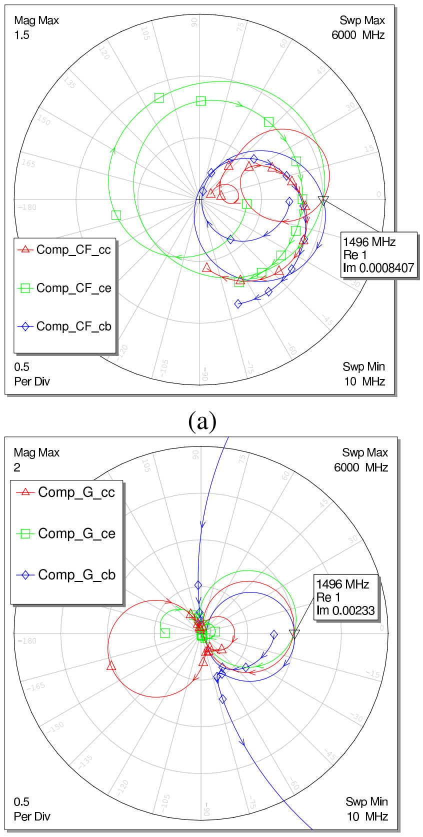


Fig. 30. Oscillator models Q_L obtained from its NDF/RR_T.



(a)

(b)

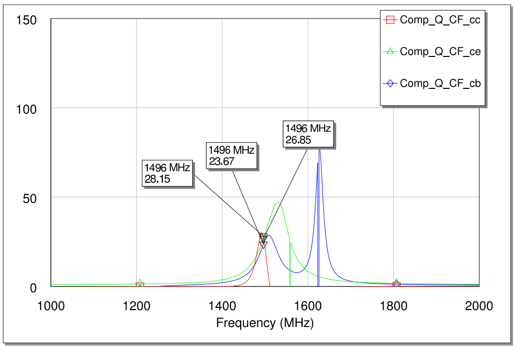
Fig. 31. CF(a) and G_L(b) Nyquist plots of the g_m compressed circuit models.

g_m is only a scale factor of RR_T and NDF, so the crossing frequency (oscillation frequency) and the Q_L factor of the RR_T model does not depend on g_m. NDF and RR_T provide directly the first harmonic approximation solution. And Q_L factor is also a good approximation of the real phase noise of the oscillator, Fig. 19. The frequency at which the phase noise is 3 dB above the noise floor is 10 MHz. According to Lesson model[9] the Q_L is 75, it is quite similar to 64, NDF/RR_T result.

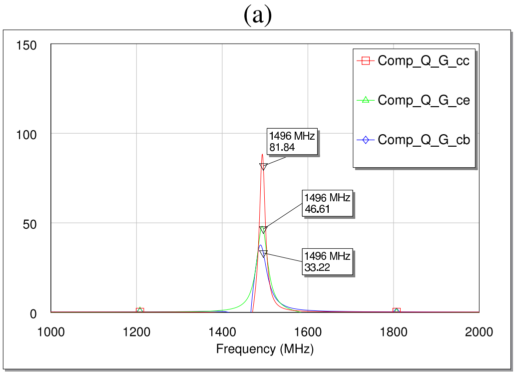
The third example is shown in Fig. 33. It represents a typical negative conductance oscillator with base inductive feedback. The selected active device is BFR380F biased with $I_C = 40$ mA and $V_{CE} = 5$ V. This oscillator topology is usually analyzed with negative conductance method and not with open loop gain one, because the base-emitter capacitor is included into the feedback path. But, however it is possible to analyze it using the *NDF* or RR_T method. These methods provide the first harmonic approximation, the gain margin, the Q_L factor and the start-up time, which are not possible to obtain when the negative conductance method is used. The *NDF* and Q_L of this example are shown in Fig. 34.

The spectrum, oscillation frequency and phase noise of the negative conductance oscillator have been calculated using *HB*, and they are shown in Figs. 35a) and 35b).

The NDF/RR_T oscillation frequency is 1314 MHz, which is very close to the *HB* result, 1346 MHz. And, the NDF/RR_T Q_L is 39, while the obtained one using the *HB* phase noise is 29.



(a)



(b)

Fig. 32. Oscillator g_m compressed models Q_L obtained from its CF(a) and G_L(b).

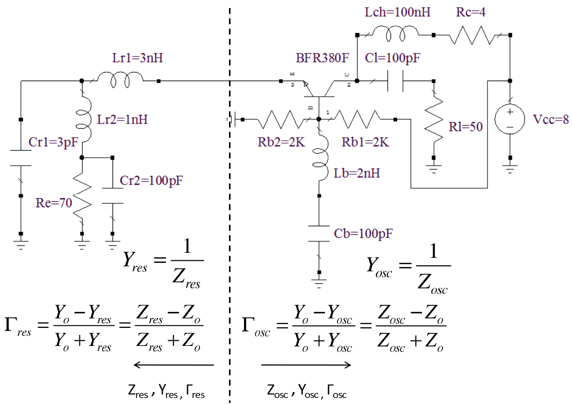
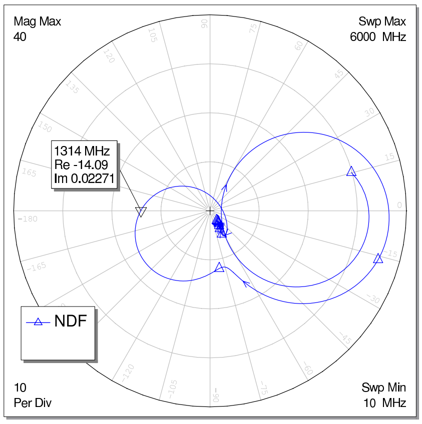
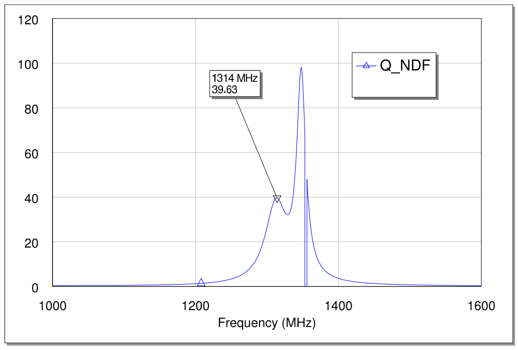


Fig. 33. Negative conductance oscillator.

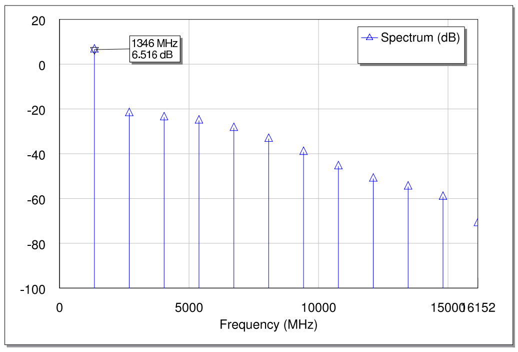


(a)

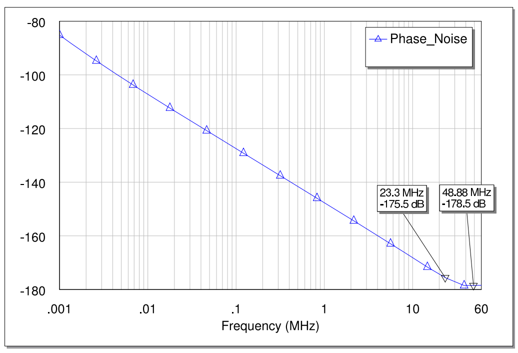


(b)

Fig. 34. Negative conductance oscillator: a) NDF. b) Q_L.



(a)



(b)

Fig. 35. Negative conductance oscillator: a) Spectrum. b) Phase Noise (dBc/Hz).

5. Conclusions

This paper has reviewed the classic open-loop-gain method for oscillators analysis and design. It has been shown, through examples, that it is not suitable for all topologies and the obtained solutions in some cases are wrong if previous conditions are not fulfilled. These required conditions have also been defined and explained throughout this paper. To use Characteristic Function (CF) of the system, it is required to apply the NDF analysis in order to verify if the transfer function is suitable for being analyzed, otherwise the obtained results may be wrong. On the other hand, the widely used Randall and Hock open-loop-gain method does not only need the NDF analysis but it also needs an additional "test function" to assure proper results. It has also been pointed out that both methods, CF and open loop-gain, provide different solutions for the same circuit when the Alechno virtual ground theorem is used.

The NDF method provides information about the RHP poles of the network without requiring any additional approximation or verification. A good-conditioned oscillator must only have a pair of conjugated poles in the RHP. This article presents the NDF expression as a direct method for oscillators design and analysis. It has been demonstrated that the NDF solutions are independent of the virtual ground location, while they depend on the position for the CF and open loop gain methods. This NDF independence is based on its relation with the Return Relations (and RR_T), as they provide the "true open-loop-gain".

The authors of this article propose the RR_T method as an accurate and direct technique for linear oscillator design, which provides the first harmonic approximation (as Kurokawa defined) without requiring transistor compression. This method is suitable for calculating the real Q_L factor of the circuit. The RR_T technique is also suitable to apply the loop-gain concept to any oscillator topologies that were not previously analyzed in this way, but they were analysed by reference plane methods, like negative conductance and negative resistance. Also an example of RR_T analysis has been provided for a negative conductance oscillator. To sum up, the NDF/RR_T method is stated as an optimum tool for the necessary quasi-lineal oscillator analysis before using the HB or TD techniques to obtain a complete and correct solution.

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About Authors...

José Luis JIMÉNEZ-MARTÍN was born in Madrid, Spain, in 1967. He received the Radio-Communication Technical Engineering (B.S. in electrical engineering), Telecommunications Engineering (M.S.), and Ph.D. degrees from the Universidad Politďż˝cnica de Madrid, Madrid, Spain, in 1991, 2000, and 2005, respectively, and the Masters degree in high strategic studies from CESEDEN (organization pertaining to the Spanish Ministry of Defense), Madrid, Spain, in 2007. He is currently an Associate Professor with the Technical University of Madrid, Madrid, Spain. He has authored or coauthored over 60 technical conference, letters, and journal papers. His interests are related to oscillators, amplifiers, and microwave technology.

Vicente GONZÁLEZ-POSADAS was born in Madrid (Spain) in 1968. He received the Ing. Técnico in radiocommunication engineering degree (BS) from the Polytechnic University of Madrid (UPM) in 1992, M.S. degree in physics in 1995, from the UNED and Ph.D degree in telecommunication engineering in 2001 from the Carlos III University of Madrid. Masters degree in high strategic studies from CESEDEN (organization pertaining to the Spanish Ministry of Defense), Madrid, Spain, in 2009. He is working now as an Associate Professor at the Technical Telecommunication School in the Polytechnic University of Madrid. His interests are related to active antennas, microstrip antennas, CRLH lines and metamaterials, microwave technology and RFID. He has authored or coauthored over 60 technical conference, letter and journal papers.

Ángel PARRA-CERRADA was born in Madrid (Spain) in 1975. He received the Technical Ing. in radiocommunication engineering degree (BS) from the Polytechnic University of Madrid (UPM) in 1997, M.S. degree in telecommunication from the Polytechnic University of Madrid (UPM) in 2003. He is now working as an Assistant Professor at the Technical Telecommunication School in the Polytechnic University of Madrid. His interests are related to active antennas, oscillators, micro-strip antennas, CRLH lines and metamaterials, and microwave technology.

Luis Enrique GARCA-MUÑOZ received the telecom engineer degree from Universidad Politécnica de Madrid, Spain, in 1999 and the Ph.D. degree in telecommunication from the Universidad Politécnica de Madrid, Madrid, Spain, in 2003. He is an Associate Professor with the Department of Signal Theory and Communications, University Carlos III, Madrid. His main research interests include radioastronomy receivers, radiotelescopes, microstrip patch antennas and arrays as well as periodic structures applied to electromagnetics.

Daniel SEGOVIA-VARGAS was born in Madrid in 1968. He received the Telecommunication Engineering Degree and the Ph.D. from the Polytechnic University of Madrid in 1993 and in 1998. From 1993 to 1998 he was an Assistant Professor at Valladolid University. From 1998 he is an Associate Professor at Carlos III University in Madrid where he is in charge of the Microwaves and Antenna courses. He has authored and co-authored over 90 technical conference, letters and journal papers. His research areas are printed antennas and active radiators and arrays and smart antennas, LH metamaterials, passive circuits and RFID systems. He has also been member of the European Projects Cost260, Cost284 and COST IC0603.