FREQUENCY CHARACTERISTICS OF THE INDUCTION MACHINE

Martin Kroupa

Doctoral Degree Programme (1), FEEC BUT

E-mail: xkroup07@stud.feec.vutbr.cz

Supervised by: Čestmír Ondrůšek

E-mail:ondrusek@feec.vutbr.cz

Abstract: The paper focuses on the induction machine as the transfer member and on its behavior in the frequency domain. The linearized mathematical model of induction machine is introduced and the issue of its frequency transmissions is discussed. Finally, the appropriateness of the motor use as diagnostic tool for analysis of time-varying load torque is mentioned.

Keywords: Induction machine, transmission function, frequency characteristic, control theory

1. INTRODUCTION

Since the 80s of last century, the issue of dealing with the diagnosis of electromechanical objects is very often discussed topic. Each electromechanical system usually consists of a drive (electric motor) and the driven load. Driven load can be for example: fun, hydrodynamic pump, screw compressor and other. The task of diagnosis is evaluation the behavior of one subsystem from analysis of second subsystem. In most cases, this means evaluating driven load by the analysis of the electric motor quantities.

At present, the major phenomenon of diagnostic method is analysis named Motor Current Signature Analysis (MCSA). This method commonly allows to perform the diagnosis of electric motor or diagnosis of driven load today. There are already a lot of literature in which this diagnostic methodology have been published and papers which this method use for functional diagnosis. However, almost any literature does not view on the motor as a part of a larger system, as on “the coupling element” which has its inputs (power grid voltage, load torque), “outputs“ (stator and rotor currents, electromagnetic torque) and its characteristic behavior in the frequency domain. The characteristic behavior affects produced variables and input quantities of motor. Therefore, the aim of this article is to introduce the behavior of the motor as the transmission element.

From the point of view of simple construction and high efficiency, the majority of all installed motors are induction machines (IM) today, this is why attention is paid to the induction motor in this paper.

2. INDUCTION MACHINE FROM THE PERSPECTIVE OF CONTROL THEORY

2.1. MATHEMATICAL MODEL OF INDUCTION MACHINE

The induction machine is multi parametric nonlinear dynamic system which can be described by a differential equation of order *n*. For the numerical solution, the differential equation is often converted on *n* differential equations of the first order – into state equations. These equations can be obtained from the two-phase equivalent mathematical model of IM based on the theory of general machine. All derivatives are converted to one side of the equation and dependent variables are eliminated using mathematical adjustments. The state equations for mathematical description of squirrel cage IM in coordinate system rotating angular velocity – ω are [1]:

$$\frac{d\psi_{Rd}}{dt} = -\frac{R_r}{L_r} \cdot \psi_{Rd} + (\omega - \omega_e) \cdot \psi_{Rq} + \frac{L_m \cdot R_r}{L_r} \cdot i_{sd} \tag{1}$$

$$\frac{d\psi_{Rq}}{dt} = -(\omega - \omega_e) \cdot \psi_{Rd} - \frac{R_r}{L_r} \cdot \psi_{Rq} + \frac{L_m \cdot R_r}{L_r} \cdot i_{sq} \tag{2}$$

$$\begin{aligned}\frac{di_{sd}}{dt} = {} & \frac{L_m \cdot R_r}{L_r \cdot \left(L_s \cdot L_r - {L_m}^2\right)} \cdot \psi_{Rd} + \frac{L_m}{L_s \cdot L_r - {L_m}^2} \cdot \omega_e \cdot \psi_{Rq} - \\ & - \frac{{L_r}^2 \cdot R_s + {L_m}^2 \cdot R_r}{L_r \cdot \left(L_s \cdot L_r - {L_m}^2\right)} \cdot i_{sd} + \omega \cdot i_{sq} + \frac{L_r}{L_s \cdot L_r - {L_m}^2} \cdot u_{sd}\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{di_{sq}}{dt} = {} & -\frac{L_m}{L_s \cdot L_r - {L_m}^2} \cdot \omega_e \cdot \psi_{Rd} + \frac{L_m \cdot R_r}{L_r \cdot \left(L_s \cdot L_r - {L_m}^2\right)} \cdot \psi_{Rq} - \\ & - \omega \cdot i_{sd} - \frac{{L_r}^2 \cdot R_s + {L_m}^2 \cdot R_r}{L_r \cdot \left(L_s \cdot L_r - {L_m}^2\right)} \cdot i_{sq} + \frac{L_r}{L_s \cdot L_r - {L_m}^2} \cdot u_{sd}\end{aligned} \tag{4}$$

$$\frac{d\omega_e}{dt} = \frac{3}{2} \cdot \frac{L_m}{L_r} \cdot \frac{{p_p}^2}{J} \cdot \left(\psi_{Rd} \cdot i_{sq} - \psi_{Rd} \cdot i_{sq}\right) - \frac{p_p}{J} \cdot M_z \tag{5}$$

Where R_s and R_r are resistances of the stator and rotor circuit, L_s a L_r are total inductances of the stator and rotor circuit. L_m represents the magnetizing inductance. p_p – number of pole pairs and J is moment of inertia. The parameters are listed in the documentation for the motor. ω_e is mechanical angular velocity in electrical degrees and M_z represents load torque. Expressions on the left sides of equations are derivatives of state variables – rotor and stator flux linkage and rotor and stator current. Finally, equations still contain supply voltage – u_{sd}, u_{sq}. As shown for a complete description of IM in the state space, (five equations after distribution to the „*d*“ and „*q*“). In matrix form, the mentioned system of five differential equations can be written as Eq. (6).

$$\dot{\bar{x}} = \boldsymbol{A}\bar{x} + \boldsymbol{B}\bar{u} \tag{6}$$

$$\bar{y} = \boldsymbol{C}\bar{x} + \boldsymbol{D}\bar{u} \tag{7}$$

Where vector $\bar{x}$ is state vector, $\boldsymbol{A}$ is characteristic matrix of systems which determines behavior of the entire system. For stable systems, all eigenvalues of matrix are negative. In the case of complex numbers, eigenvalues have to be with negative real part. Matrix $\boldsymbol{B}$ is input matrix of system, $\boldsymbol{C}$ is output matrix of system and matrix $\boldsymbol{D}$ expresses direct effect of inputs to output variables, $\bar{u}$ is vector of variables which enter to system and $\bar{y}$ is vector of output variables. This vector is often identical with the state vector. Note: Matrix $\boldsymbol{D}$ is zero for evaluation of motor frequency characteristics.[2]

2.2. LINEARIZED MATHEMATICAL MODEL OF INDUCTION MACHINE

Like most real systems, IM is nonlinear system and its dynamic behavior changes with increasing loads or slip. In order to the motor could be correctly described, range of speed will be divided into several intervals – from zero load to rated load. The linear behavior of IM is supposed at these intervals and with small changes of system variables. This is a local approximative linearization.

If the system properties are studied from linearized model - in certain steady state, it will have to take in mind the local validity and the possibility of major changes by the larger deviations of the system variables from the "operating point". [3] The local linearization is carried out by so called

„perturbation variable”. Then the system of linearized differential state equations in matrix form can be written as [2]:

$$\dot{\bar{x}} + \Delta\dot{\bar{x}} = \boldsymbol{A}(\bar{x} + \Delta\bar{x}) + \boldsymbol{B}(\bar{u} + \Delta\bar{u}) \tag{8}$$

$$\bar{y} = \boldsymbol{C}\bar{x} + \boldsymbol{D}\bar{u} \tag{9}$$

It means that all state and input variables that are not constant and are able to be changed are replaced by constant characterizing the linearized point and perturbation variable (labeled with „Δ“).

3. FREQUENCY CHARACTERISTICS OF INDUCTION MACHINE

It is evident that the frequency characteristics of IM are dependent on the parameters of the equivalent mathematical model. Therefore, each motor used as a diagnostic „tool“ may have different frequency characteristics. The characteristics may be different in: cutoff frequency, resonance amplification, number of resonance amplification and other. In this paper, the frequency characteristics of 3 kW IM will be investigated. The parameters and some values of IM are listed in the Tab. 1.

$P = 3$ kW	
R_s	1.993 Ω
R_r'	1.735 Ω
L_s	0.01134 H
L_r'	0.01134 H
L_m	0.21 H
p_p	2
J	0.0062 kg/m^2
M_n	19.967 Nm
n_n	1437 min^{-1}
U_n	400 V

Table 1: The parameters of IM

It is possible to compile 18 combinations of motor transfer functions from equations of state. However in the article, only stated transmissions of motor are investigated, concretely:

$$F_1 = \frac{i_{sd}}{m_z} \quad ; \quad F_2 = \frac{i_{sq}}{m_z} \quad ; \quad F_3 = \frac{\omega_{mec\,h}}{m_z} \quad ; \quad F_1 = \frac{m_i}{m_z} \tag{10}$$

The transfer functions are chosen in correlation with the use of the motor for detecting the load torque oscillation. Another interesting transfer functions are dependent on the voltage change – u_{sd}, u_{sq}.

3.1. RESULTS

The frequency characteristics of the selected IM are drawn for zero load motor and for its rated load. Of course in practice, it is necessary to know all the characteristics at any load. (It should be noted that the frequency characteristics in this article provide information on the dynamic changes of the examined variables. Medium values of variables respecting the operating point of the motor appear to be zero. This fact is based on Eq. (8) and Eq (9), where derivatives of constants by time are zero.

On Fig. 1 there are frequency characteristics and time responses to a step change for four variables of motor – i_{sd}, i_{sq}, ω_{mech} a m_i. The frequency characteristics of all the studied variables are related to a step change in load torque, therefore three of the named variables begin with negative amplification. It is clearly seen from the diagrams that the motor behaves as a low pass filter with a cutoff frequency of about 28 Hz. The amplification of all values fall sharply after this frequency. The cutoff frequency is exactly 28.3 Hz for zero load motor, but after a load up to the rated torque, the cutoff frequency decreases slightly to about 27.2 Hz. In detail, this fact is shown in Fig. 2. The frequency characteristics of motor variables include the resonant increase at the cutoff frequency. This fact is really unpleasant and entails a certain risk. Not only that the stator currents will be greatly distorted at frequencies around 28 Hz, but the modulations of stator currents will achieve uncomfortably high levels. This can significantly affect machine life. Therefore, this has to be taken into account.

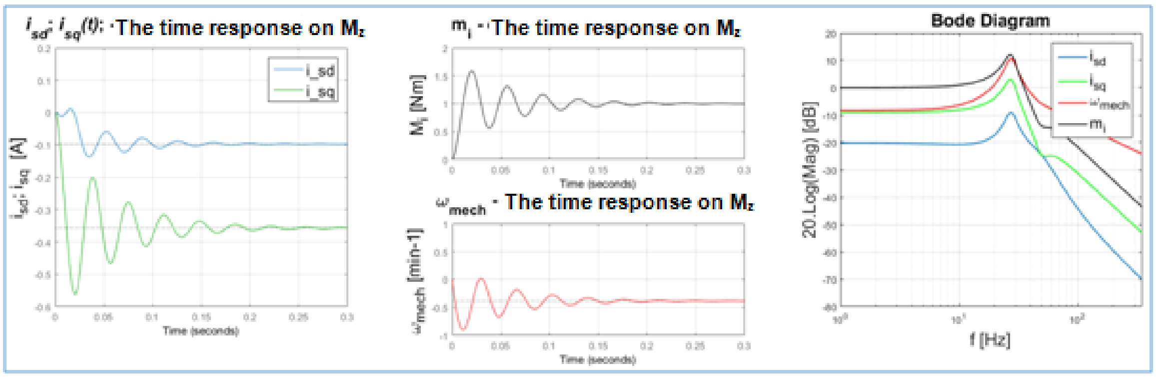

Figure 1: Motor response to a step change M_z [$M_z = (0.0, 1.0)\ M_n$]

To complement, the dependence of resonant increase of motor variables is showed on the following figure. The cutoff frequency of all motor variables decreases due to the average load.

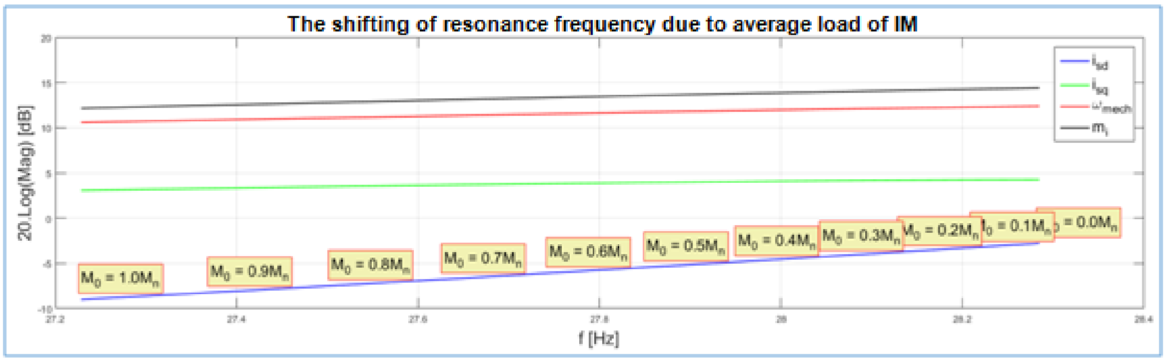

Figure 2: The shifting of resonance frequency due to average load

Frequency transfers of IM are dependent on all the parameters of the equivalent mathematical model, the number of poles and the moment of inertia. The parameters of equivalent motor model are constant after neglecting the temperature dependences, notably winding resistance, neglecting of the magnetizing inductance dependence on the average load and assuming steady state motor operation. All frequency characteristics of the motor were created for these simplifications. The only thing that may change is the moment of inertia. From the electrical point of view, the moment of inertia forms with resistance and inductance of IM analogy resonant RLC circuit. In fact, the moment of inertia is naturally bigger after connecting the load. It must be calculated with this for the purpose of diagnosis and for increased accuracy of frequency characteristics.

This fact is visible on Fig. 3. The diagrams are created for rated load but the moment of inertia of the rotating masses is changed. This time, the amplifications of motor quantities do not relate to 1 Nm. On the left, the moment of inertia is the same as moment of inertia specified in the catalog for the selected motor. Toward the right, the moment of inertia is raised up on two times greater value than rated one. The cutoff frequency is slightly shifted downward from the initial 27.2 Hz to about

23 Hz and pass band of IM was restricted even more. More importantly, the resonance peaks don't reached already such amplification and the effects of variable load torque in all investigated motor variables are mitigated at cutoff frequency.

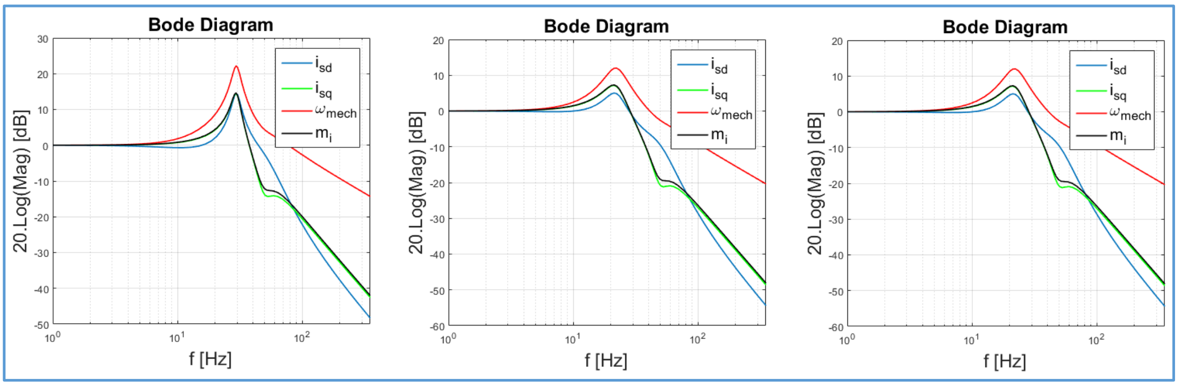

Figure 3: The frequency characteristics of 3 kW IM [J = (1.0, 1.5, 2.0) J_n]

4. CONCLUSION

The article introduced induction machine from a different perspective. Its characteristics in the frequency domain were described in detail, namely the behavior of the motor at a time variable load. The study foresees a further extension, where it will be examined the behavior of IM depending on supply voltage variation and the properties of several motors similar type of performance will be compared.

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