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MINISTRY OF EDUCATION,  
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# Introduction to Electrical Engineering

Self-study textbook

Assoc. Prof. M. Steinbauer, Ph.D., 2026

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# Instructions


- Go through and study this material. Each chapter contains answers to questions that will be the subject of ongoing testing via quizzes
- You must go through all the slides in order; you cannot skip any.
- Some slides include more detailed notes—view them by clicking the “Notes” button in the top right corner
- The presentation contains **6 milestones—quizzes** that you must complete to proceed to the next chapter; if you fail a quiz, you will return to the beginning of the previous chapter
- **Successful completion (passing all quizzes) is considered fulfillment of the BPC-ELS course**




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AND COMMUNICATION


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# PART 1

# INTRODUCTION



# BASIC ELECTRICAL QUANTITIES

## SI PREFIXES

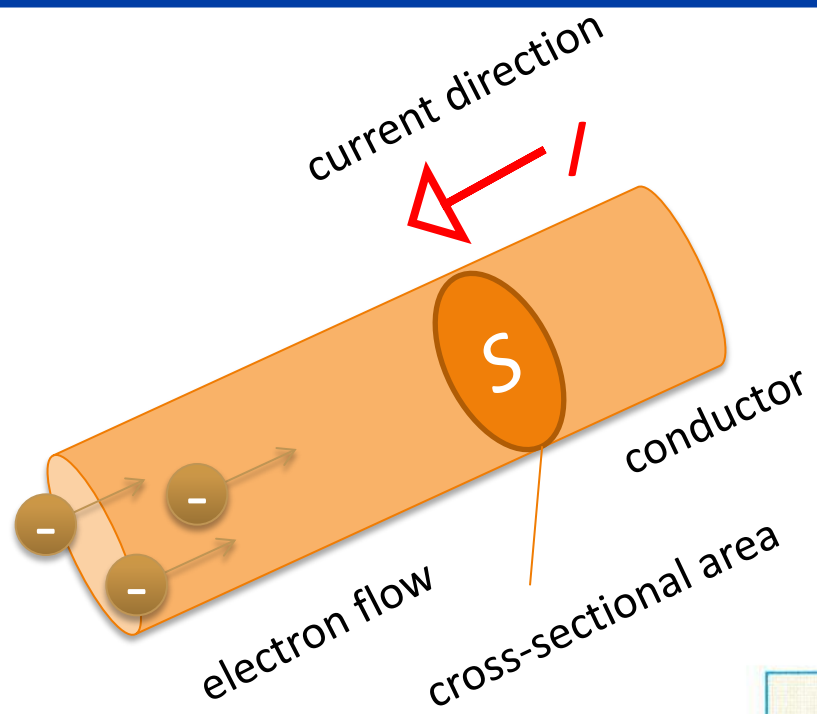
Charge	Current	Voltage	Power	Energy	Frequency
<b>Q</b>	<b>I</b>	<b>U</b>	<b>P</b>	<b>E</b>	<b>f</b>
Coulomb	Ampere	Volt	Watt	Joule	Hertz
(C)	(A)	(V)	(W)	(J)	(Hz)

Resistance/ Conductivity	Capacitance	Inductance
<b>R / G</b>	<b>C</b>	<b>L</b>
Ohm / Siemens	Farad	Henry
( $\Omega$ ) / (S)	(F)	(H)

giga	mega	kilo	deci	centi	mili	= mikro	nano	piko
G	M	k	d	c	m	$\mu$	n	p
$10^9$	$10^6$	$10^3$	$10^{-1}$	$10^{-2}$	$10^{-3}$	$10^{-6}$	$10^{-9}$	$10^{-12}$
1 000 000 000	1 000 000	1 000	0,1	0,01	0,001	0,000 001	0,000 000 001	0,000 000 000 001

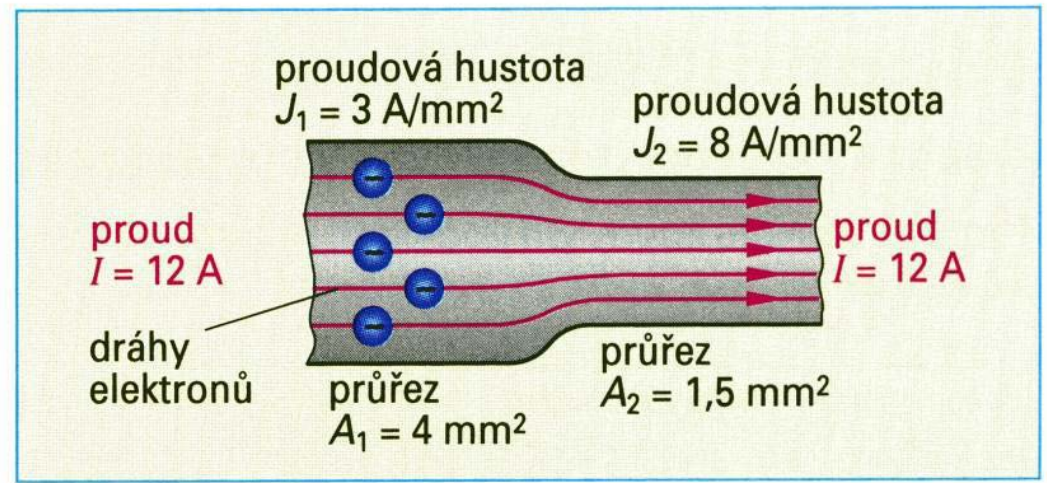


# ELECTRIC CURRENT



$$I = \frac{Q}{t} \text{ (A)}$$

$$J = \frac{I}{S} \text{ (A/m}^2\text{)}$$

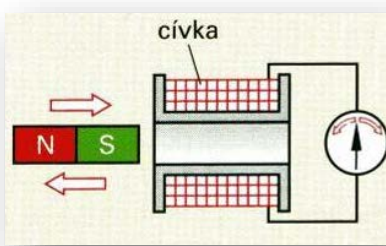




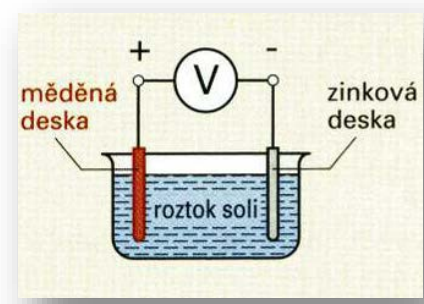
# THE GENERATION OF ELECTRIC CURRENT

Electricity is always generated by converting another form of energy

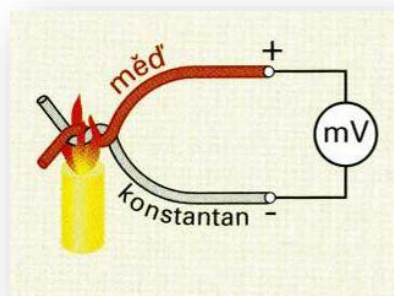
## Mechanical (use of induction)



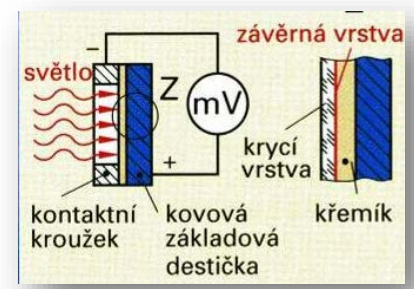
## Chemical (galvanic cell)



## Thermal (Seebeck effect)



## Light (photocell)

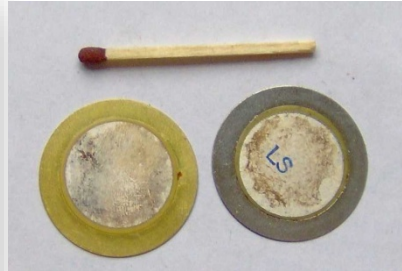
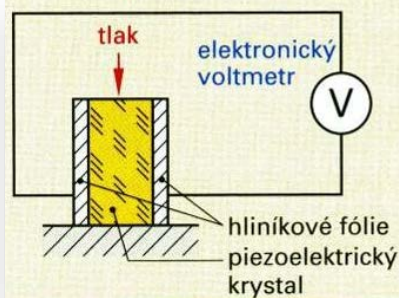


Thermocouple for temperature measurement

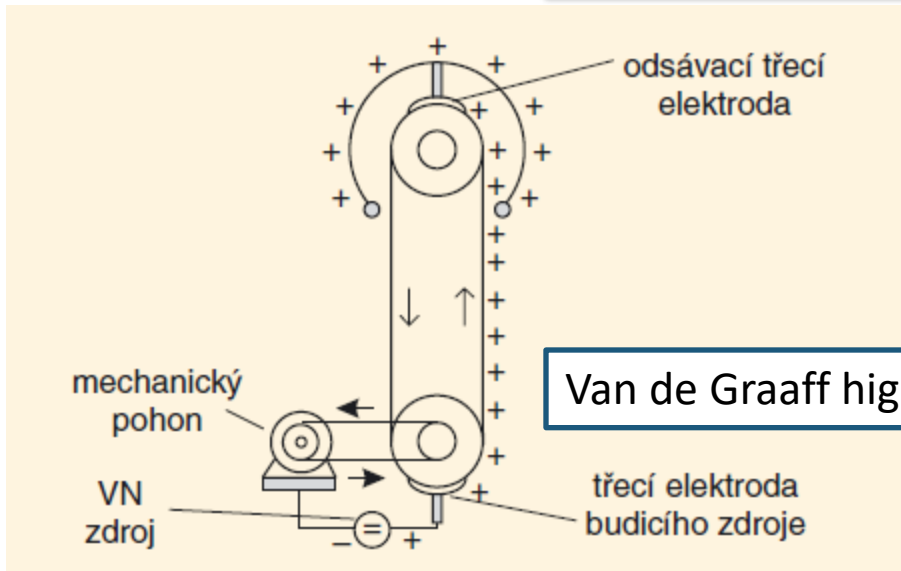


# GENERATION OF ELECTRIC CURRENT

## Mechanical (piezoelectric effect)



## Mechanical (triboelectric effect)

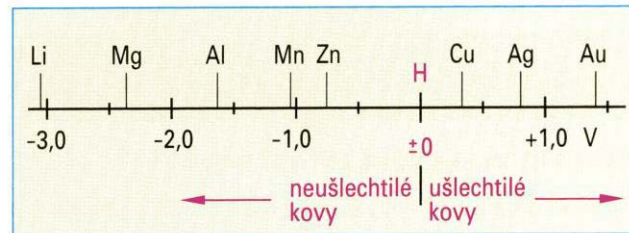
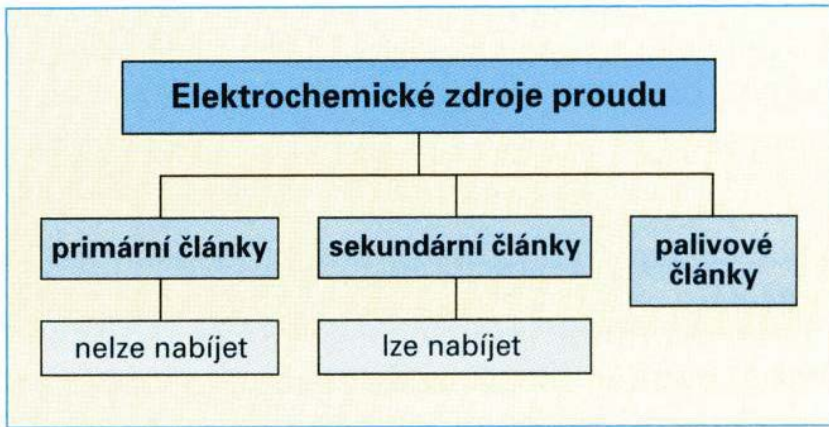


Van de Graaff high-voltage generator

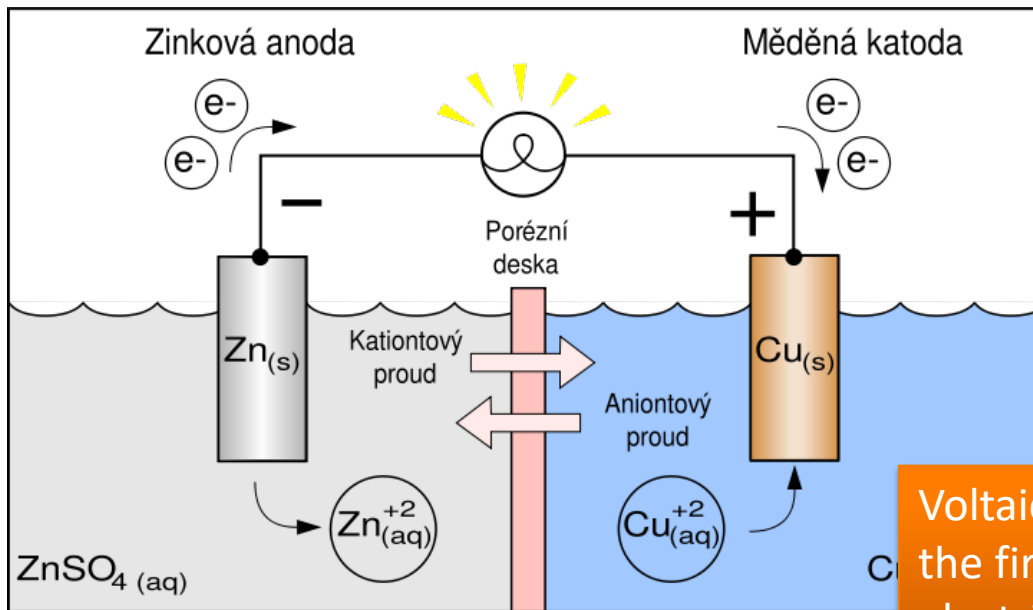
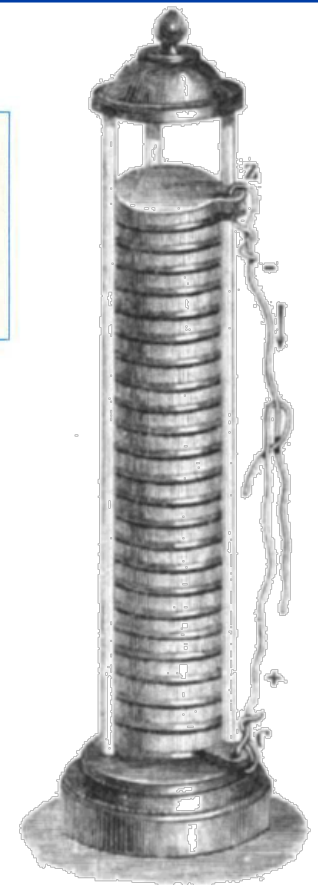




# ELECTROCHEMICAL CURRENT SOURCES

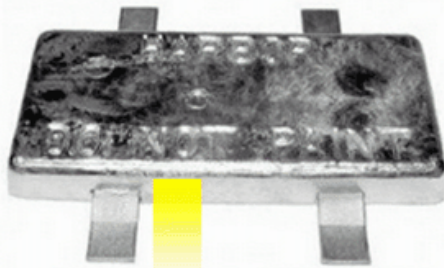


Obr. 2: Elektrochemické potenciály kovů vůči vodíku (resp. vůči vodě jako elektrolytu)



Voltaic pile (A. Volta, 1800) the first chemical source of electric current

## Other Applications of Electrochemical Cells

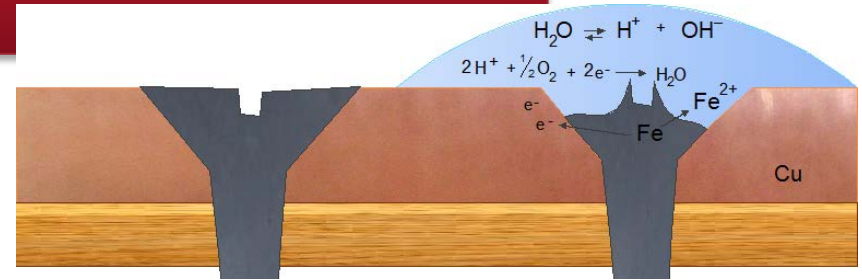


Obr. 7: Obětovaná anoda k ochraně trupů lodí

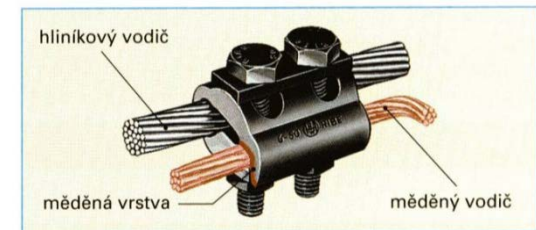


Obr. 8: Elektrochemická ochrana kormidla lodí.

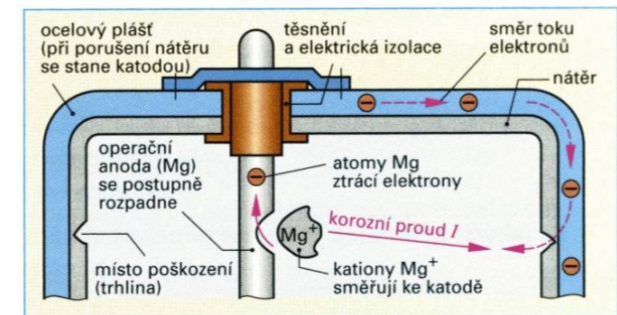
Electrochemical corrosion can also be used for corrosion protection, for example, of steel pipes or the hulls of seagoing vessels. This is achieved by connecting the protected steel body to a so-called sacrificial anode made of an alloy of less noble metals, such as magnesium, aluminum, and zinc. This electrode corrodes preferentially in an aqueous environment. Once it has dissolved, another one is connected in its place.



Why can't copper sheet be fastened with steel screws?



Obr. 4: Přechodová svorka (Cupal)

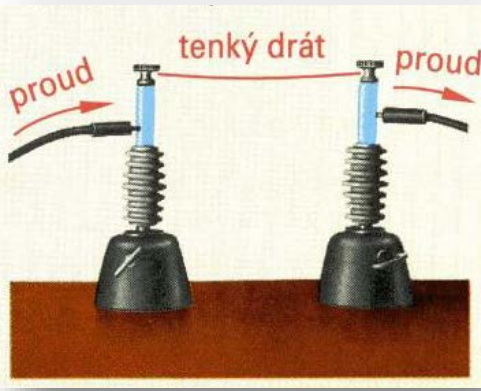


Obr. 5: Katodová antikoroziční ochrana zásobníku teplé vody s operační anodou



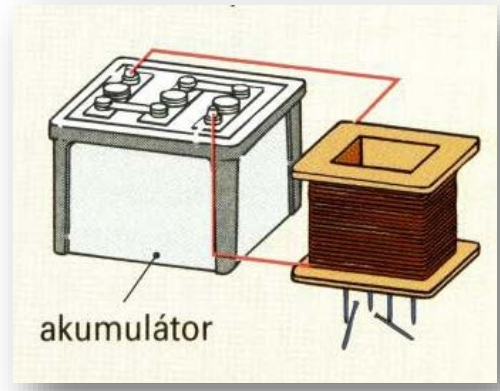
# EFFECTS OF ELECTRIC CURRENT

## Thermal effects – Joule heat (always)



- Heating appliances
- iron
- water heater
- Soldering iron
- Fuses
- (Light bulb)

## Generation of a magnetic field (always)



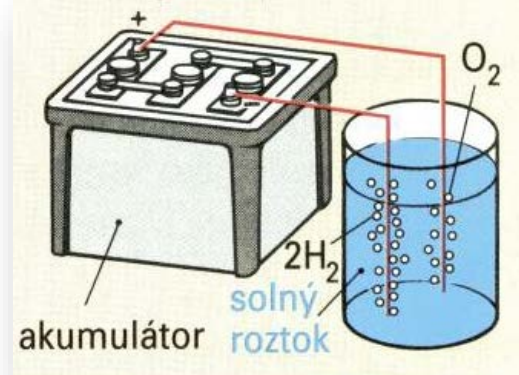
- Relays
- Electric motors
- Electromagnets
- Speakers
- Bells

## Generation of light (in gases and semiconductor)



- Glow plugs
- LEDs
- Fluorescent lamps
- Discharge lamps
- Electric arc

## Chemical effects (in conductive liquids)



- Electrolysis
- Galvanic plating
- Battery charging



# EFFECTS OF ELECTRIC CURRENT

## Physiological effects (on living organisms)



- Electric fences  
Medical devices:
- pacemakers
  - defibrillators
  - electrotherapy

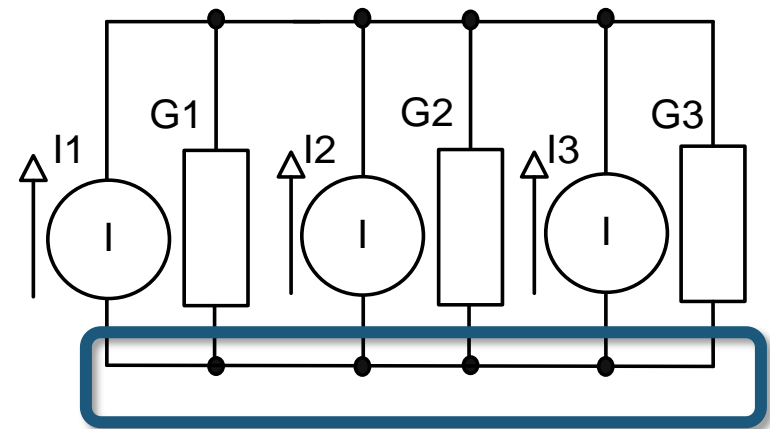
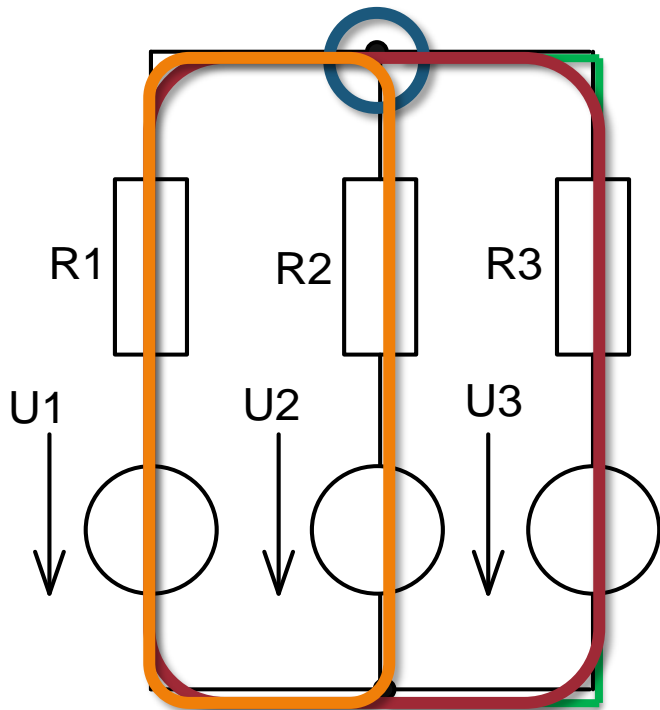
### Electric shock

Electric shock – will be covered in electrical safety training.



# ELECTRICAL CIRCUITS – BASIC CONCEPTS

- **Node** – equipotential point, conductive connection between circuit elements
- **Branch** – a section of a circuit between two nodes
- **Loop** – a closed section of a circuit
- **Minimum loop (mesh)** – not divided by any branch

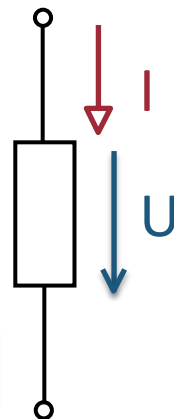


This is a single node!

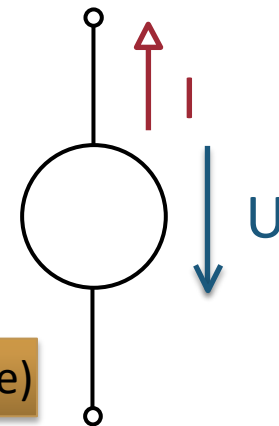


# DIRECTION ARROWS

- In the diagram, we represent voltage and current using arrows that indicate the direction of the quantity  $U$  or  $I$
- To make it easier to distinguish between them, we use different arrow shapes for current and voltage
- The orientation of the arrows with the direction of the quantities in the equations follows K. z, not necessarily the actual directions of the currents and voltages. When choosing the direction of the arrows, we respect
  - the **load** (passive) orientation, where the arrows point in the same direction
  - **source** (active) orientation, where the  $U$  and  $I$  arrows point in opposite directions



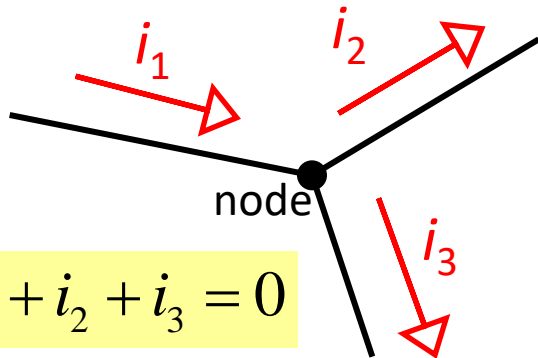
Device (passive)



Source (active)



# ELECTRIC CIRCUITS – KIRCHHOFF'S LAWS



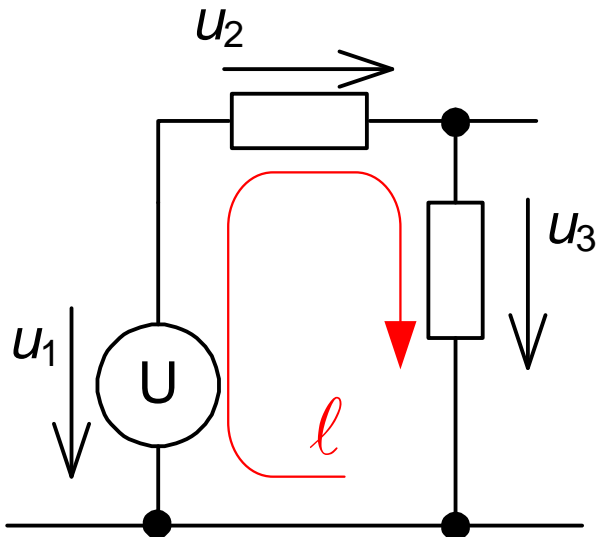
$$-i_1 + i_2 + i_3 = 0$$

## Kirchhoff's First Law (Current)

The algebraic sum of currents flowing into (-) and flowing out (+) of a node is equal to zero.

$$\sum_j \pm i_j = 0$$

It is based on the law of conservation of charge.



$$-u_1 + u_2 + u_3 = 0$$

## 2. Kirchhoff's Law (Voltage)

The algebraic sum of voltages along a closed oriented loop  $\ell$  is equal to zero.

$$\sum_j \pm u_j = 0$$

It follows from the law of electromagnetic induction for the steady-state case.

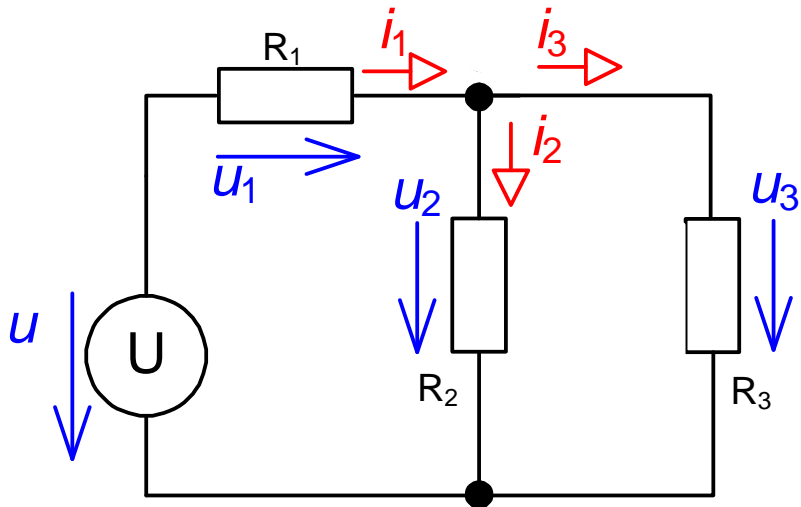
$$u = -\frac{d\Phi}{dt} \rightarrow 0$$



# ELECTRIC CIRCUITS – KIRCHHOFF'S LAWS

## Example:

Set up the equations using Kirchhoff's laws for the circuit shown in the figure.



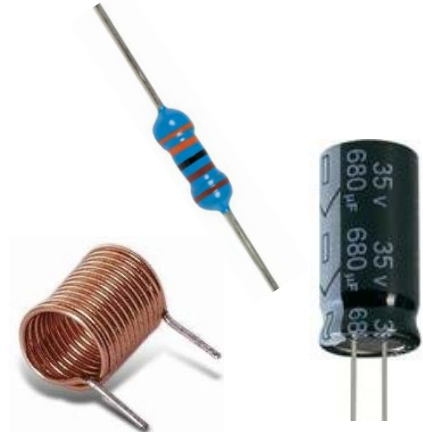
Node 1:  $-i_1 + i_2 + i_3 = 0$

Loop  $S_1$ :  $-u + u_1 + u_2 = 0$

Loop  $S_2$ :  $-u_2 + u_3 = 0$

Based on **the direction of energy flow**:

- **Passive** components: consume (or store) energy
  - Resistors **R**
  - capacitors **C**
  - inductors **L**
- **Active** components: supply energy
  - voltage sources **U**
  - Current sources **I**
  - transistors, ICs, ...



By number of poles (terminals):

- two-pole, three-pole, four-pole, etc.

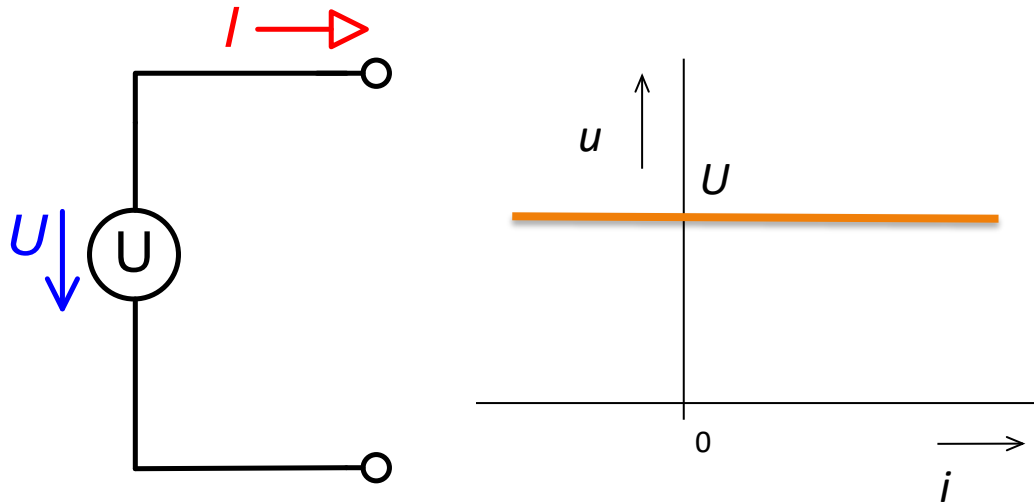
By **type of circuit equations** determining the mathematical procedures for their solution:

- linear
- nonlinear



# INDEPENDENT VOLTAGE SOURCE

**An ideal independent voltage source** maintains a constant voltage across its terminals regardless of the magnitude of the current drawn.

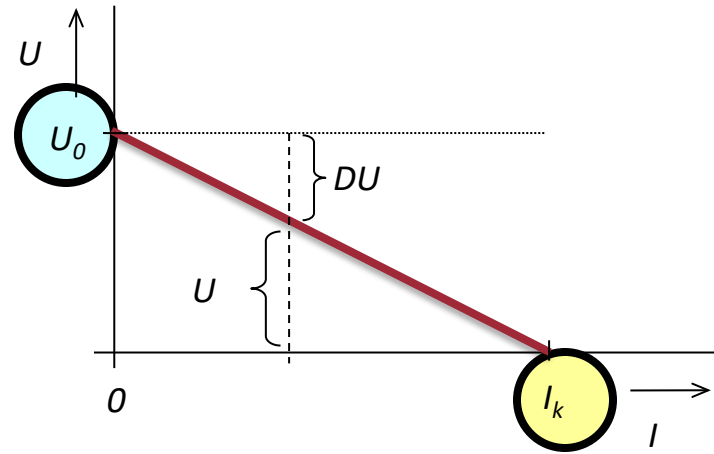
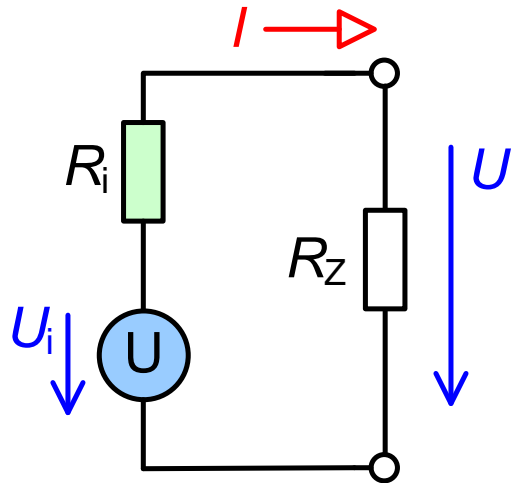


An ideal voltage source and its load characteristic

- The only parameter of an ideal voltage source is its voltage  $U$
- An ideal voltage source is capable of supplying any output current and therefore has an infinite energy supply



# MODEL OF A REAL VOLTAGE SOURCE (LINEAR)



$R_i$  – internal resistance

$U_i$  – internal voltage

**Terminal voltage**  $U = U_i - I \cdot R_i = U_i - \Delta U$

$U_0$  – open-circuit voltage ( $I = 0$ )

$$U_0 = U_i$$

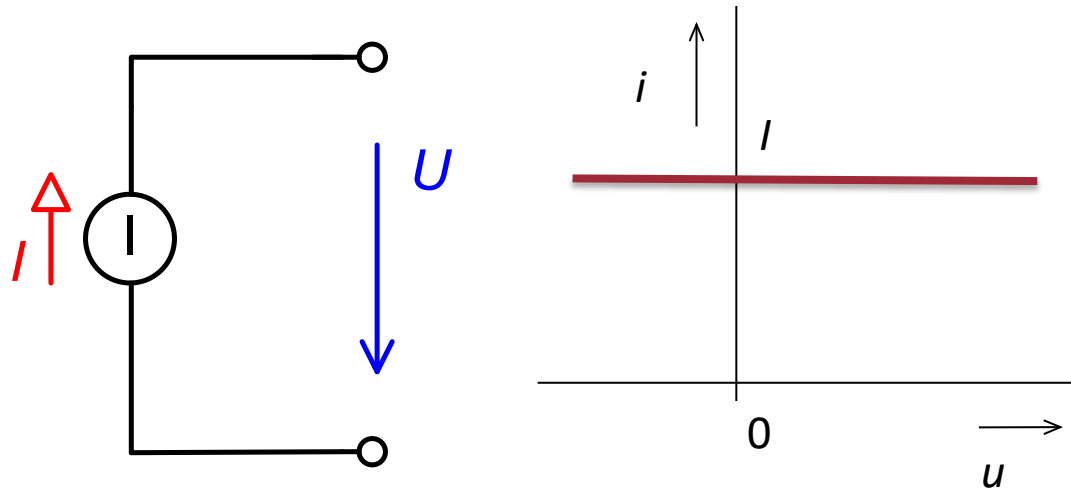
$I_k$  – short-circuit current ( $U = 0$ )

$$I_k = \frac{U_i}{R_i}$$



# INDEPENDENT CURRENT SOURCE

An **ideal independent current source** maintains a constant current at its terminals regardless of the voltage.

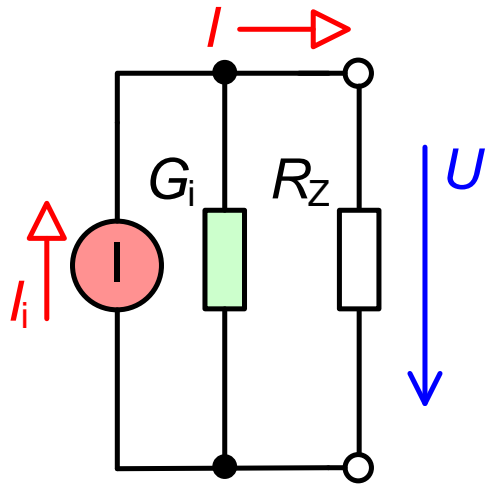


Ideal current source and its load characteristic

- The only parameter of an ideal current source is its current  $I$
- An ideal current source is capable of supplying a constant current regardless of the terminal voltage and therefore has an infinite energy supply

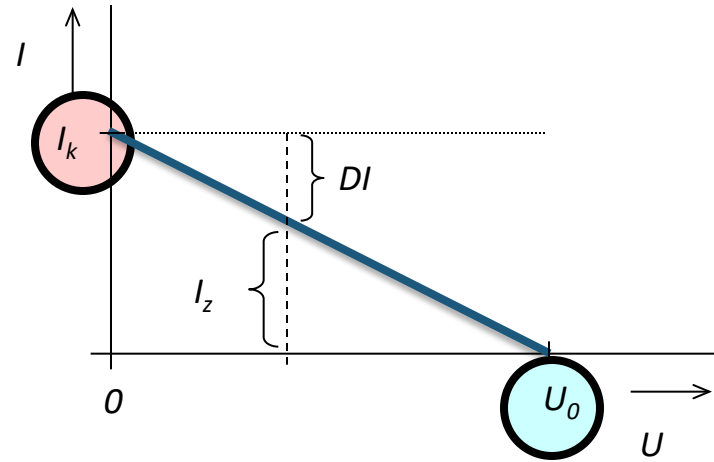


# REAL CURRENT SOURCE MODEL (LINEAR)



$I_i$  – internal current

$G_i$  – internal conductivity



Output current

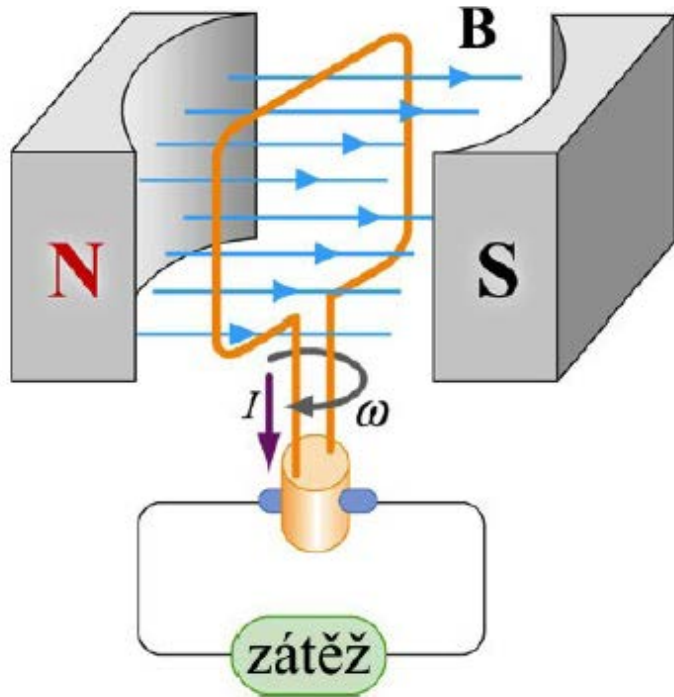
$$I = I_i - U \cdot G_i = I_i - \Delta I$$

$U_0$  – open-circuit voltage ( $I = 0$ )

$$U_0 = \frac{I_i}{G_i}$$

$I_k$  – short-circuit current ( $U = 0$ )

$$I_k = I_i$$



Automotive alternator

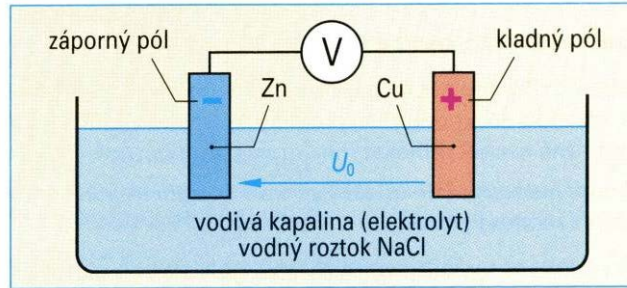
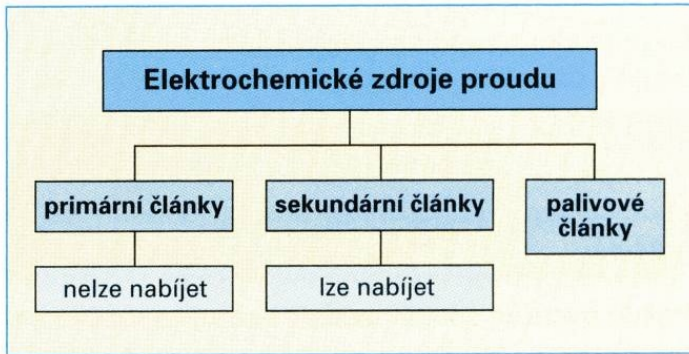


## Rotary generator

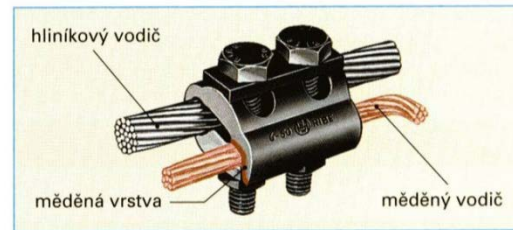
- Conversion from mechanical energy
- Uses Faraday's law of induction
- **Alternator** (AC source, can be rectified)
- **Dynamo** (DC source)



# THE MOST COMMON PRACTICAL SOURCES—ELECTROCHEMICAL SOURCES (BATTERIES)



Single cells

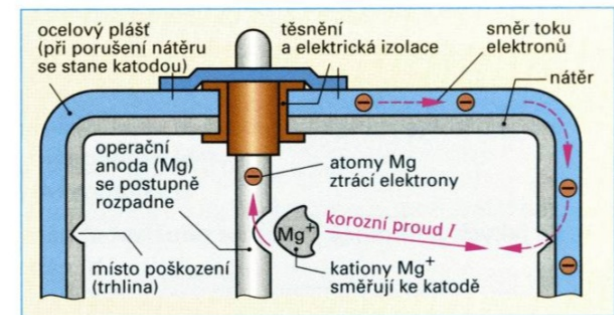


Obr. 4: Přechodová svorka (Cupal)

## Chemical cell

- Conversion from chemical energy
- **Primary cell** (battery, irreversible process)
- **Secondary cell** (rechargeable battery, reversible process, can be recharged)
- **Fuel cell** (electrodes are not consumed; fuel includes hydrogen, methanol, etc.)

## Other uses of electrical cells



Obr. 5: Katodová antikoroziční ochrana zásobníku teplé vody s operační anodou



# EXAMPLE

An R14-type single cell has an open-circuit voltage of  $U_0 = 1.5 \text{ V}$ . If we draw a current  $I_z = 0.5 \text{ A}$  from it, the output voltage drops to  $1.3 \text{ V}$ . Determine the components of the equivalent circuit for the single cell.



**Solution:**

$$\Delta U = 1.5 - 1.3 = 0.2 \text{ V}$$

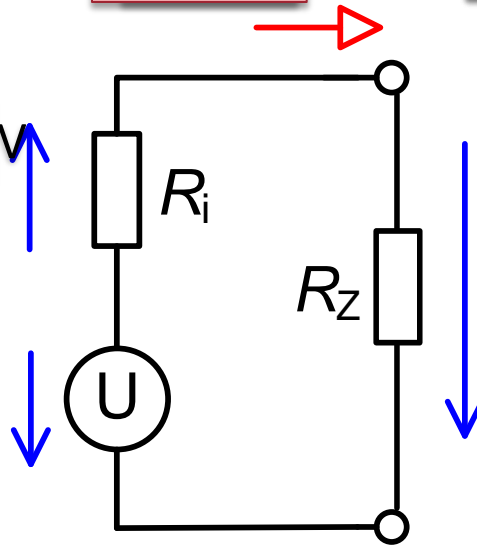
$$R_i = 0.4 \Omega$$

$$U_i = 1.5 \text{ V}$$

$$I = 0.5 \text{ A}$$

Under load

$$U = 1.3 \text{ V}$$

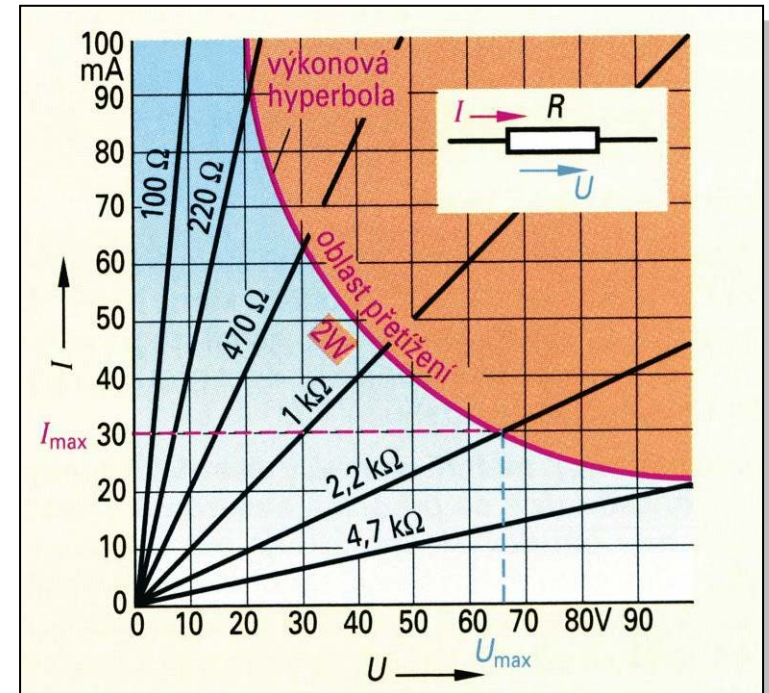
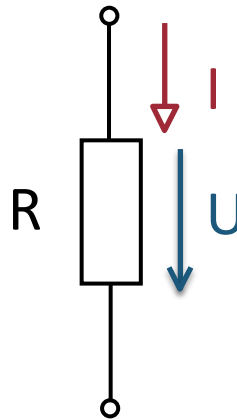


$$R_i = \frac{\Delta U}{I} = \frac{0,2}{0,5} = 0,4 \Omega$$



$$P = \frac{W}{t} = U \cdot I \quad (\text{W})$$

$$P = U \cdot I = \frac{U^2}{R} = I^2 \cdot R$$



**Tabulka: Výkony elektrospotřebičů a pohonu (příklady)**

zařízení	výkon	zařízení	výkon
vysoušeč vlasů	2 000 W	žárovka	100 W
televizor	150 W	páječka	60 W
pohon vlaku	8 000 kW	hodinky	90 μW



## NOTE - TELLEGEN'S THEOREM

Tellegen's theorem is a mathematical formulation of the law of conservation of energy in electrical circuits

- the energy supplied to the circuit by active elements is equal to the sum of the energy accumulated in the circuit in the form of electric and magnetic fields and the energy that is irreversibly converted in the circuit into energy of another type
- using instantaneous power values: the sum of the instantaneous power consumption values of the elements must equal the sum of the instantaneous power values of the passive elements

$$\sum_{k=1}^v u_k i_k = 0$$

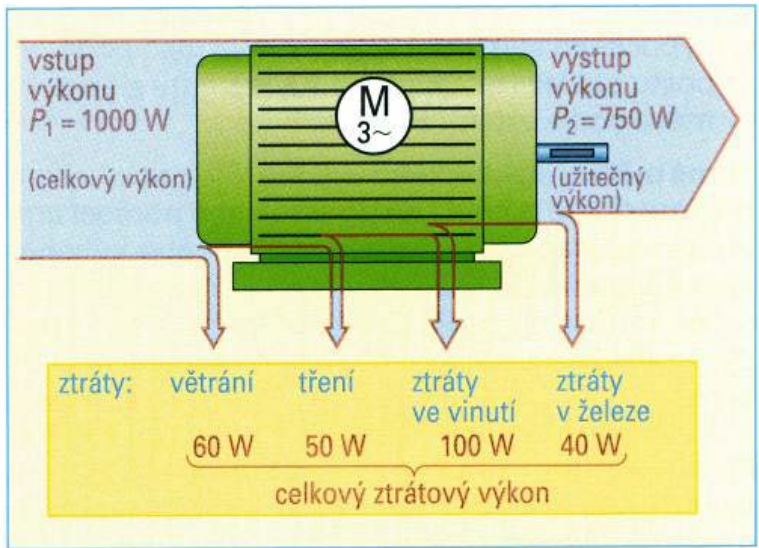
where  $v$  denotes the number of all branches of the circuit,  $u_k$  are the branch voltages, and  $i_k$  are the branch currents.

## Ztrátový výkon a účinnost

$$P_z = P_1 - P_2$$

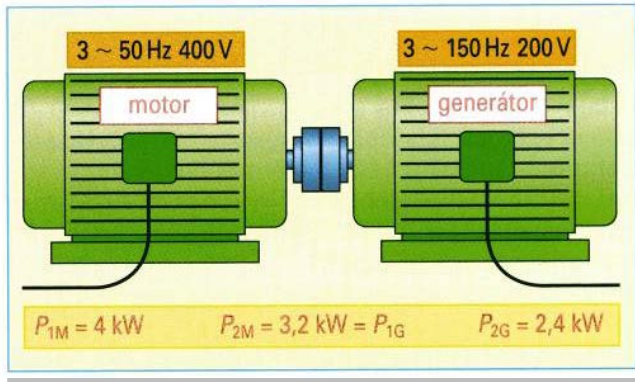
$$\eta = \frac{P_2}{P_1}; \eta = \eta_1 \cdot \eta_2 \quad [\eta] = 1$$

- $\eta$  účinnost (poměr výkonů)
- $\eta_1, \eta_2$  dílčí účinnosti
- $P_2$  výstupní (užitečný) výkon
- $P_1$  vstupní výkon (příkon)
- $P_z$  ztrátový výkon



Tabulka: Účinnosti elektrických zařízení (příklady)

zařízení		účinnost $\eta$
trojfázový motor	1 kW	0,75
transformátor	1 kVA	0,90
varná konvice	2 kW	0,95
žárovka	40 W (570 lm)	0,02



$$\eta_M = \frac{3,2 \text{ kW}}{4 \text{ kW}} = 0,8$$

$$\eta_G = \frac{2,4 \text{ kW}}{3,2 \text{ kW}} = 0,75$$

$$\eta = \eta_M \cdot \eta_G = 0,8 \cdot 0,75 = 0,6$$

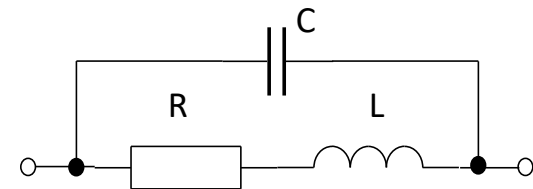
$$\eta = \frac{2,4 \text{ kW}}{4 \text{ kW}} = 0,6$$

Ideal circuit elements  
(defined by one main parameter)

- Resistor – (resistance  $R$ )
- Capacitor – (capacitance  $C$ )
- Inductor – (inductance  $L$ )

Real electrical components  
(have parasitic properties = additional parameters and their dependencies)

- Resistor (resistance)
- Capacitor
- Coil

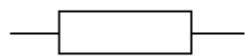


We model real components by appropriately connecting ideal components  
(e.g., using RLC)



# RESISTORS

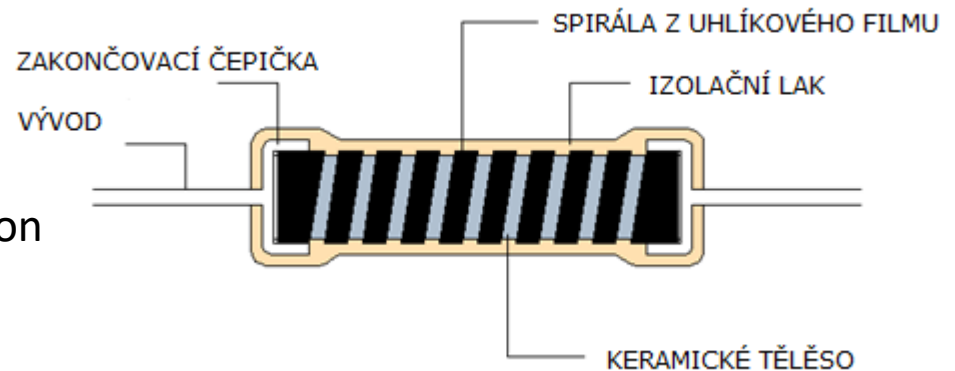
- Resistors are passive electrical components whose fundamental property is electrical resistance [ $\Omega$ ].
- They are made from materials with a very low temperature coefficient of resistance  $\alpha$ , so that changes in temperature have as little effect as possible on the resistance value.
- When current flows through a resistor, Joule heat is generated, which must be dissipated through the component's surface. Therefore, the rated power [W] is an important specification for every resistor; it indicates the maximum power at which the resistor can be continuously loaded.



Resistor Marking  
(IEC)



Resistor designation  
(ANSI)

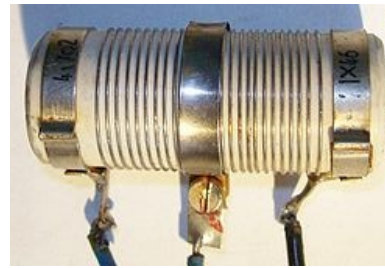




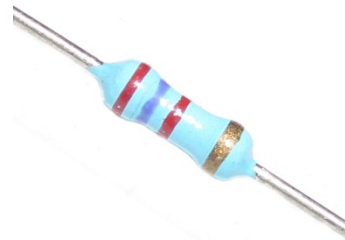
# RESISTORS

There are various types of fixed resistors:

- **wire-wound**—wound from resistance wire for high power ratings (up to 100 W)



- **Laminated** - a layer of carbon (carbon resistor) or metal (metallic resistor) on a ceramic body for power ratings up to several watts



- **SMD** – film type for surface mounting



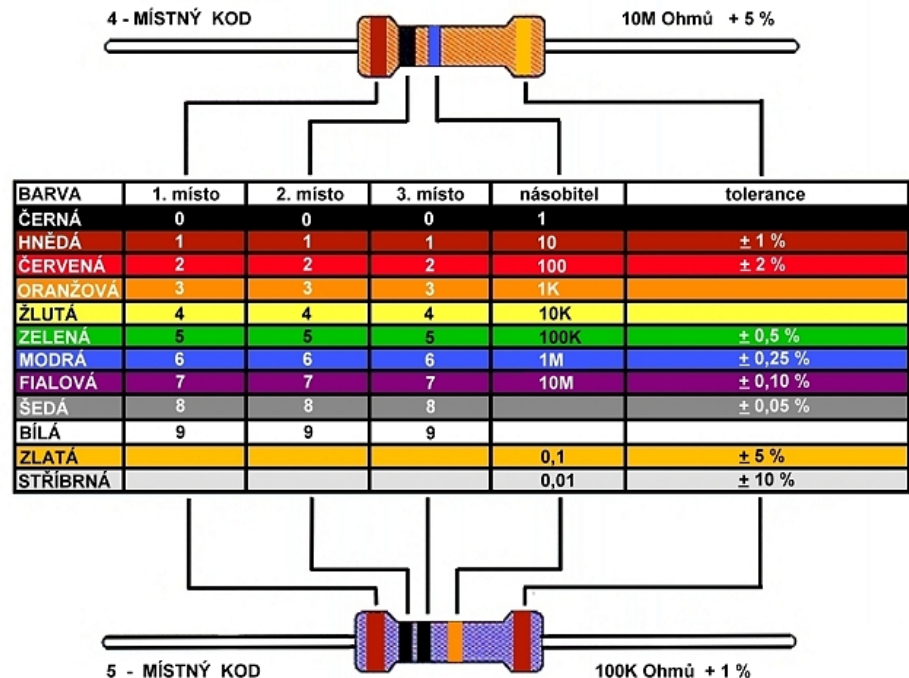


# RESISTORS - MARKINGS

- Resistors are manufactured in standardized resistance value series: E6, E12, E24, E48, E96, and E192.
- The most commonly used series is E12, which contains the following values in a single decade:  
1 1.2 1.5 1.8 2.2 2.7 3.3 3.9 4.7 5.6 6.8 8.2
- A color code is most commonly used for marking



100 Ω ±5%

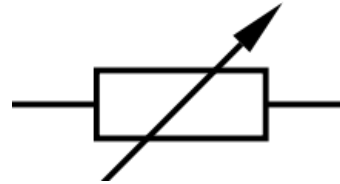




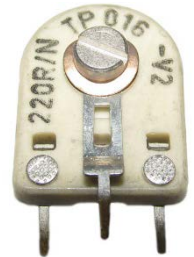
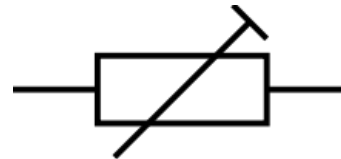
# RESISTORS

There are also variable resistors:

- **potentiometers** – rotary or slide variable resistors, most commonly used as adjustable resistive voltage dividers



- **trimmers** – variable resistors designed for precise adjustment of resistance values, e.g., when tuning electronic circuits.



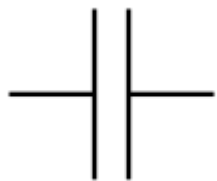
- **rheostats** – essentially power potentiometers made of power-rated resistive wire, used for current regulation



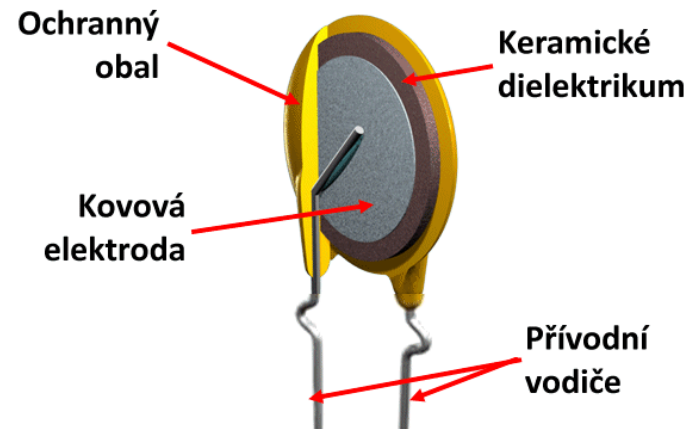


# CAPACITORS

- Capacitors are passive electronic components whose defining characteristic is capacitance [F].
- Another important parameter is the rated voltage [V], which specifies the maximum voltage at which the capacitor can be operated continuously.
- A capacitor consists of two electrodes separated by an electrically non-conductive material—a dielectric. This can be air, paper, plastic film, mica, or ceramic.



Capacitor model

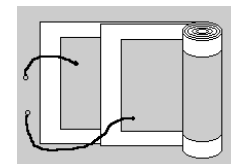
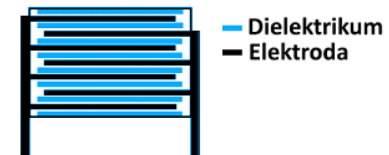




# CAPACITORS

There are various types of capacitors:

- **ceramic** – the dielectric is a special ceramic material, with a capacitance ranging from 1 pF to 1  $\mu$ F and a voltage rating from 10 V to 10 kV
- **foil** – the dielectric consists of a plastic film with metal electrodes vapor-deposited on both sides. The capacitor is formed by several such connected layers
- **tube** – the dielectric consists of a plastic film (PS, PE) with metal foil electrodes on both sides. The assembly is then rolled into a cylinder. Capacitance 1 nF to 10  $\mu$ F, voltage 100 V – 10 kV

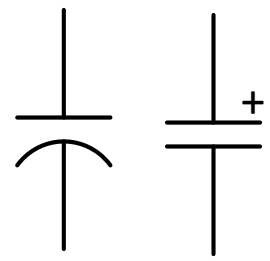




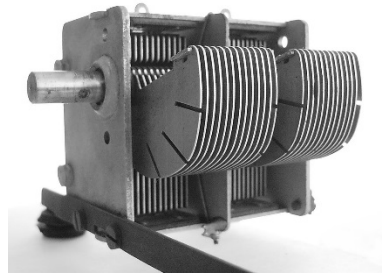
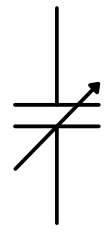
# CAPACITORS

There are various types of capacitors:

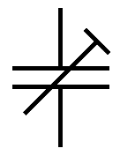
- **electrolytic** – the positive electrode consists of an aluminum foil whose surface area is significantly increased by etching. The dielectric consists of a thin layer of  $Al_2O_3$  ( $< 1 \mu m$ ); the negative electrode consists of an electrolyte that maintains a layer of oxide. The electrolytic capacitor has a high capacitance due to the thin dielectric and large electrode area. **It must not be reversed**, as this would cause electrolysis of the dielectric layer.  
Capacitance  $1 \mu F$  to  $1 F$ , voltage  $5 V - 500 V$



- **Tuning** – most commonly used for tuning resonant circuits, e.g., in older radio receivers



- **Tuning** – miniature versions of tuning capacitors, used, for example, in oscillators





# COILS

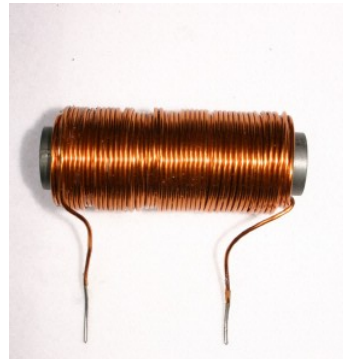
- Coils are passive components whose primary circuit characteristic is inductance [H]. Inductance depends on the number of turns, the geometric configuration, and the magnetic properties of the environment both inside and outside the coil.
- Core-less (air-core) coils are linear, while coils with a ferromagnetic core have nonlinear properties—their inductance depends on the current through the coil, the temperature, and the frequency.



cívka



cívka s jádrem



# Quiz

Click the **Quiz** button to edit this object

## Quiz 1 Introduction

- Read each question carefully
- To submit your answers, click the **Submit** button
- To pass, you must answer at least 75% of the questions correctly



## PART 2

# GRADUAL SIMPLIFICATION



## Methods of Analysis

### For special cases

- **Method of successive simplification**
- method of proportional quantities
- Transfiguration
- the principle of superposition
- Thévenin's and Norton's theorems

### General methods

- Direct application of Kirchhoff's laws
- Mesh current method (MCM)
- Node voltage method (NVM)
- Modified node voltage method (MNVM)



- **Principle** - step-by-step simplification of the circuit down to a circuit containing a single source and a single resistor
- **Solution** - gradual replacement
  - of series-connected elements
  - parallel-connected elements

### Pros:

- simple method
- uses basic mathematical operations
- suitable for "manual calculations"

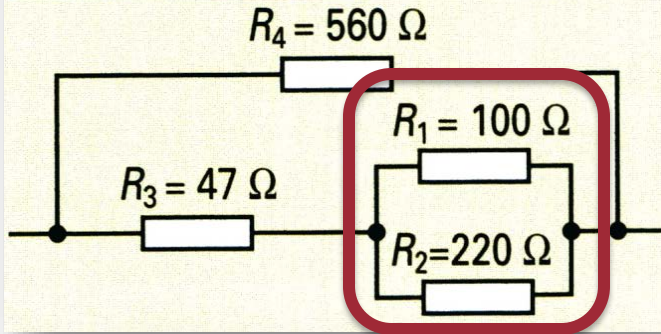
### Cons:

- **time-consuming and laborious method**
- analysis of only simpler circuits with a single source
- the solution procedure is "individual" (requires experience)
- Some circuits cannot be solved this way (they require, for example, the application of the circuit transformation method)

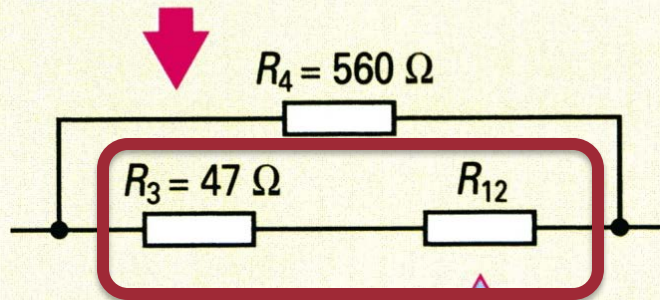


# THE METHOD OF GRADUAL SIMPLIFICATION

Výchozí smíšené zapojení

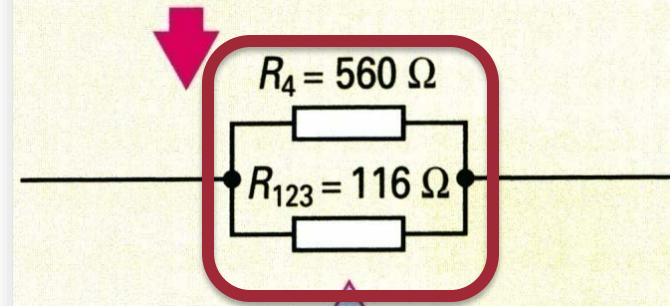


1. krok: spojení  $R_1$  a  $R_2$



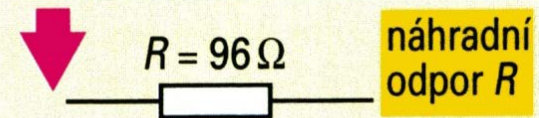
$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{100 \Omega \cdot 220 \Omega}{100 \Omega + 220 \Omega} = 69 \Omega$$

2. krok: spojení  $R_3$  a  $R_{12}$



$$R_{123} = R_3 + R_{12} = 47 \Omega + 69 \Omega = 116 \Omega$$

3. krok: spojení  $R_{123}$  a  $R_4$



$$R = \frac{R_{123} \cdot R_4}{R_{123} + R_4} = \frac{116 \Omega \cdot 560 \Omega}{116 \Omega + 560 \Omega} = 96 \Omega$$



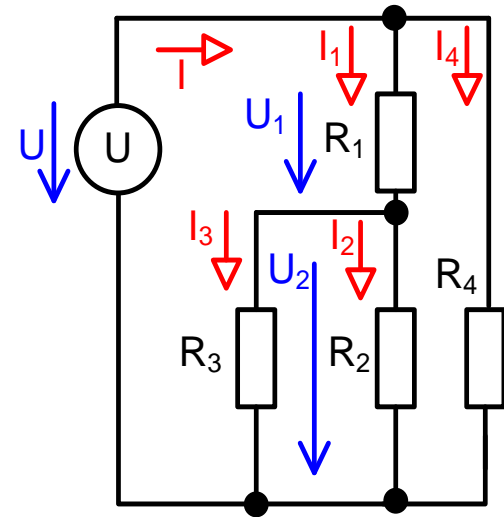
# STEP-BY-STEP SIMPLIFICATION METHOD

Use the method of successive simplification to determine all currents in the circuit shown in the figure.

$$R_1 = 150 \, \Omega, R_2 = 450 \, \Omega$$

$$R_3 = 370 \, \Omega, R_4 = 220 \, \Omega$$

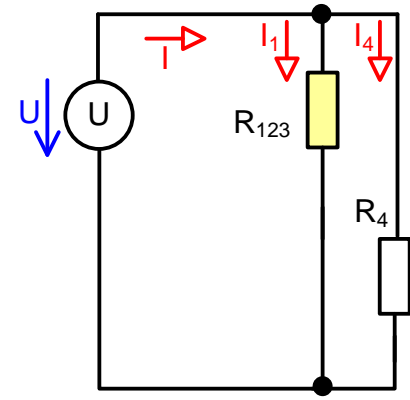
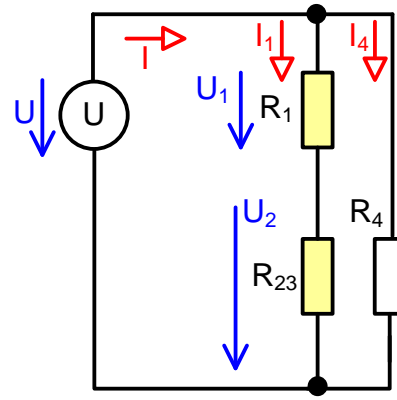
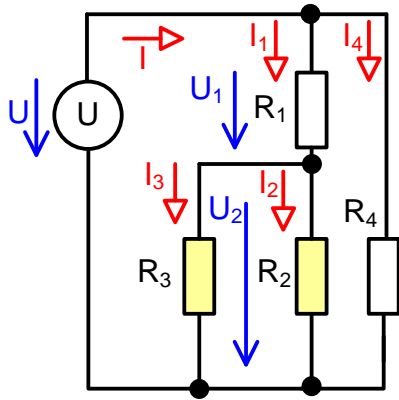
$$U = 24 \, V$$





# THE METHOD OF GRADUAL SIMPLIFICATION

- $R_1 = 150 \Omega$
- $R_2 = 450 \Omega$
- $R_3 = 370 \Omega$
- $R_4 = 220 \Omega$
- $U = 24 V$



## Part 1 – “Forward method”

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3} = 203 \Omega$$

$$R_{123} = R_1 + R_{23} = 353 \Omega$$

## Part 2 – “Backward approach”

$$I_2 = \frac{U_2}{R_2} = \underline{\underline{30,67 \text{ mA}}}$$

$$U_2 = I_1 R_{23} = 13,8 V$$

$$I_4 = \frac{U}{R_4} = \underline{\underline{109,1 \text{ mA}}}$$

$$I_1 = \frac{U}{R_{123}} = \underline{\underline{67,98 \text{ mA}}}$$

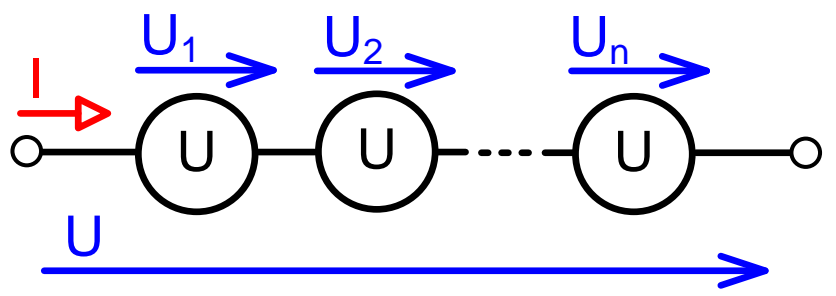
$$I_3 = \frac{U_2}{R_3} = \underline{\underline{37,31 \text{ mA}}}$$

$$I = I_1 + I_4 = \underline{\underline{177,1 \text{ mA}}}$$

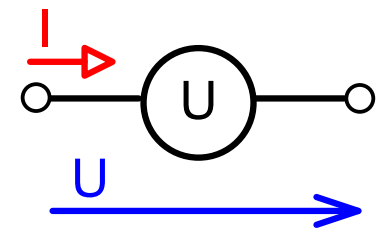


# CONNECTING POWER SUPPLIES

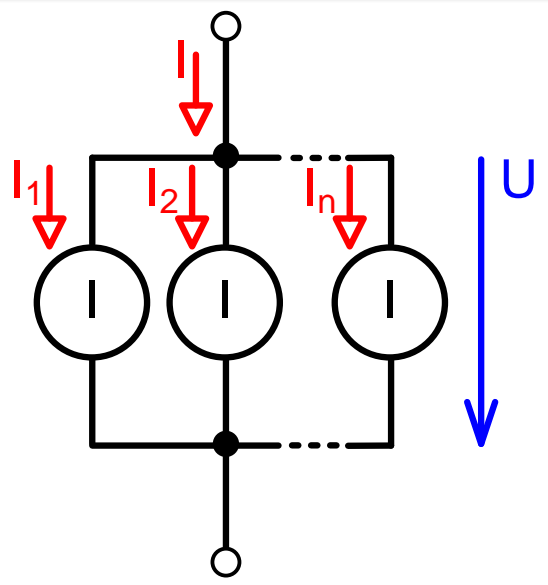
Series connection of voltage sources



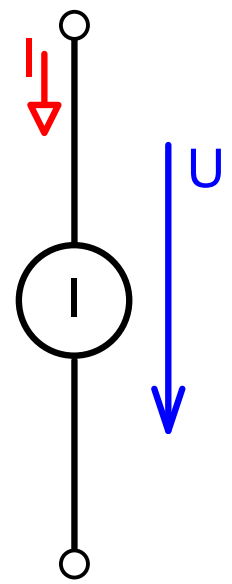
$$U = U_1 + U_2 + \dots + U_n$$



Parallel connection of current sources



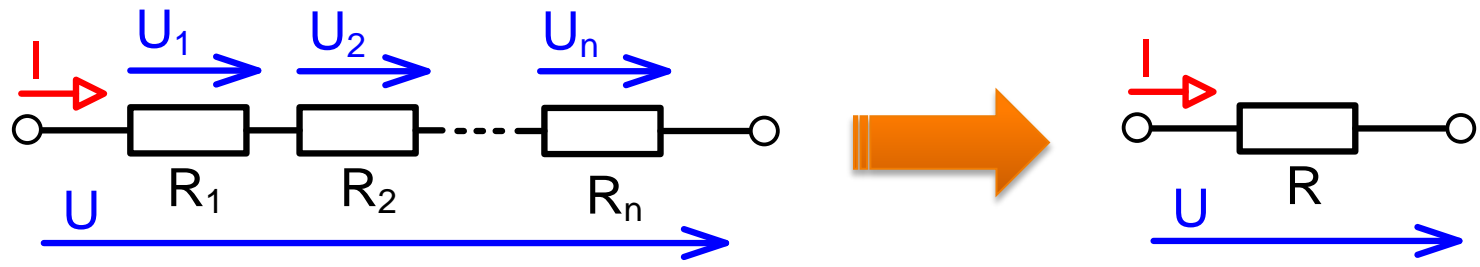
$$I = I_1 + I_2 + \dots + I_n$$





# CONNECTING RESISTORS

## Series connection of resistors



$$U = U_1 + U_2 + \dots + U_n = I(R_1 + R_2 + \dots + R_n) = I \cdot R$$

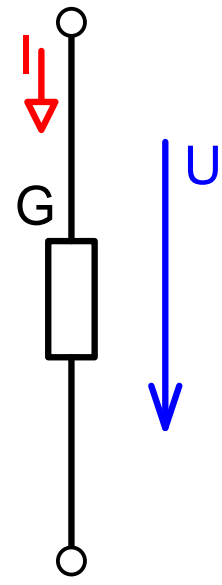
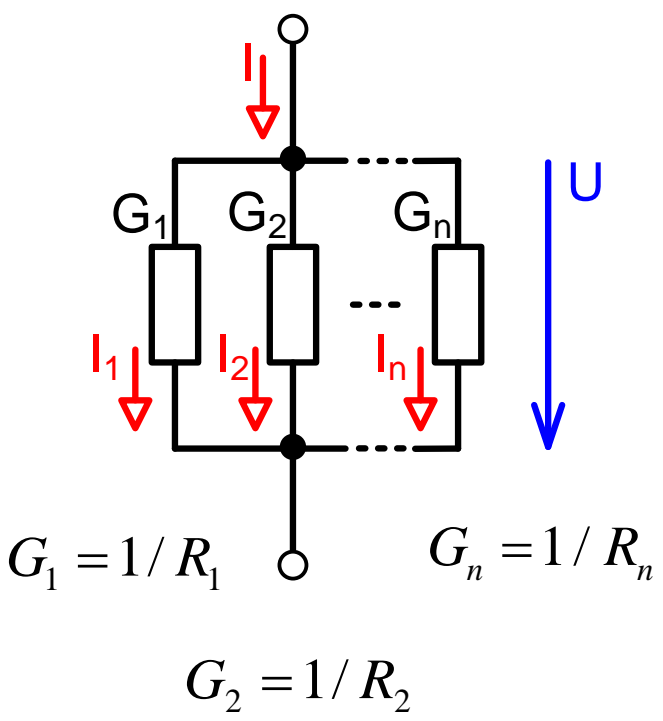
$$R = \sum_{j=1}^n R_j$$

$$R = R_1 + R_2 + \dots + R_n$$



# CONNECTING RESISTORS

## Parallel connection of resistors



$$G = G_1 + G_2 + \dots + G_n$$

Abbreviated notation:

$$R = R_1 \parallel R_2 \parallel \dots \parallel R_n$$

$$G = \sum_{j=1}^n G_j$$

$$G = 1/R$$

$$R = \frac{1}{\sum_{j=1}^n \frac{1}{R_j}}$$

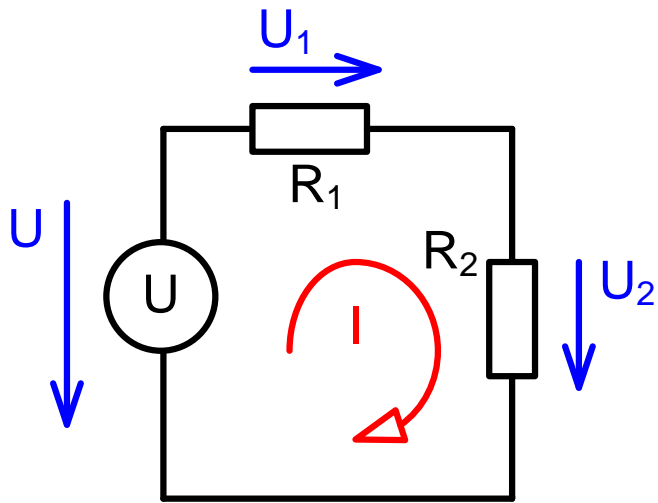
$$I = I_1 + I_2 + I_3 = U (G_1 + G_2 + \dots + G_n) = U \cdot G$$

Specifically for 2 resistors:

$$R = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$$



## Resistor voltage divider



$$U_1 = ? \quad U_2 = ?$$

The total resistance of the circuit is  $R = R_1 + R_2$

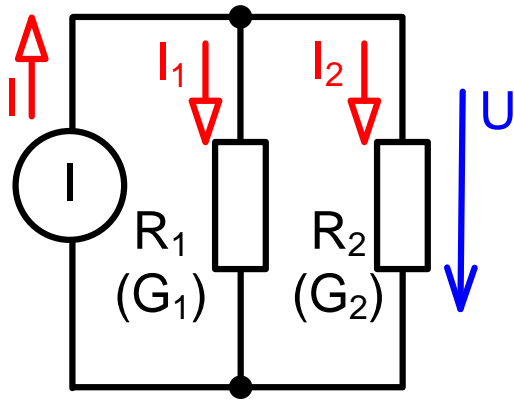
and current through the circuit  $I = \frac{U}{R_1 + R_2}$

Therefore, the voltage  $U_1 = I \cdot R_1 = U \frac{R_1}{R_1 + R_2}$

$$U_2 = I \cdot R_2 = U \frac{R_2}{R_1 + R_2}$$



## Resistor current divider



$$I_1 = ? \quad I_2 = ?$$

Since

$$U = I \cdot R = I \frac{R_1 R_2}{R_1 + R_2}$$

the currents are branches:  $I_1 = \frac{U}{R_1} = I \frac{R_2}{R_1 + R_2} = I \frac{G_1}{G_1 + G_2}$

$$I_2 = \frac{U}{R_2} = I \frac{R_1}{R_1 + R_2} = I \frac{G_2}{G_1 + G_2}$$

# Quiz

Click the **Quiz** button to edit this object

## Quiz 2

### Simplification method

- Read each question carefully
- To submit your answers, click the **Submit** button
- To pass, you must answer at least 75% of the questions correctly



## PART 3

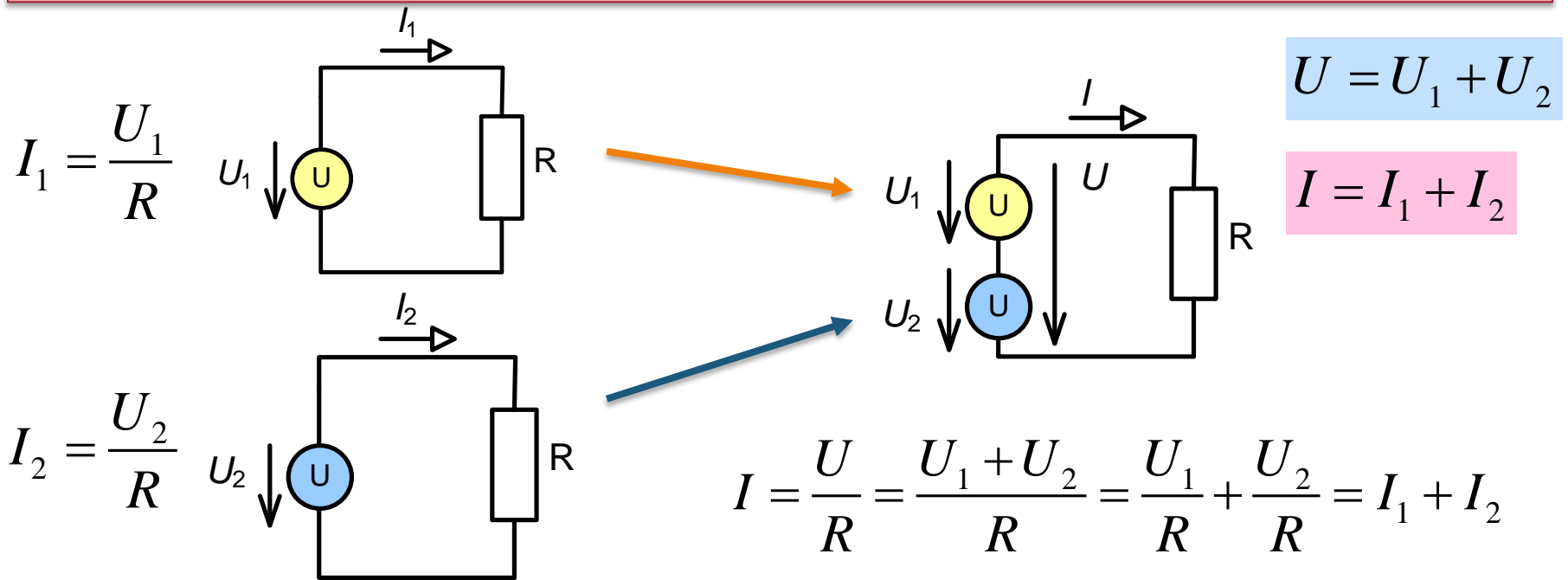
# THE PRINCIPLE OF SUPERPOSITION



# THE PRINCIPLE OF SUPERPOSITION

**Principle of superposition:** The response to the sum of stimuli is equal to the sum of the responses to the individual stimuli acting separately.

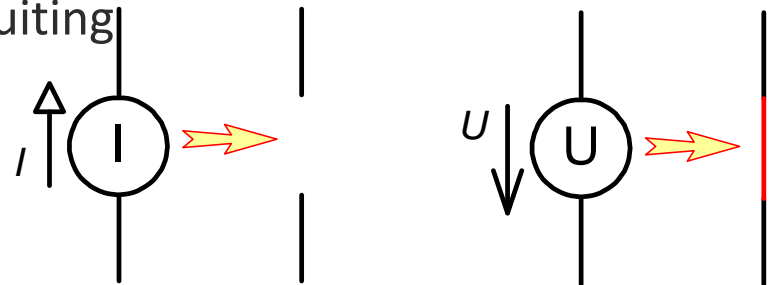
**Electrical circuits:** stimuli = voltages/currents of independent sources  
responses = voltages/currents of circuit elements



**THE EFFECTS OF SOURCES ARE LINEARLY ADDED IN LINEAR CIRCUITS.**

## The Principle of Superposition

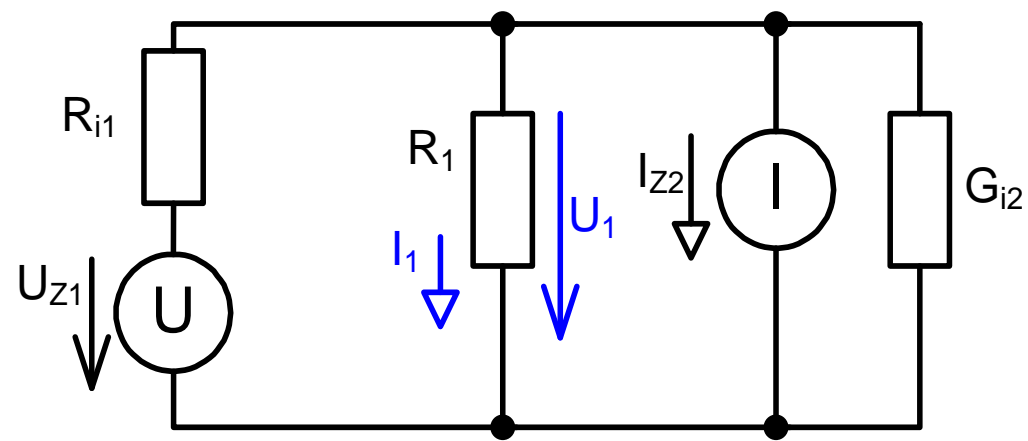
- Applies **only to linear circuits**
- The effects of the sources are added together (superimposed)
- The circuit variables ( $U$ ,  $I$ ) are calculated separately for each source; the other sources are replaced by an internal resistance:
  - ideal **source  $I$**  ( $R \rightarrow \infty$ ) **by open-circuiting**
  - ideal source  $U$  ( $R = 0$ ) by short-circuiting





# THE PRINCIPLE OF SUPERPOSITION

Determine the current and voltage across resistor  $R_1$  .

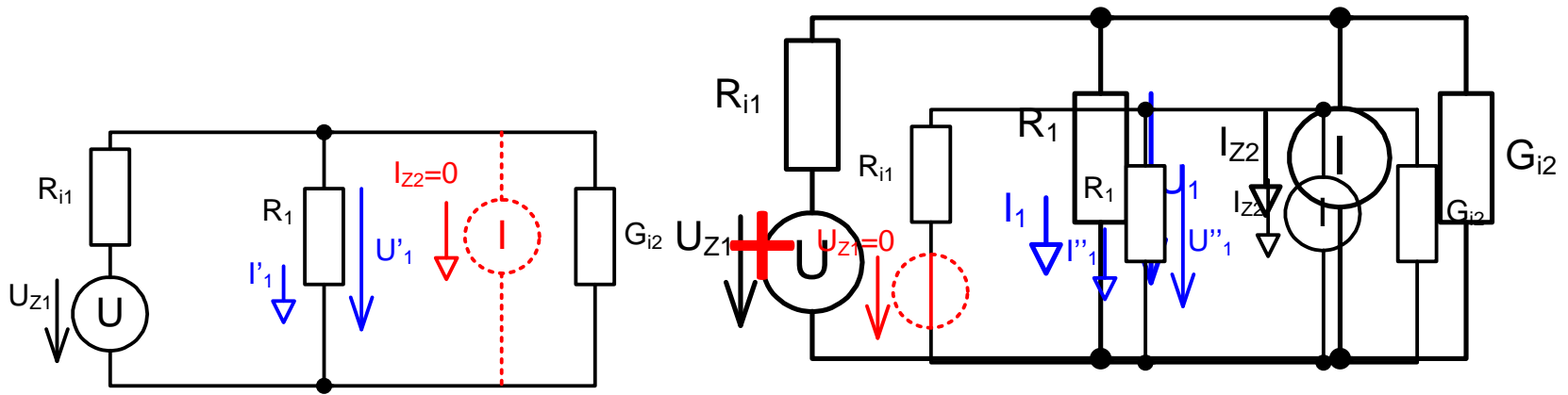


$$U_{Z1} = 12 \text{ V}, R_{i1} = 2 \Omega$$
$$I_{Z2} = 4 \text{ A}, G_{i2} = 0,5 \text{ S}$$
$$R_1 = 4 \Omega$$



# THE PRINCIPLE OF SUPERPOSITION

For  $U_1$  and  $I_1$ , we determine by superposition



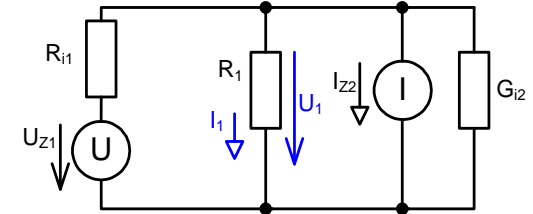
$$U_1 = U'_1 + U''_1$$

$$I_1 = I'_1 + I''_1$$



# POST FROM THE FIRST SOURCE

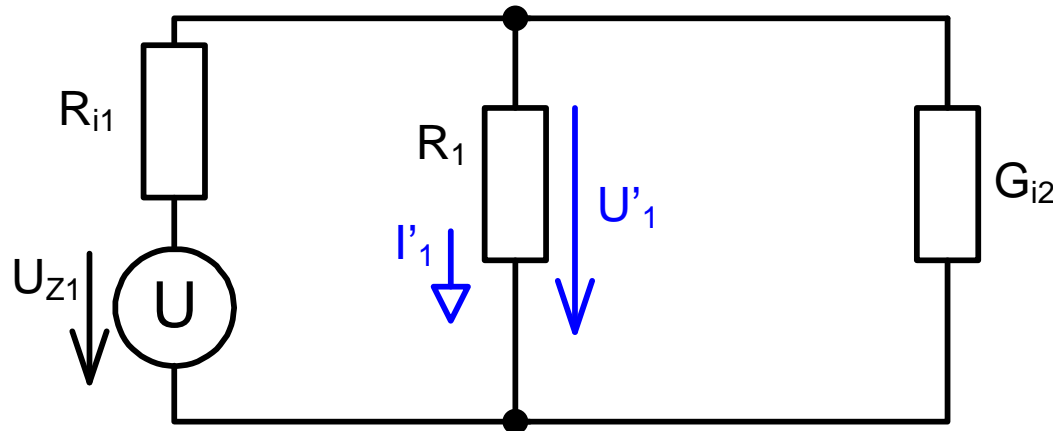
- Source  $I_{Z2}$  is disconnected
- We solve this, for example, by simplifying, replacing the voltage source with a current source, ...



$$U_{Z1} = 12 \text{ V}, R_{i1} = 2 \Omega$$

$$I_{Z2} = 4 \text{ A}, G_{i2} = 0,5 \text{ S}$$

$$R_1 = 4 \Omega$$

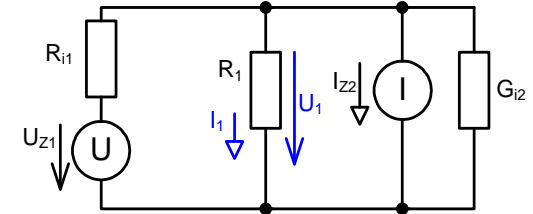


$$U'_1 = \underline{4,8 \text{ V}}, I'_1 = \underline{1,2 \text{ A}}$$



# CONTRIBUTION FROM THE SECOND SOURCE

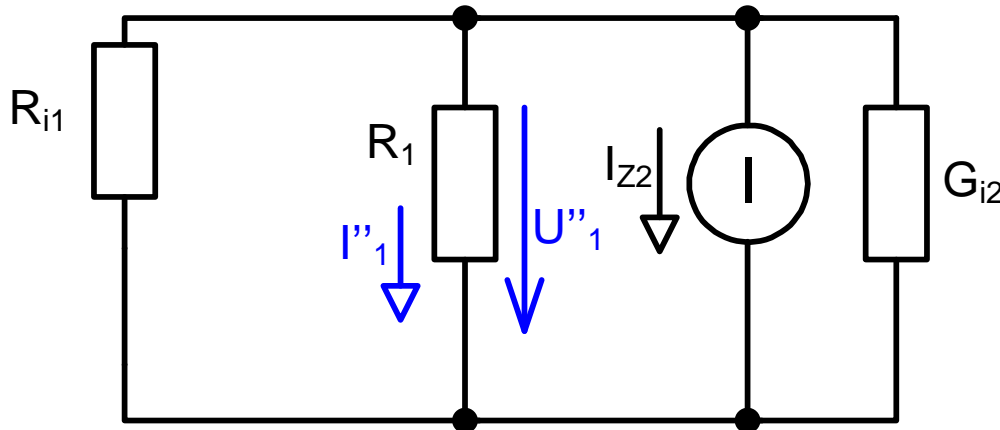
- The source  $U_{Z1}$  is replaced by an abbreviation
- We solve this, for example, by simplifying, ...



$$U_{Z1} = 12 \text{ V}, R_{i1} = 2 \Omega$$

$$I_{Z2} = 4 \text{ A}, G_{i2} = 0,5 \text{ S}$$

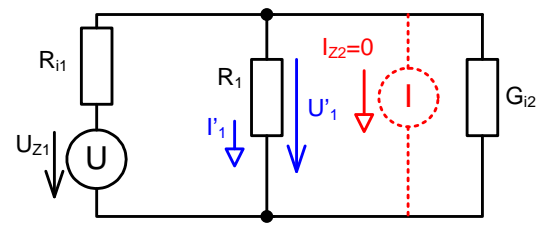
$$R_1 = 4 \Omega$$



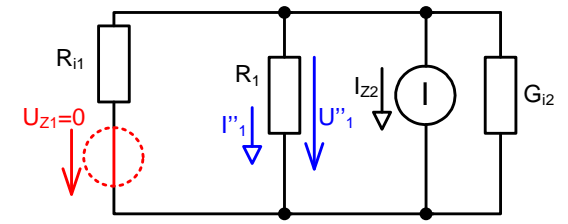
$$U''_1 = \underline{\underline{-3,2 \text{ V}}}, I''_1 = \underline{\underline{-0,8 \text{ A}}}$$



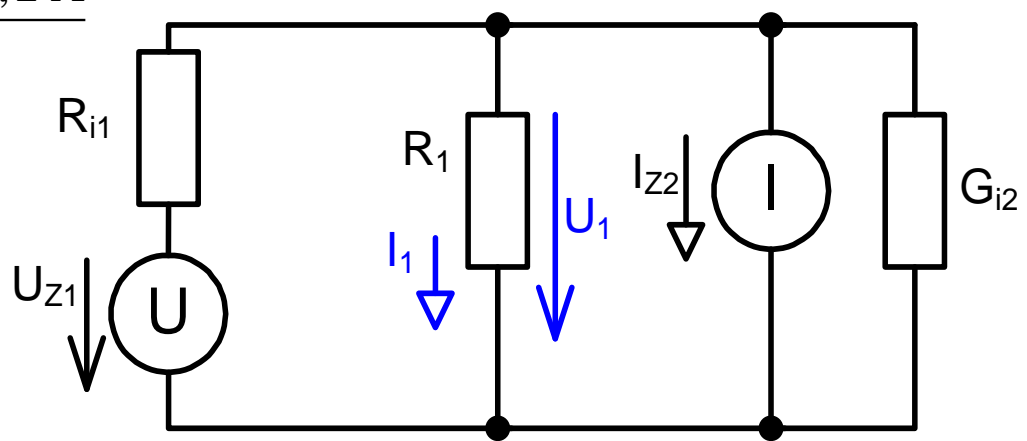
- For  $U_1$  and  $I_1$ , we determine by superposition



$$U'_1 = \underline{4,8 \text{ V}}, I'_1 = \underline{1,2 \text{ A}}$$



$$U''_1 = \underline{-3,2 \text{ V}}, I''_1 = \underline{-0,8 \text{ A}}$$



$$U_1 = U'_1 + U''_1 = 4,8 + (-3,2) = \underline{\underline{1,6 \text{ V}}}$$

$$I_1 = I'_1 + I''_1 = 1,2 + (-0,8) = \underline{\underline{0,4 \text{ A}}}$$

# Quiz

Click the **Quiz** button to edit this object

## Quiz 3

### The Principle of Superposition

- Read each question carefully
- To submit your answers, click the **Submit** button
- To pass, you must answer at least 75% of the questions correctly



## PART 4

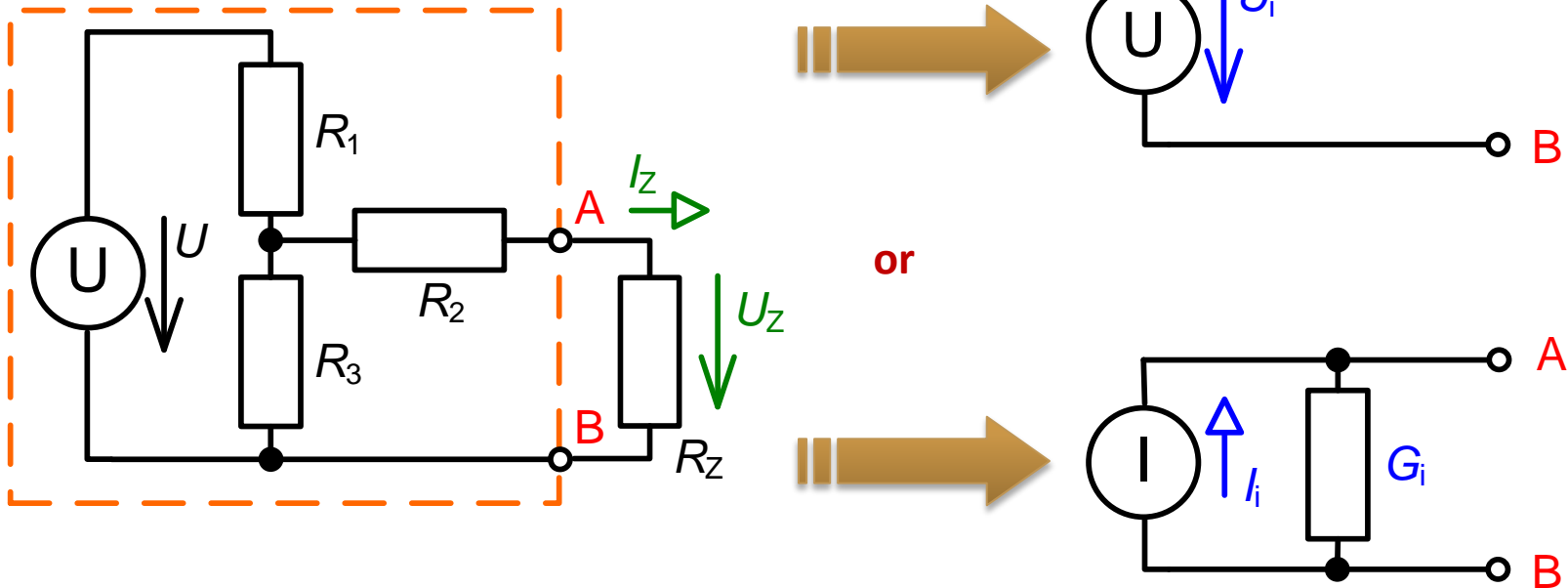
# THE SUBSTITUTE SOURCE METHOD



# EQUIVALENT SOURCE METHOD

Replacing part of a circuit with an equivalent source:

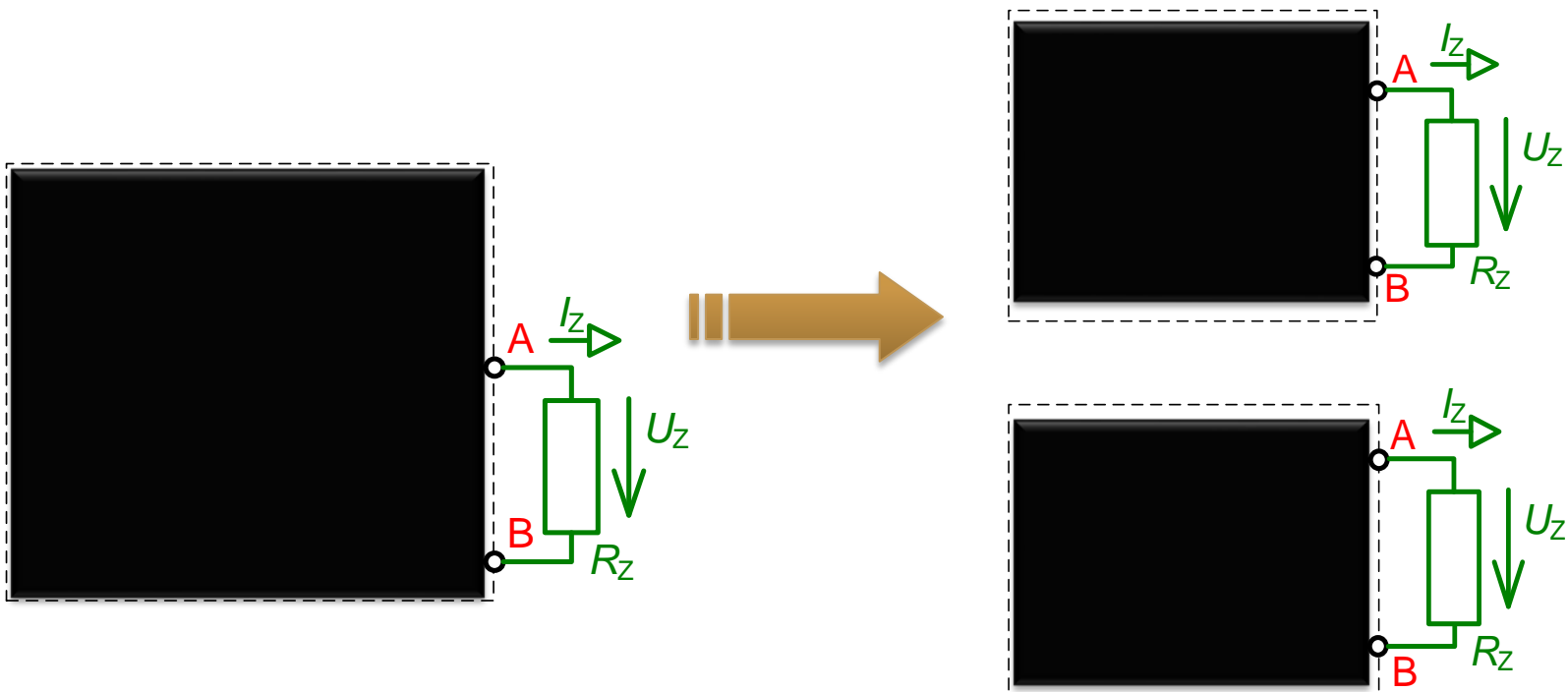
- **voltage** ( $U_i$ ,  $R_i$  —Thevenin's theorem)
- **current** ( $I_i$ ,  $G_i$  —Norton's theorem)





# APPLICATIONS OF BACKUP POWER SOURCES

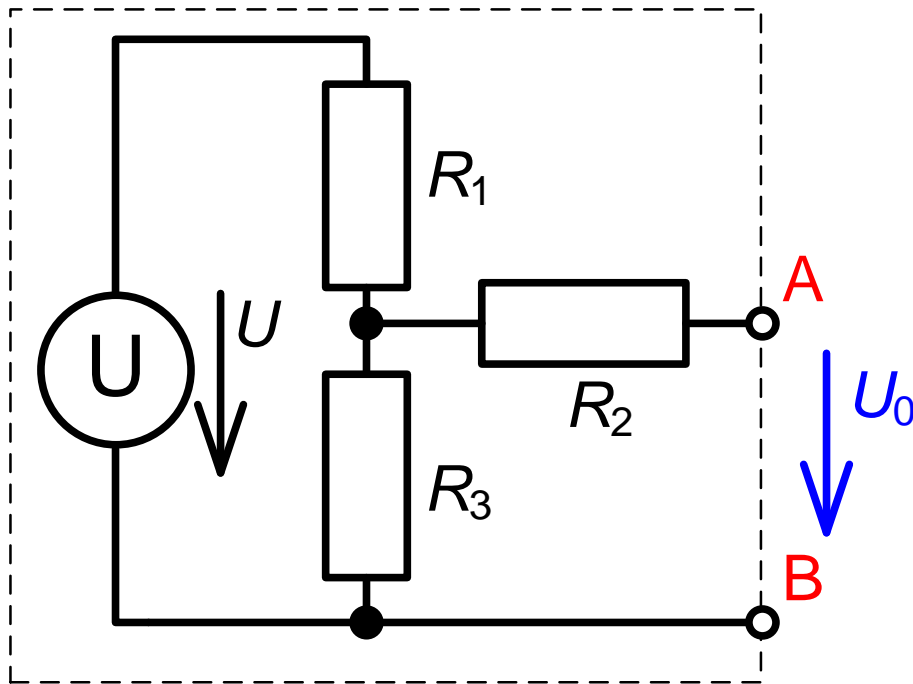
- With respect to terminals AB, the replacement sources are equivalent to the original circuit
- Same  $U_Z$  and  $I_Z$  for the same  $R_Z$



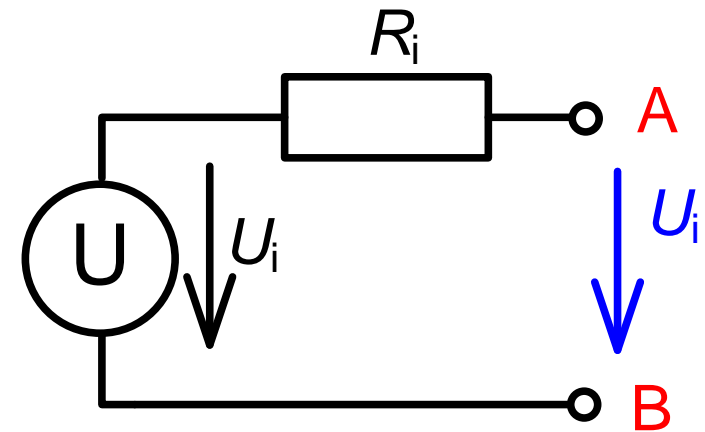


# DETERMINING THE VOLTAGE OF THE SUBSTITUTE SOURCE

- The voltage of the substitute source is equal to the terminal voltage of the original unloaded circuit



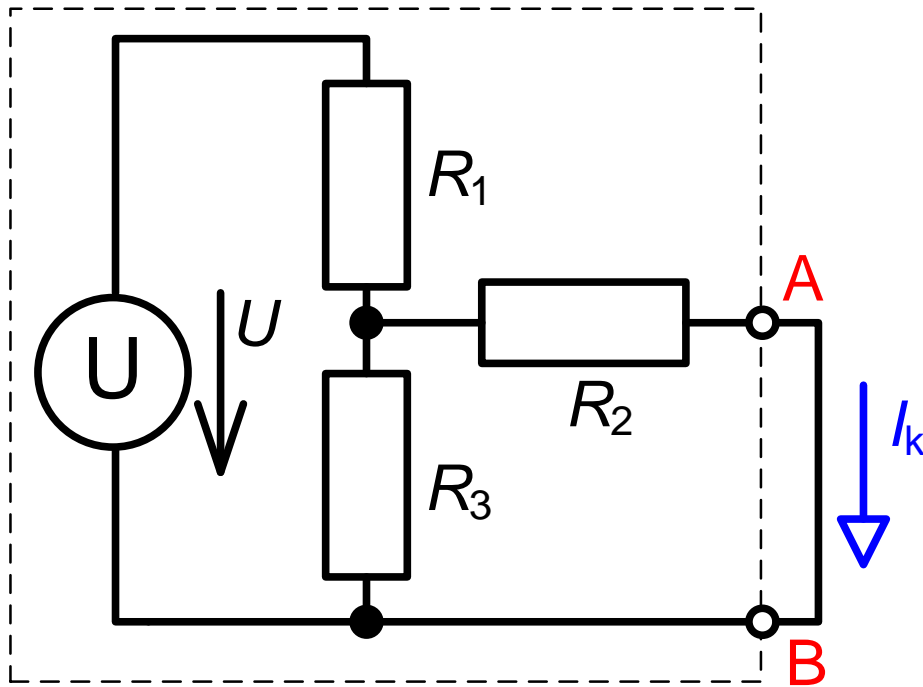
$$U_i = U_0$$



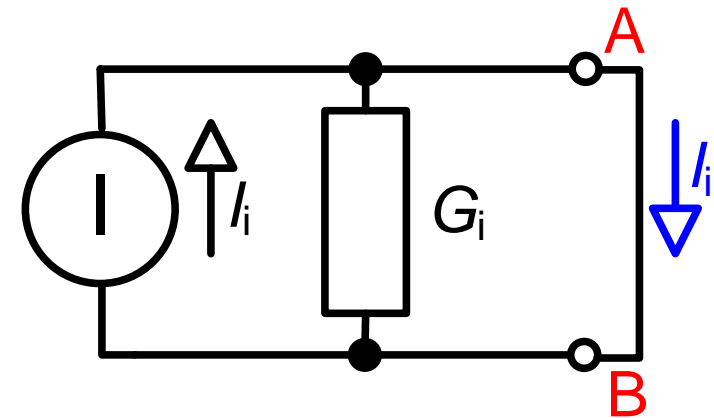


# DETERMINING THE CURRENT OF THE REPLACEMENT POWER SOURCE

- The current of the replacement source is equal to the short-circuit terminal current of the original circuit



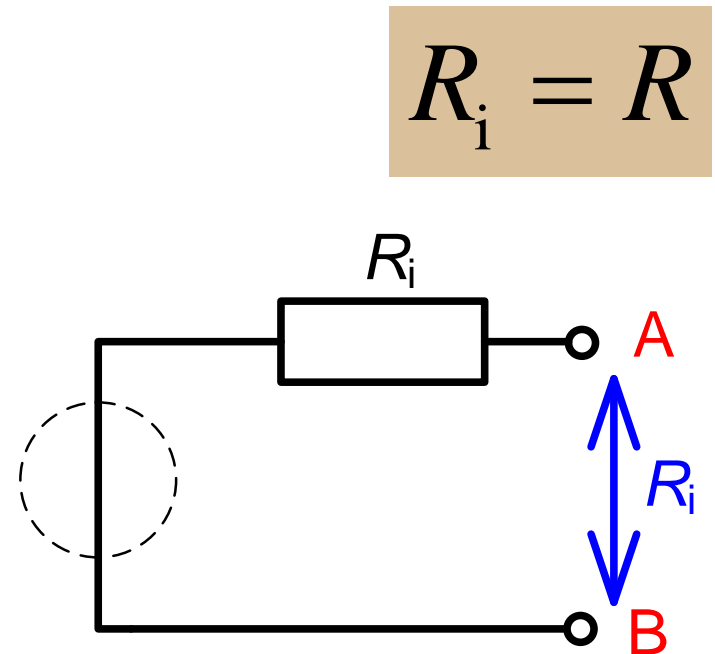
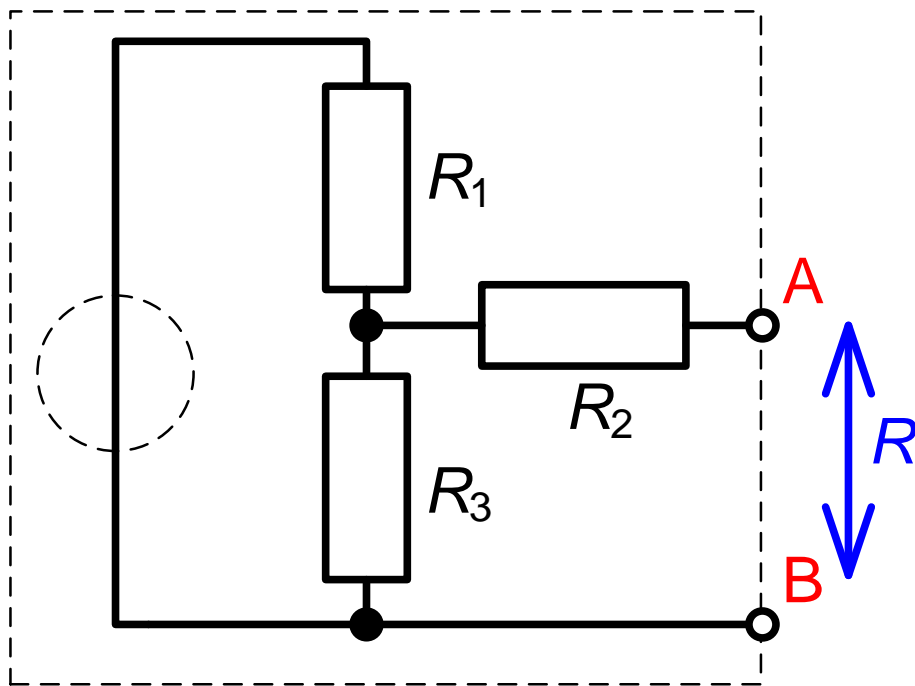
$$I_i = I_K$$





# DETERMINING THE INTERNAL RESISTANCE OF A SUBSTITUTE SOURCE

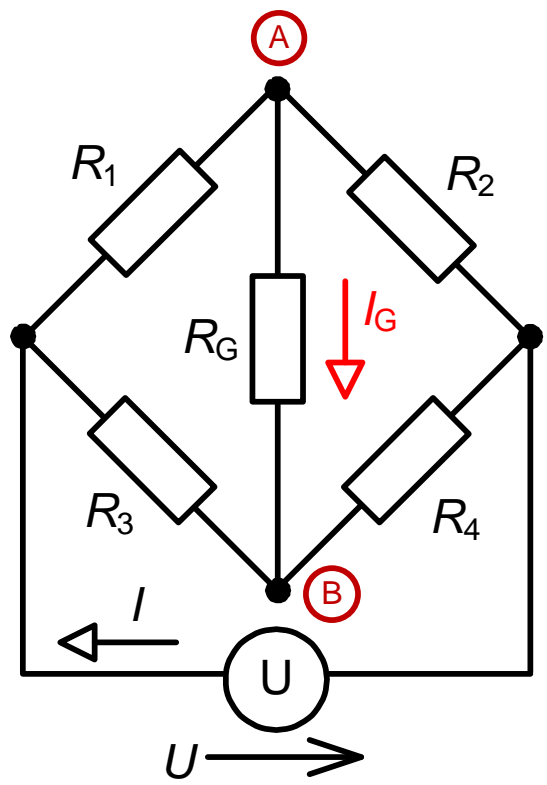
- Current sources are disconnected; voltage sources are replaced by a short circuit



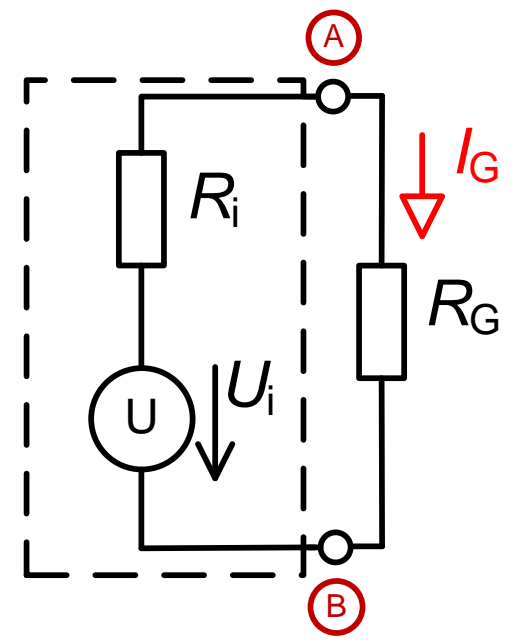


# EXAMPLE 1

Use the virtual source method to determine the current  $I_G$



- $U_Z = 12 \text{ V}$
- $R_1 = 10 \text{ } \Omega$
- $R_2 = 50 \text{ } \Omega$
- $R_3 = 20 \text{ } \Omega$
- $R_4 = 40 \text{ } \Omega$
- $R_G = 2 \text{ } \Omega$

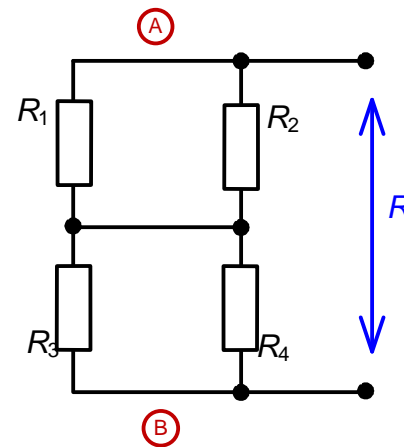
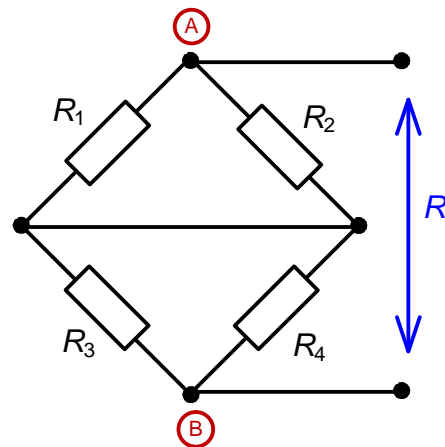
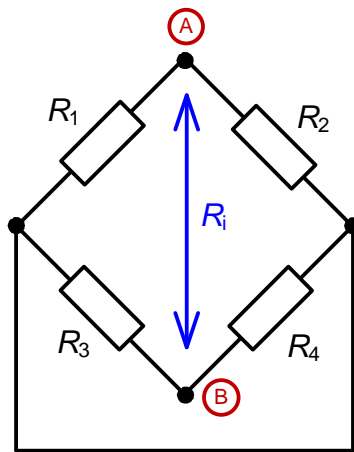
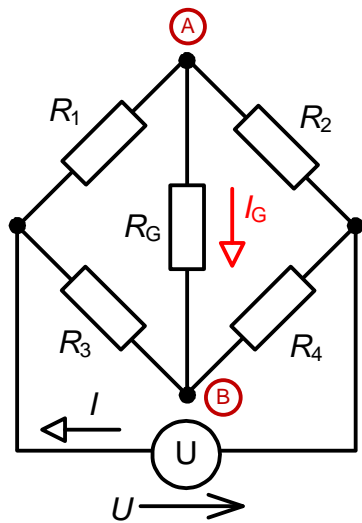


$$I_G = \frac{U_i}{R_i + R_G}$$



# EXAMPLE 1

## DETERMINING THE RESISTANCE OF A SUBSTITUTE SOURCE



$$U_Z = 12 \text{ V}$$

$$R_1 = 10 \text{ } \Omega$$

$$R_2 = 50 \text{ } \Omega$$

$$R_3 = 20 \text{ } \Omega$$

$$R_4 = 40 \text{ } \Omega$$

$$R_G = 2 \text{ } \Omega$$

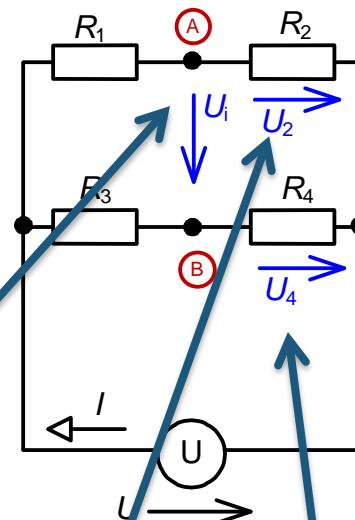
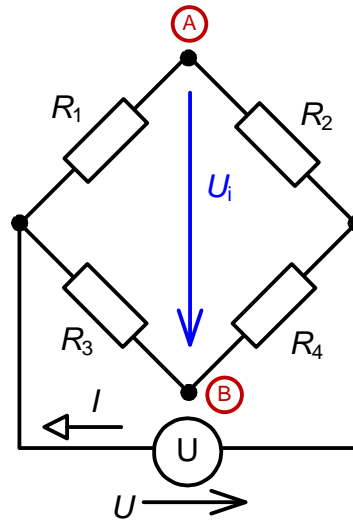
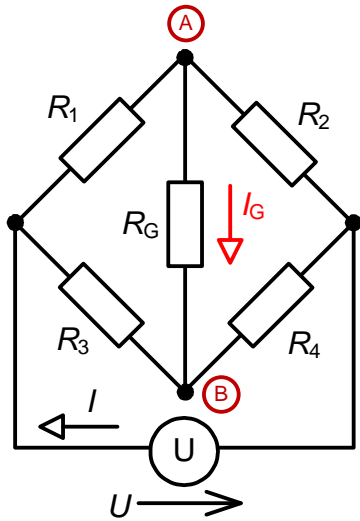
$$R_i = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4}$$

$$R_i = \frac{10 \cdot 50}{10 + 50} + \frac{20 \cdot 40}{20 + 40} = \underline{\underline{21,67 \text{ } \Omega}}$$



# EXAMPLE 1

## DETERMINING THE VOLTAGE OF THE SUBSTITUTE SOURCE



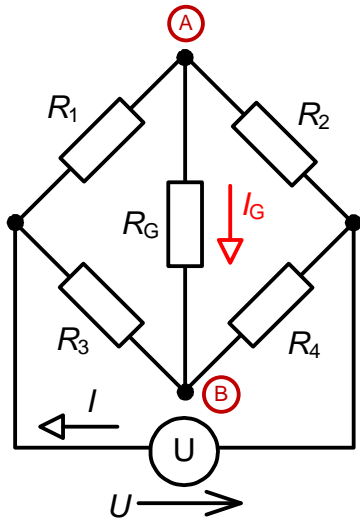
- $U_Z = 12 \text{ V}$
- $R_1 = 10 \text{ } \Omega$
- $R_2 = 50 \text{ } \Omega$
- $R_3 = 20 \text{ } \Omega$
- $R_4 = 40 \text{ } \Omega$
- $R_G = 2 \text{ } \Omega$

$$U_i = U_2 - U_4 = U \frac{R_2}{R_1 + R_2} - U \frac{R_4}{R_3 + R_4}$$

$$U_i = 12 \left[ \frac{50}{10 + 50} - \frac{40}{20 + 40} \right] = \underline{2 \text{ V}}$$



# EXAMPLE 1 COMPLETION



$$U_Z = 12 \text{ V}$$

$$R_1 = 10 \text{ } \Omega$$

$$R_2 = 50 \text{ } \Omega$$

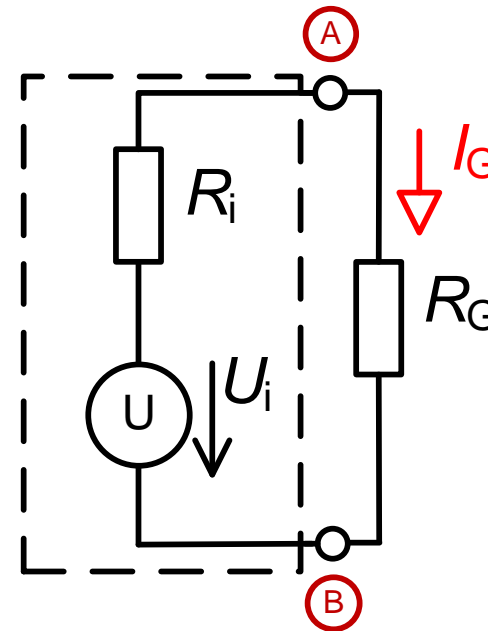
$$R_3 = 20 \text{ } \Omega$$

$$R_4 = 40 \text{ } \Omega$$

$$R_G = 2 \text{ } \Omega$$

$$R_i = \underline{21,67 \text{ } \Omega}$$

$$U_i = \underline{2 \text{ V}}$$



$$I_G = \frac{U_i}{R_i + R_G}$$

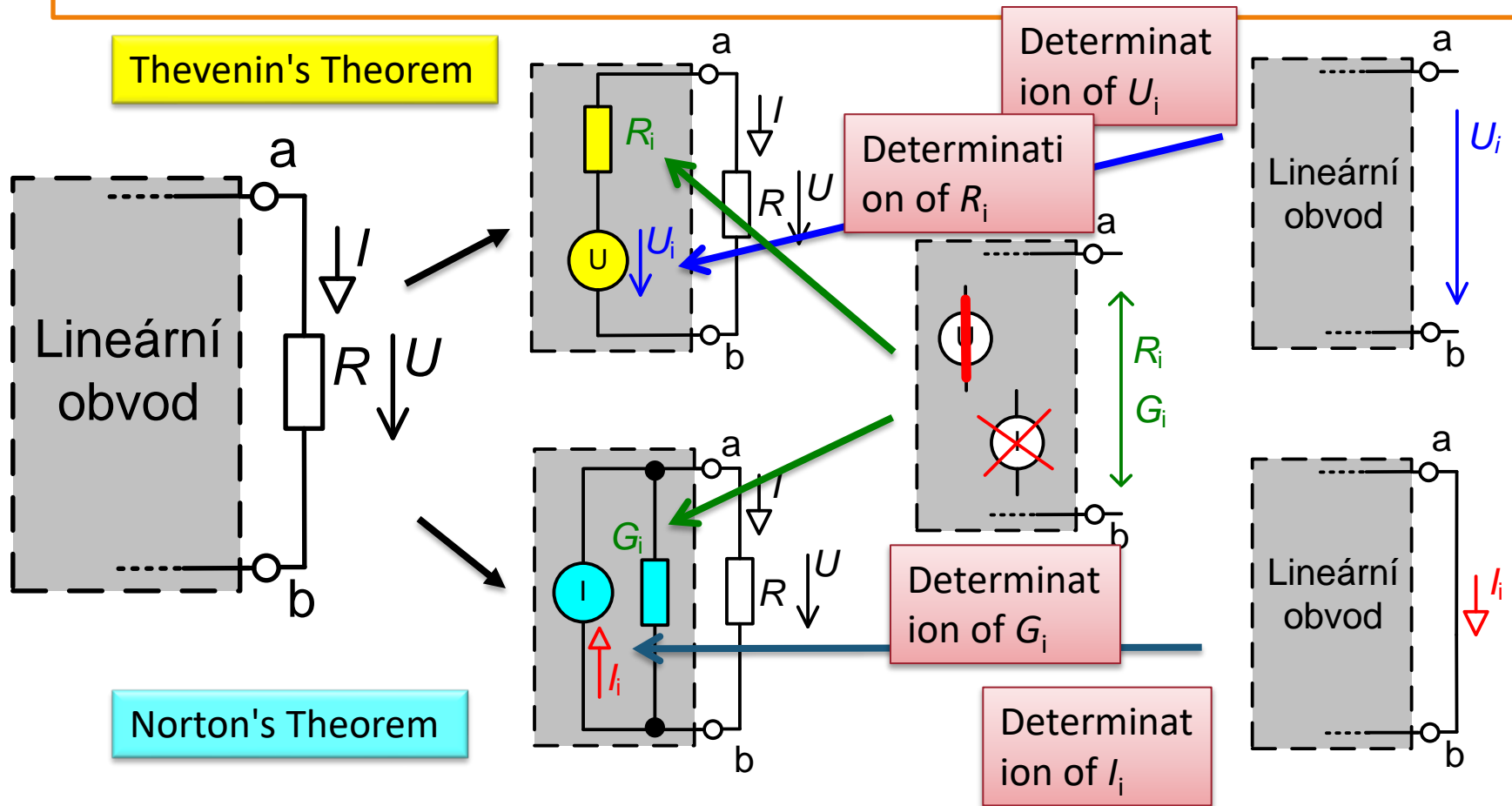
$$I_G = \frac{2}{21,67 + 2} = \underline{\underline{84,51 \text{ mA}}}$$



# EQUIVALENT CIRCUIT METHOD - SUMMARY

This method is suitable when analyzing **a single branch of a circuit**

- finding the internal resistance of the circuit (e.g., for power matching)
- suitable for solving transient phenomena (1 storage element)
- can also be used for nonlinear R



# Quiz

Click the **Quiz** button to edit this object

## Quiz 4

### The substitute source method

- Read each question carefully
- To submit your answers, click the **Submit** button
- To pass, you must answer at least 75% of the questions correctly



# PART 5

## UNIVERSAL METHODS

MESH CURRENT METHOD

NODE VOLTAGE METHOD



# MCM

# MESH CURRENT METHOD



## MESH CURRENT METHOD (MCM)

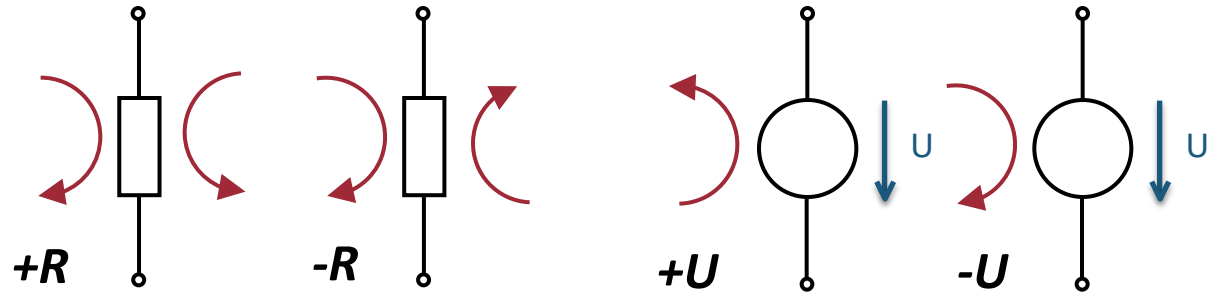
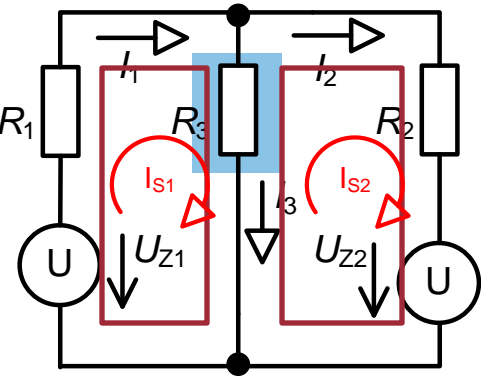
- is based on Kirchhoff's second law
- The circuit may contain only voltage sources; current sources must be replaced
- The following matrix equation is solved:

$$\mathbf{R} \cdot \mathbf{I} = \mathbf{U}$$



# MESH CURRENT METHOD (MCM)

- WITH DIRECT FORMULATION OF THE MATRIX EQUATION



MSP – direct matrix construction:

- 1) Select independent meshes
- 2) We plot the mesh currents
- 3) Let's prepare the matrix notation
- 4) Write down the resistance matrix
- 5) We write down the source voltages
- 6) Solve the matrix equation
- 7) We will determine the currents and voltages in the circuit

$$\begin{matrix} & \begin{matrix} 1 & 2 \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \end{matrix} \cdot \begin{matrix} I_{S1} \\ I_{S2} \end{matrix} = \begin{matrix} U_{Z1} \\ -U_{Z2} \end{matrix}$$

$$I_{S1} = \frac{\Delta_1}{\Delta}$$

$$I_{S2} = \frac{\Delta_2}{\Delta}$$

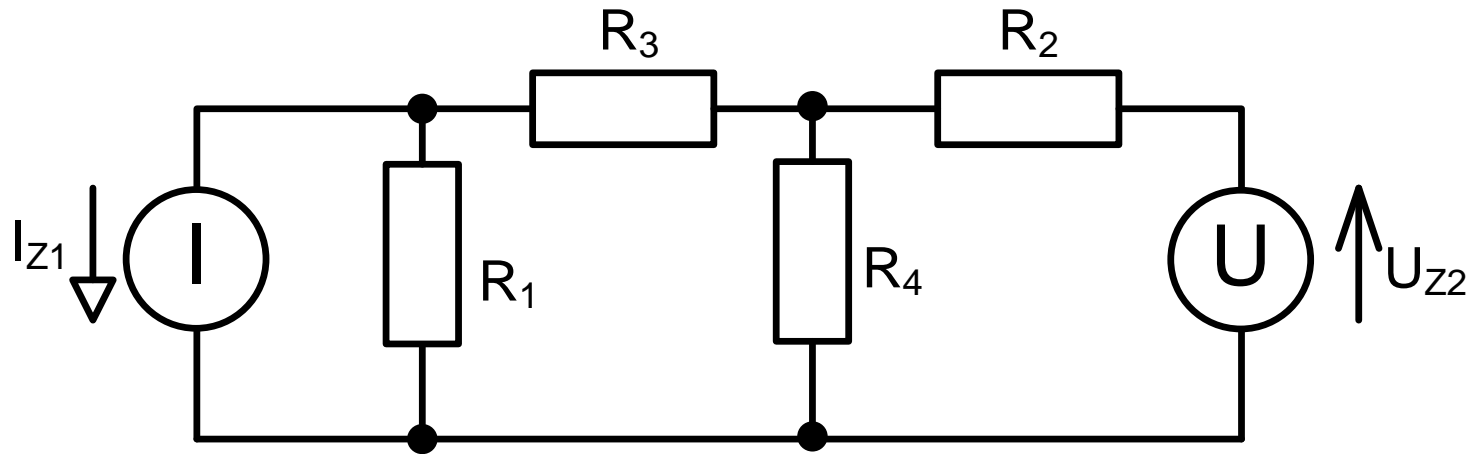
$$\begin{aligned}
 I_1 &= I_{S1} \\
 I_2 &= I_{S2} \\
 I_3 &= I_{S1} - I_{S2}
 \end{aligned}$$

$$\begin{aligned}
 U_1 &= R_1 I_1 \\
 U_2 &= R_2 I_2 \\
 U_3 &= R_3 I_3
 \end{aligned}$$



# LOOP CURRENT METHOD (LCM)

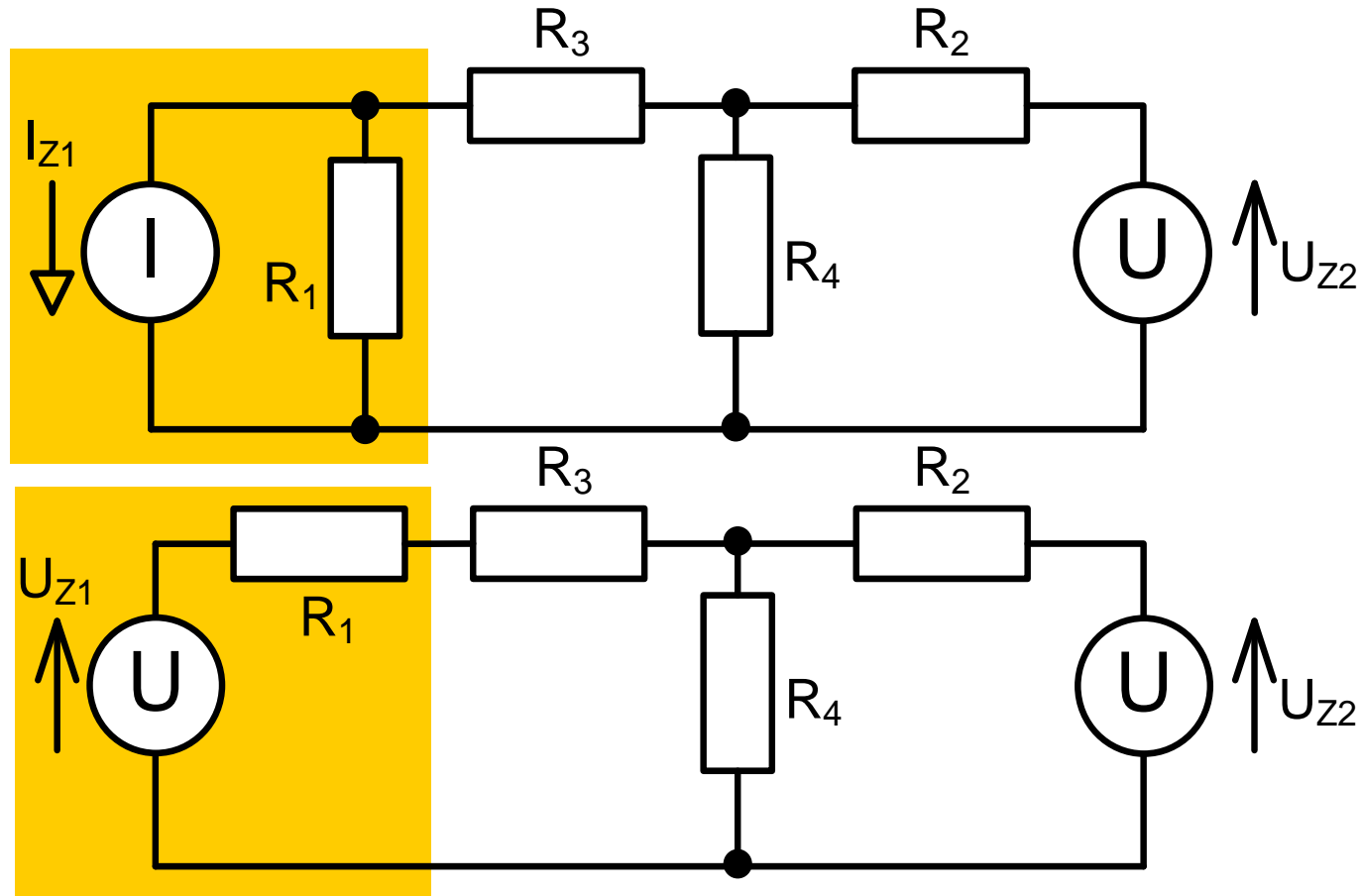
Calculate the currents through all resistors in the circuit.



$$I_{z1} = 200 \text{ mA}, U_{z2} = 300 \text{ V}, R_1 = 2,4 \text{ k}\Omega,$$
$$R_2 = 6 \text{ k}\Omega, R_3 = 1,6 \text{ k}\Omega, R_4 = 5 \text{ k}\Omega$$



# REPLACEMENT OF THE POWER SUPPLY



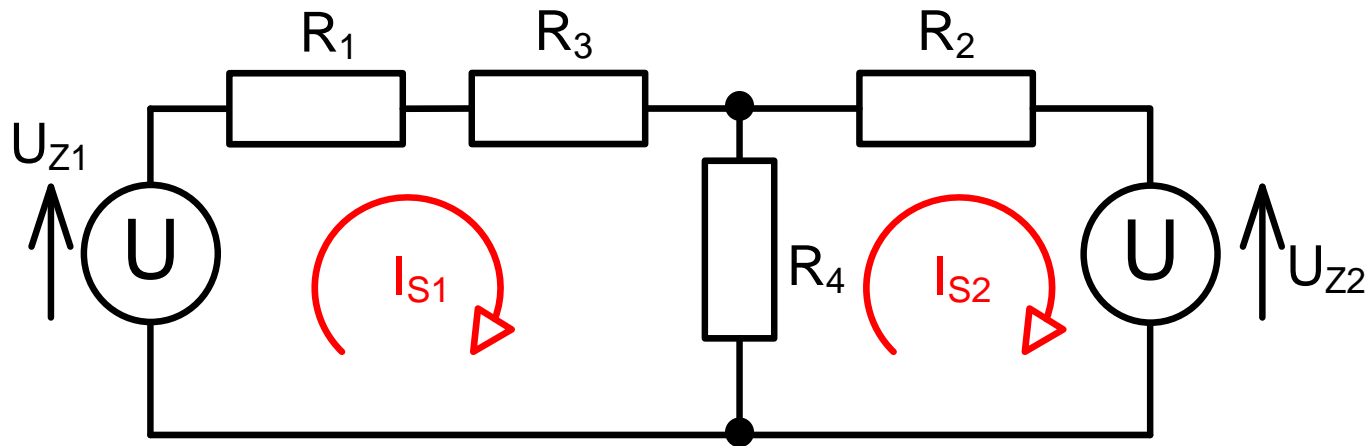
$$U_{z1} = I_{z1} \cdot R_1 = 0,2 \cdot 2400 = 480 \text{ V}$$

$$I_{z1} = 200 \text{ mA}, U_{z2} = 300 \text{ V}, R_1 = 2,4 \text{ k}\Omega, \\ R_2 = 6 \text{ k}\Omega, R_3 = 1,6 \text{ k}\Omega, R_4 = 5 \text{ k}\Omega$$



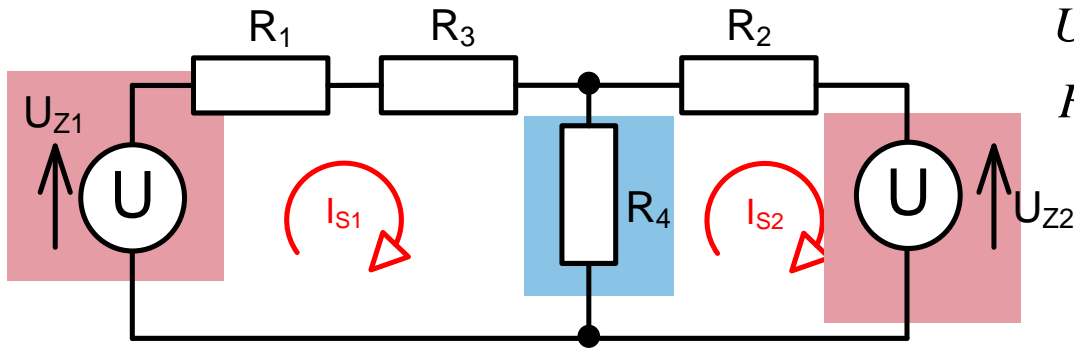
# SELECTION OF MESH CURRENTS

- We select independent meshes and mark **the mesh currents  $I_s$** 
  - We select the so-called "mesh nodes"
  - One direction of current





# FORMULATING AN MSP MATRIX EQUATION



$U_{z1} = 480 \text{ V}$ ,  $U_{z2} = 300 \text{ V}$ ,  $R_1 = 2,4 \text{ k}\Omega$ ,  
 $R_2 = 6 \text{ k}\Omega$ ,  $R_3 = 1,6 \text{ k}\Omega$ ,  $R_4 = 5 \text{ k}\Omega$

In general:

$R_1 + R_3 + R_4$	$-R_4$
$-R_4$	$R_2 + R_4$

 $\cdot$ 

$I_{S1}$
$I_{S2}$

 $=$ 

$-U_{z1}$
$U_{z2}$

Substitution:

9000	-5000
-5000	11000

 $\cdot$ 

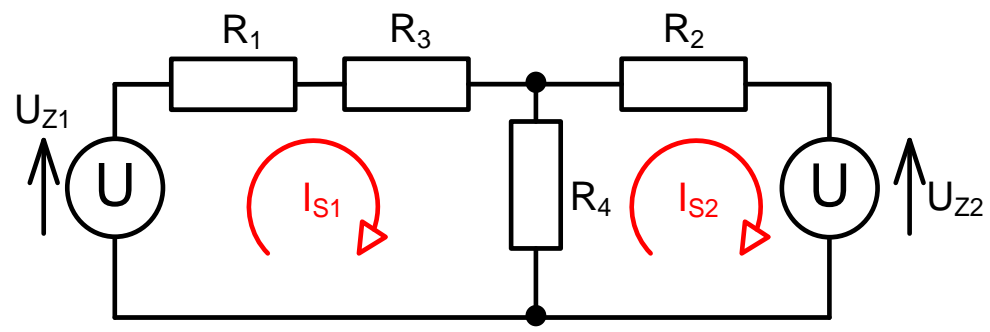
$I_{S1}$
$I_{S2}$

 $=$ 

-480
300



# SOLUTION



Result:

$$I_{S1} = \frac{\Delta_1}{\Delta} = \frac{-3,78 \cdot 10^6}{7,4 \cdot 10^7} = \underline{\underline{-51,08 \text{ mA}}}$$

$$I_{S2} = \frac{\Delta_2}{\Delta} = \frac{3 \cdot 10^5}{7,4 \cdot 10^7} = \underline{\underline{4,054 \text{ mA}}}$$

$$\begin{pmatrix} 9000 & -5000 \\ -5000 & 11000 \end{pmatrix} \cdot \begin{pmatrix} I_{S1} \\ I_{S2} \end{pmatrix} = \begin{pmatrix} -480 \\ 300 \end{pmatrix}$$

$$\Delta = \begin{vmatrix} 9000 & -5000 \\ -5000 & 11000 \end{vmatrix} = 7,4 \cdot 10^7$$

$$\Delta_1 = \begin{vmatrix} -480 & -5000 \\ 300 & 11000 \end{vmatrix} = -3,78 \cdot 10^6$$

$$\Delta_2 = \begin{vmatrix} 9000 & -480 \\ -5000 & 300 \end{vmatrix} = 3 \cdot 10^5$$



# BRANCH CURRENTS

loop currents:

$$I_{S1} = \underline{\underline{-51,08 \text{ mA}}}$$

$$I_{S2} = \underline{\underline{4,054 \text{ mA}}}$$

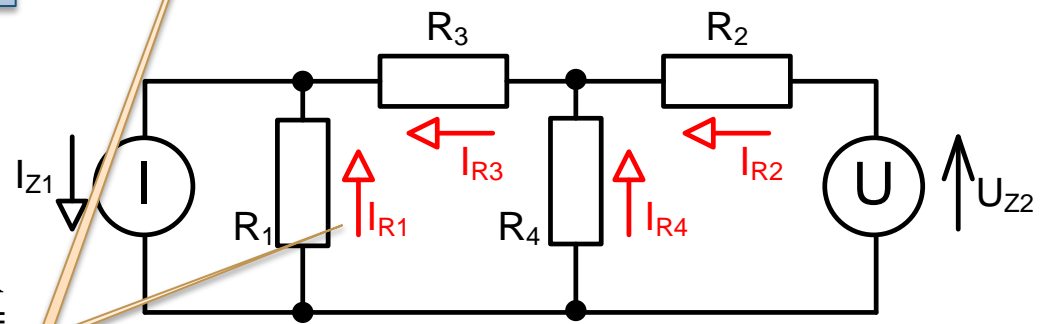
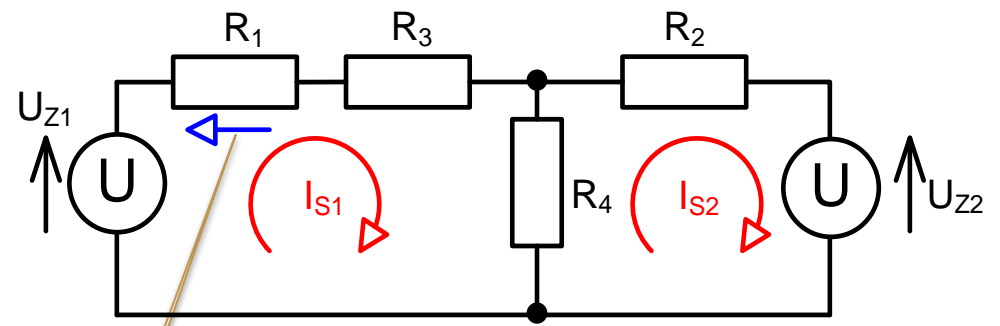
branch currents:

$$I_{R2} = -I_{S2} = \underline{\underline{-4,054 \text{ mA}}}$$

$$I_{R3} = -I_{S1} = \underline{\underline{51,08 \text{ mA}}}$$

$$I_{R4} = I_{S2} - I_{S1} = \underline{\underline{55,14 \text{ mA}}}$$

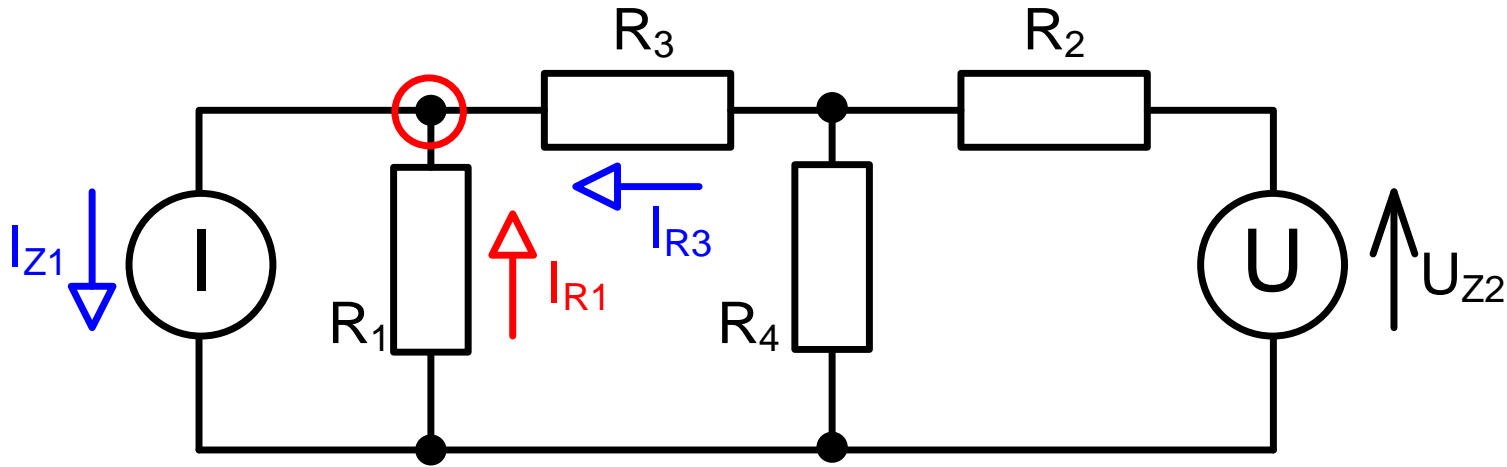
$$I_{R1} = ?$$



Note – the current through resistor **R1** in the equivalent circuit is not equal to the desired  $I_{R1}$ !



# A STREAM OF BRANCHES $R_1$ , USING THE 1ST K. ST.



$$I_{Z1} - I_{R3} - I_{R1} = 0$$

$$I_{R1} = I_{Z1} - I_{R3} = \underline{\underline{148,9 \text{ mA}}}$$

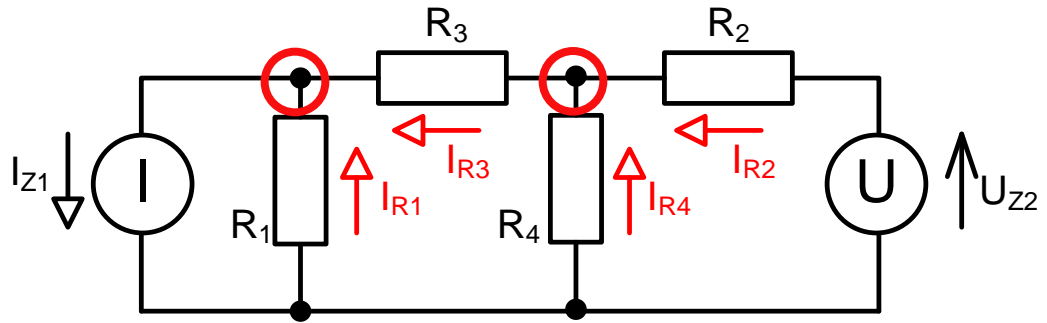
Currents:

$$I_{R1} = \underline{\underline{148,9 \text{ mA}}}$$

$$I_{R2} = \underline{\underline{-4,054 \text{ mA}}}$$

$$I_{R3} = \underline{\underline{51,08 \text{ mA}}}$$

$$I_{R4} = \underline{\underline{55,14 \text{ mA}}}$$



Check (e.g.):

$$I_{R1} + I_{R3} - I_{Z1} = 0$$

$$148,9 + 51,08 - 200 = 0$$

$$I_{R2} + I_{R4} - I_{R3} = 0$$

$$-4,054 + 55,14 - 51,08 = 0$$



# NVM

## NODE VOLTAGE METHOD



## NODE VOLTAGE METHOD (NVM)

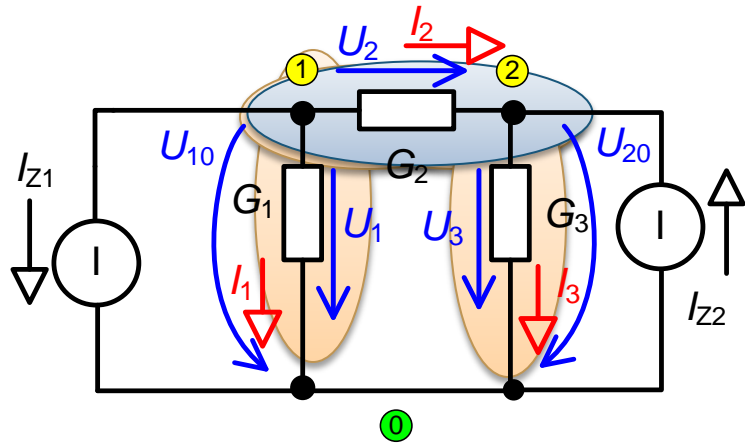
- Based on Kirchhoff's first law
- The circuit may contain only **current sources**; voltage sources must be replaced
- A matrix equation is solved:

$$\mathbf{G} \cdot \mathbf{U} = \mathbf{I}$$



# NODE STRESS METHOD (NSM)

- WITH DIRECT FORMULATION OF THE MATRIX EQUATION



$$\begin{matrix} & \begin{matrix} 1 & 2 \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \end{matrix} \cdot \begin{bmatrix} U_{10} \\ U_{20} \end{bmatrix} = \begin{bmatrix} -I_{z1} \\ I_{z2} \end{bmatrix}$$

### MUN – direct matrix construction:

- 1) We select a reference node (0) and number the independent nodes
- 2) We plot the nodal stresses
- 3) Write down the conductances
- 4) Write down the source currents
- 5) Solve the matrix equation
- 6) Calculate the remaining voltage
- 7) We determine the currents in the circuit

$$U_{10} = \frac{\Delta_1}{\Delta} \quad U_{20} = \frac{\Delta_2}{\Delta}$$

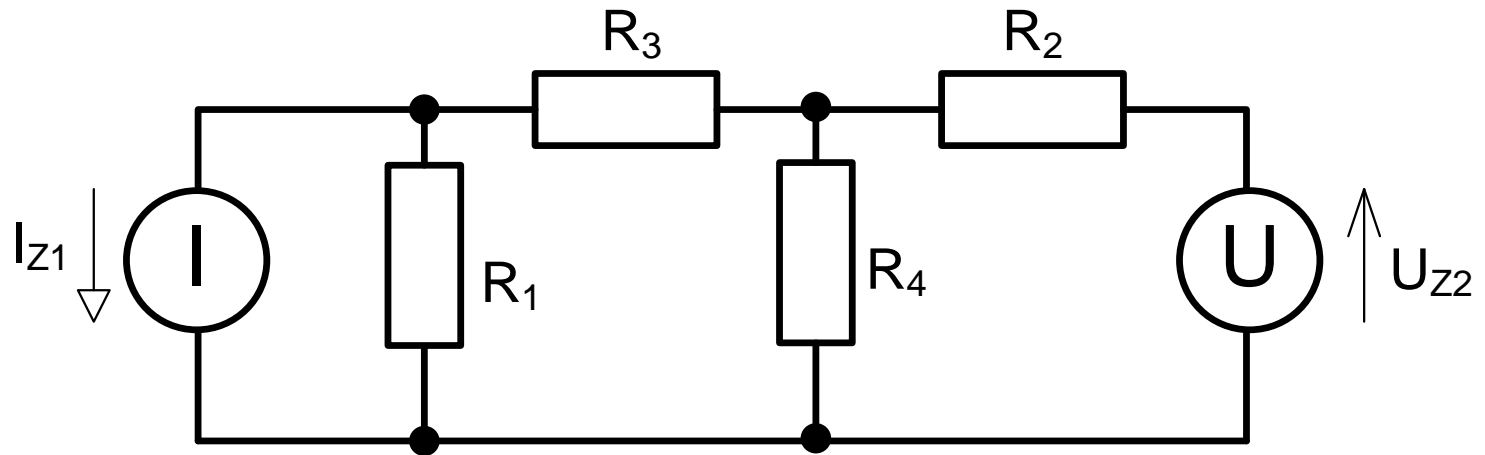
$$\begin{matrix} U_1 = U_{10} \\ U_2 = U_{10} - U_{20} \\ U_3 = U_{20} \end{matrix}$$

$$\begin{matrix} I_1 = G_1 U_1 \\ I_2 = G_2 U_2 \\ I_3 = G_3 U_3 \end{matrix}$$



# EXAMPLE

Calculate the currents through all the resistors in the circuit.

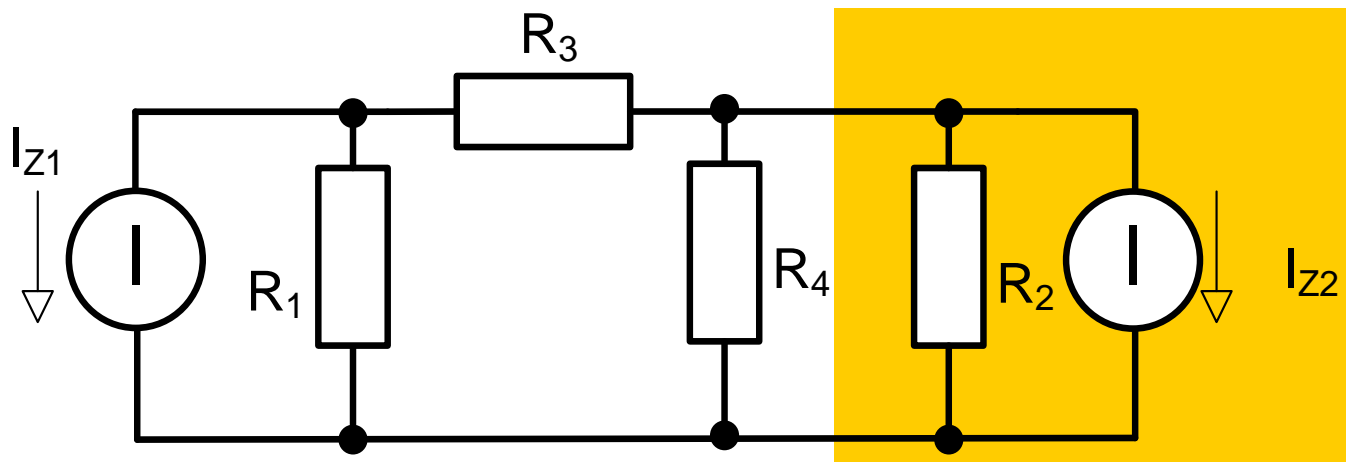
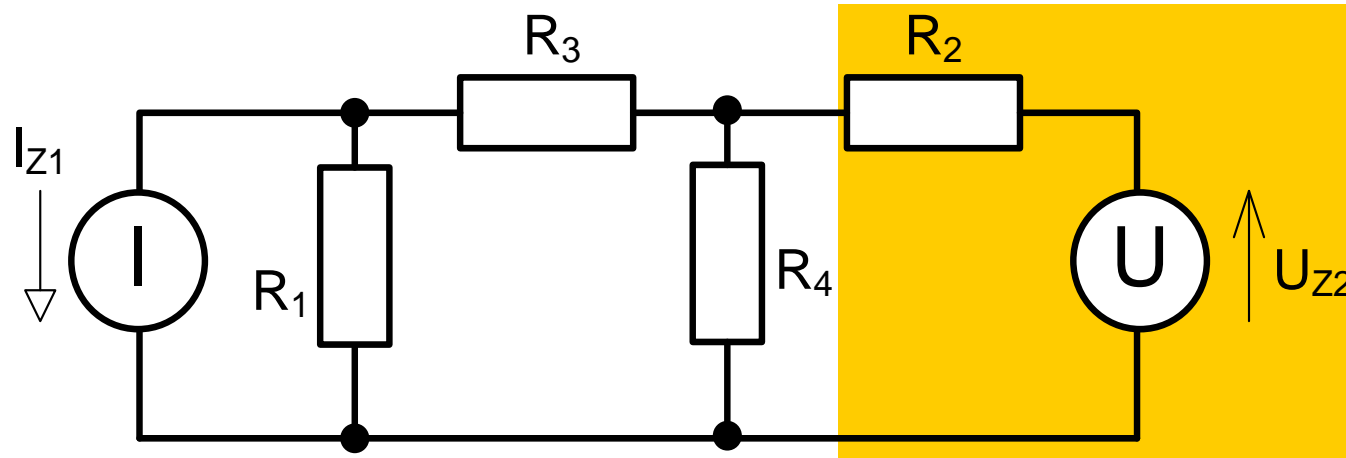


$$I_{z1} = 200 \text{ mA}, U_{z2} = 300 \text{ V}, R_1 = 2,4 \text{ k}\Omega,$$

$$R_2 = 6 \text{ k}\Omega, R_3 = 1,6 \text{ k}\Omega, R_4 = 5 \text{ k}\Omega$$



# REPLACEMENT OF THE POWER SUPPLY



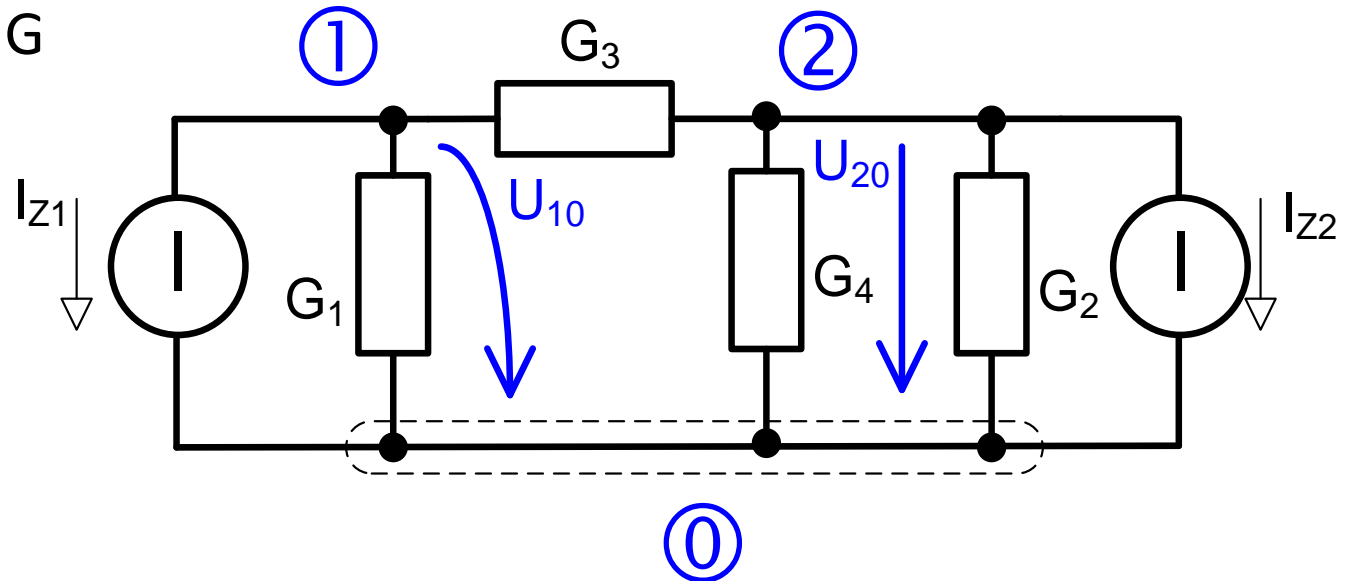
$I_{z1} = 200 \text{ mA}$ ,  $U_{z2} = 300 \text{ V}$ ,  $R_1 = 2,4 \text{ k}\Omega$ ,  
 $R_2 = 6 \text{ k}\Omega$ ,  $R_3 = 1,6 \text{ k}\Omega$ ,  $R_4 = 5 \text{ k}\Omega$

$$I_{z2} = U_{z2} / R_2 = 300 / 6000 = 50 \text{ mA}$$



# SELECTING NODES

- *Note: A node is a point where lines of equal potential meet*
- We select a reference node and label it **0**
- Number the other nodes **1, 2, ...**
- We mark the node voltages
- Convert R to G



$$I_{z1} = 200 \text{ mA}, I_{z2} = 50 \text{ mA}, G_1 = 1/2400 = 0,4167 \text{ mS},$$

$$G_2 = 1/6000 = 0,1667 \text{ mS}, G_3 = 1/1600 = 0,625 \text{ mS}, G_4 = 1/5000 = 0,2 \text{ mS}$$



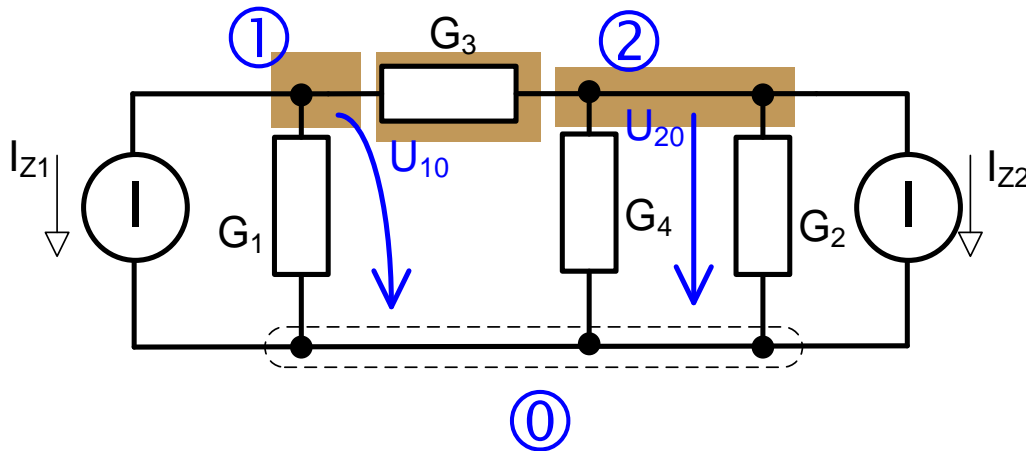
# FORMULATING THE MUN MATRIX EQUATION

In general:

$$\begin{pmatrix} G_1 + G_3 & -G_3 \\ -G_3 & G_2 + G_3 + G_4 \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \end{pmatrix} = \begin{pmatrix} -I_{Z1} \\ -I_{Z2} \end{pmatrix}$$

Substitution:

$$\begin{pmatrix} 1,0417 \cdot 10^{-3} & -0,625 \cdot 10^{-3} \\ -0,625 \cdot 10^{-3} & 0,9917 \cdot 10^{-3} \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \end{pmatrix} = \begin{pmatrix} -0,2 \\ -0,05 \end{pmatrix}$$

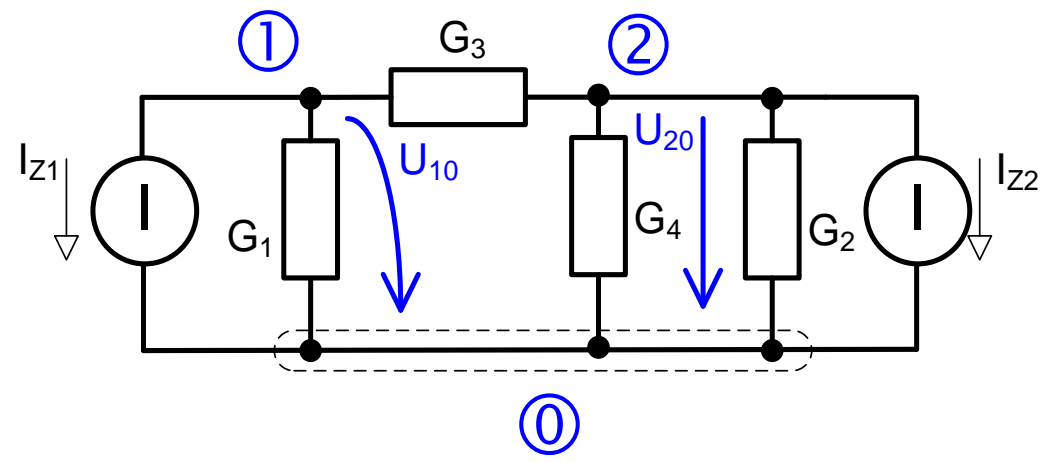


$$I_{Z1} = 200 \text{ mA}, I_{Z2} = 50 \text{ mA}, G_1 = 1/2400 = 0,4167 \text{ mS},$$

$$G_2 = 1/6000 = 0,1667 \text{ mS}, G_3 = 1/1600 = 0,625 \text{ mS}, G_4 = 1/5000 = 0,2 \text{ mS}$$



# SOLUTION



$$\begin{pmatrix} 1,0417 \cdot 10^{-3} & -0,625 \cdot 10^{-3} \\ -0,625 \cdot 10^{-3} & 0,9917 \cdot 10^{-3} \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \end{pmatrix} = \begin{pmatrix} -0,2 \\ -0,05 \end{pmatrix}$$

$$\Delta = \begin{vmatrix} 1,0417 \cdot 10^{-3} & -0,625 \cdot 10^{-3} \\ -0,625 \cdot 10^{-3} & 0,9917 \cdot 10^{-3} \end{vmatrix} = 6,424 \cdot 10^{-7}$$

$$\Delta_1 = \begin{vmatrix} -0,2 & -0,625 \cdot 10^{-3} \\ -0,05 & 0,9917 \cdot 10^{-3} \end{vmatrix} = -2,296 \cdot 10^{-4}$$

$$\Delta_2 = \begin{vmatrix} 1,0417 \cdot 10^{-3} & -0,2 \\ -0,625 \cdot 10^{-3} & -0,05 \end{vmatrix} = -1,771 \cdot 10^{-4}$$

Result:

$$U_{10} = \frac{\Delta_1}{\Delta} = \frac{-2,296 \cdot 10^{-4}}{6,424 \cdot 10^{-7}} = \underline{\underline{-357,4 \text{ V}}}$$

$$U_{20} = \frac{\Delta_2}{\Delta} = \frac{-1,771 \cdot 10^{-4}}{6,424 \cdot 10^{-7}} = \underline{\underline{-275,6 \text{ V}}}$$



# BRANCH CURRENTS

node stresses:

$$U_{10} = \underline{\underline{-357,4 \text{ V}}}$$

$$U_{20} = \underline{\underline{-275,6 \text{ V}}}$$

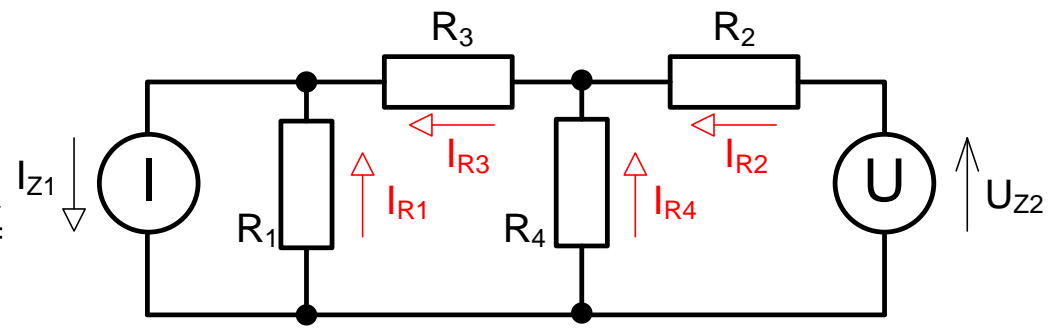
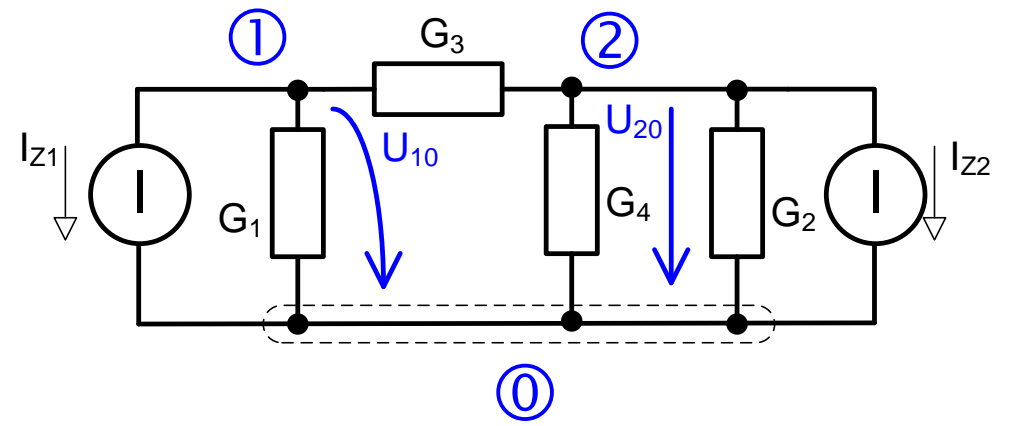
Branch currents:

$$I_{R1} = -U_{10} / R_1 = \underline{\underline{148,9 \text{ mA}}}$$

$$I_{R2} = ?$$

$$I_{R3} = (U_{20} - U_{10}) / R_3 = \underline{\underline{51,08 \text{ mA}}}$$

$$I_{R4} = -U_{20} / R_4 = \underline{\underline{55,14 \text{ mA}}}$$



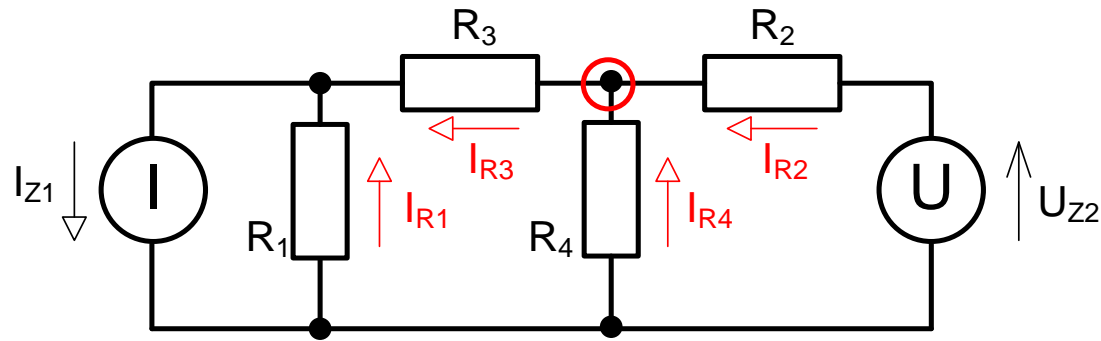


# A STREAM OF BRANCHES $R_2$ , USING THE 1ST K. ST.

$$I_{R1} = -U_{10} / R_1 = \underline{\underline{148,9 \text{ mA}}}$$

$$I_{R3} = (U_{20} - U_{10}) / R_3 = \underline{\underline{51,1 \text{ mA}}}$$

$$I_{R4} = -U_{20} / R_4 = \underline{\underline{55,14 \text{ mA}}}$$



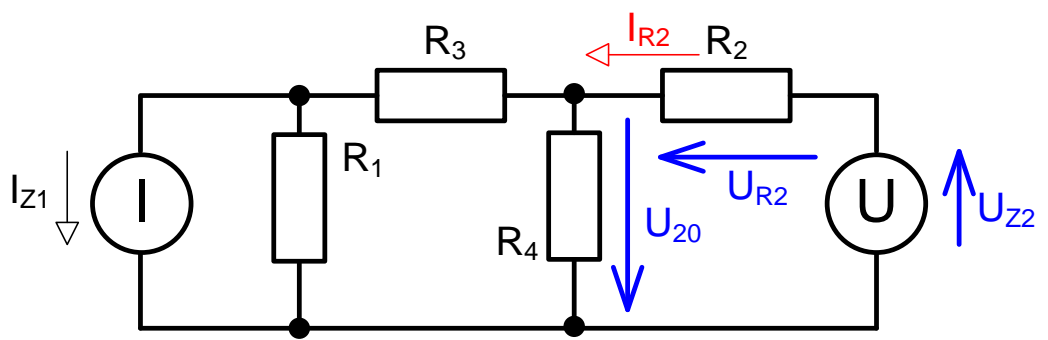
$$I_{R3} - I_{R4} - I_{R2} = 0$$

$$I_{R2} = I_{R3} - I_{R4} = \underline{\underline{-4,05 \text{ mA}}}$$



# BRANCH CURRENT $R_2$ , USING THE 2ND K. Z.

Original diagram



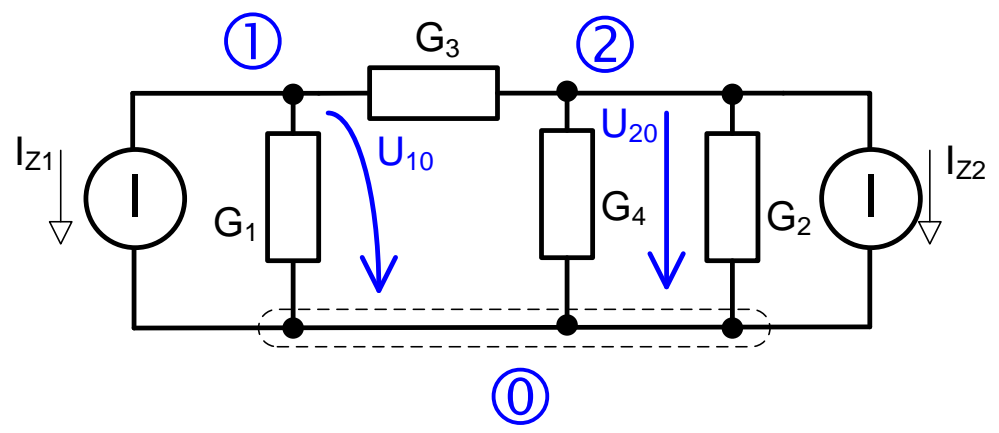
$$-U_{20} - U_{R2} - U_{Z2} = 0$$

$$U_{R2} = -U_{20} - U_{Z2} = 275,6 - 300$$

$$U_{R2} = \underline{\underline{-24,4 \text{ V}}}$$

$$I_{R2} = \frac{U_{R2}}{R_2} = \underline{\underline{-4,05 \text{ mA}}}$$

Alternative (calculated) diagram



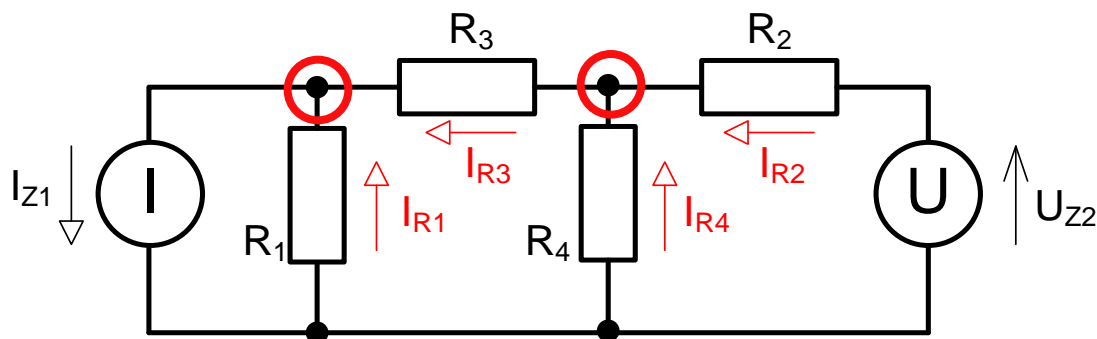
Currents:

$$I_{R1} = \underline{\underline{148,9 \text{ mA}}}$$

$$I_{R2} = \underline{\underline{-4,05 \text{ mA}}}$$

$$I_{R3} = \underline{\underline{51,08 \text{ mA}}}$$

$$I_{R4} = \underline{\underline{55,14 \text{ mA}}}$$



Check (e.g.):

$$I_{R1} + I_{R3} - I_{Z1} = 0$$

$$148,9 + 51,08 - 200 = 0$$

$$I_{R2} + I_{R4} - I_{R3} = 0$$

$$-4,054 + 55,14 - 51,08 = 0$$

# Quiz

Click the **Quiz** button to edit this object

## Quiz 5

### Universal Methods: MCM and NVM

- Read each question carefully
- To submit your answers, click the **Submit** button
- To pass, you must answer at least 75% of the questions correctly



# PART 6

# MAGNETIC CIRCUITS



# BASIC QUANTITIES AND RELATIONSHIPS OF THE MAGNETIC FIELD

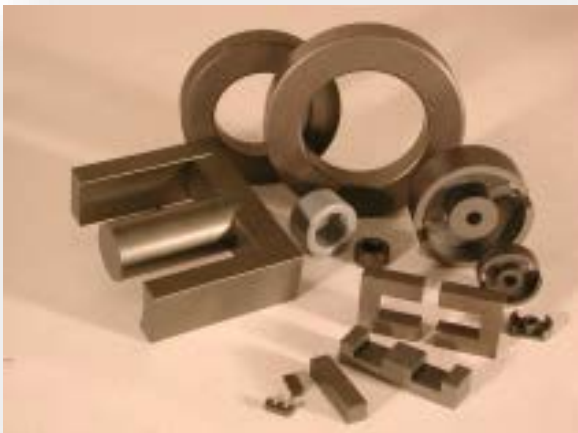
- Magnetic field intensity  $H$  (A/m)  
- The source of the field is a current

- Magnetic flux density  $B$  (T)  
- the field is influenced by the environment

$$\mathbf{B} = \mu \mathbf{H}$$

- Permeability (property of the medium)

$$\mu = \mu_0 \mu_r, \quad \mu_0 = 4\pi \cdot 10^{-7} \text{ (H/m)}$$



- Magnetic flux density  $U_m$  (A)

$$U_m = \int_{\ell} \mathbf{H} d\ell$$

$$U_m = H \cdot \ell$$

- Magnetic flux  $\Phi$  (Wb)

$$\Phi = \iint_S \mathbf{B} \cdot d\mathbf{S}$$

$$\Phi = B \cdot S$$

- Coil inductance  $L$  (H)  
( $N$  – number of turns)

$$L = \frac{\Psi}{I} = \frac{N\Phi}{I}$$



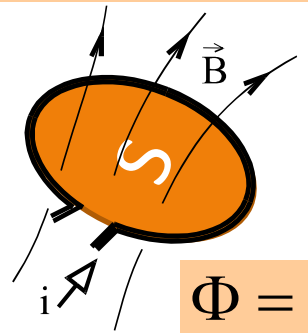
Simplification (we assume a homogeneous field, a constant cross-sectional area of the circuit, and field lines perpendicular to the cross-section)



# INDUCTANCE OF A COIL WITH A CORE

## Definition: Inductance

$$L = \frac{\Phi}{I} \quad (\text{H})$$



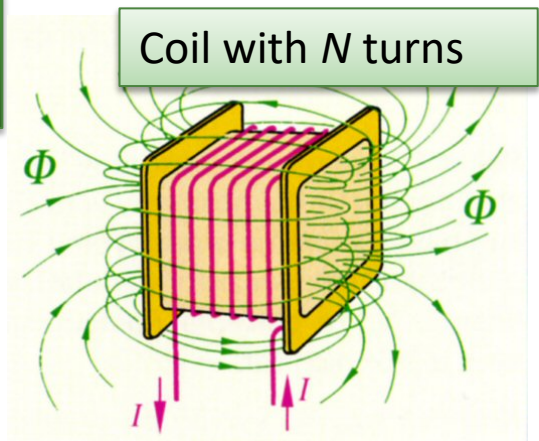
$$\Phi = B \cdot S$$

For a coil with N turns, the following relationship applies:

$$L = \frac{\Psi}{I} = \frac{N\Phi}{I} = \frac{NBS}{I} = \frac{N\mu HS}{I}$$

The intensity H is given by Ampère's law:

$$H\ell = NI$$

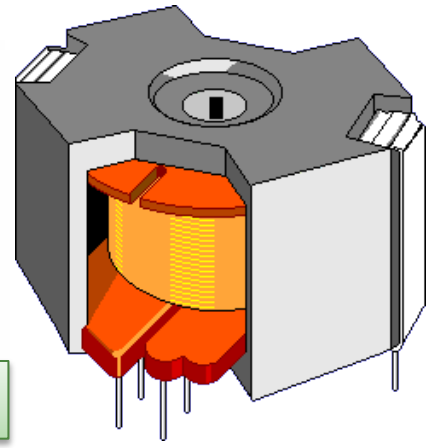


$$L = \frac{N\mu NIS}{I\ell} = \frac{N^2\mu S}{\ell} = \frac{N^2}{R_m} = A_L N^2 \quad R_m = \frac{\ell}{\mu S}$$

Inductance constant  $A_L$  (in nH/turn<sup>2</sup>)



Material – type of ferrite



- $\Phi$  inductive flux (Wb)
- $\Psi$  coupled magnetic flux (Wb)
- $N$  number of coil turns (-)
- $L$  inductance (H)
- $H$  magnetic field intensity (A/m)
- $R_m$  magnetic resistance (1/H)

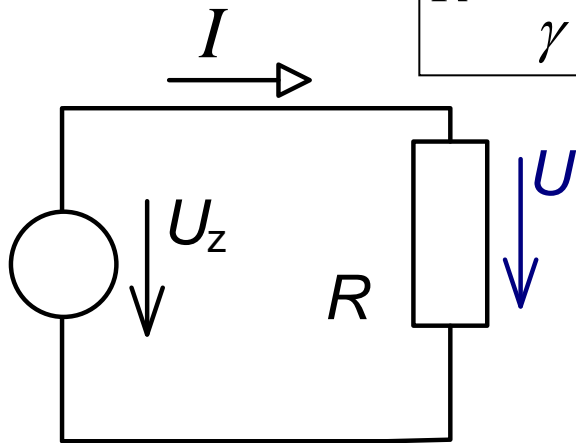


# MAGNETIC CIRCUIT – FORMAL ANALOGY

Electric circuit		Analogy	Magnetic circuit		
electric current	$I$ (A)			Magnetic flux	$\Phi$ (Wb)
Electric voltage	$U$ (V)			magnetic voltage	$U_m$ (A)
electrical resistance	$R$ ( $\Omega$ )	magnetic resistance (reluctance)		$R_m$ (1/H)	

## Electrical circuit

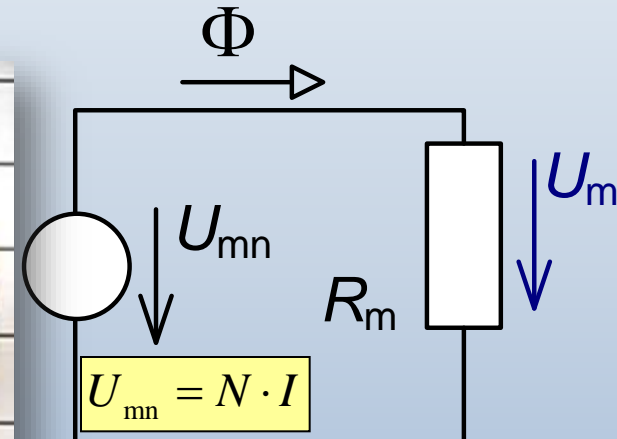
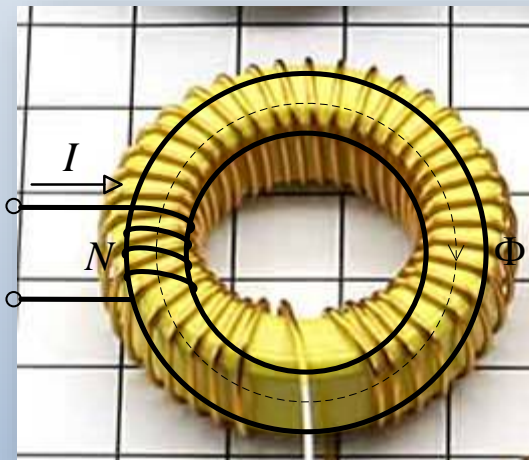
$$R = \frac{1}{\gamma} \frac{\ell}{S}$$



$$U = I \cdot R$$

## Magnetic circuit

$$R_m = \frac{1}{\mu} \frac{\ell}{S}$$

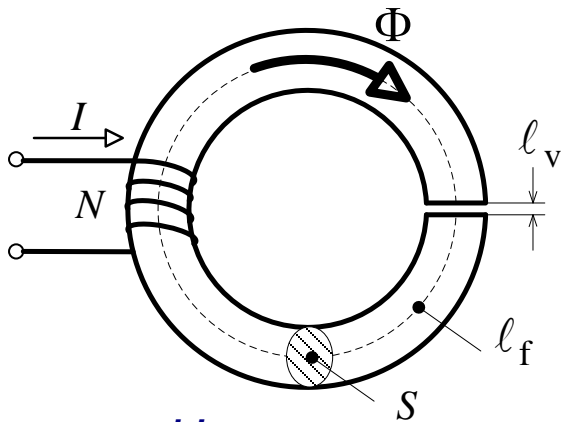


$$U_{mn} = N \cdot I$$

$$U_m = \Phi \cdot R_m$$



# MAGNETIC CIRCUITS – SOLUTION PROCEDURE



From Ampère's law:

$$\int_l \mathbf{H} \cdot d\mathbf{l} = U_m$$

The line of integration is identical with the mean field line

$$U_{mf} = H_f l_f = \frac{B_f}{\mu_0 \mu_r} l_f$$

$$U_{mv} = H_v l_v = \frac{B_v}{\mu_0} l_v$$

Option 1

$$NI = U_{mn} = U_{mf} + U_{mv}$$

2nd alternative

From Hopkinson's law:

$$U_{mf} = \Phi \cdot R_{mf}$$

$$U_{mv} = \Phi \cdot R_{mv}$$

$$R_{mf} = \frac{l_f}{\mu_0 \mu_r S_f}$$

$$\Phi = B_f \cdot S_f$$

$$R_{mv} = \frac{l_v}{\mu_0 S_v}$$

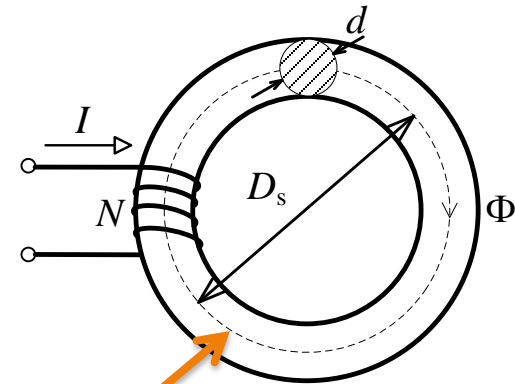
$$\Phi = B_v \cdot S_v$$



# SIMPLE MAGNETIC CIRCUIT

A coil wound on a cast steel toroidal core has  $N = 200$  turns, and a current  $I = 1$  A flows through it. Determine the magnetic flux through the core and the inductance  $L$  of the coil.

The mean diameter of the core  $D_s = 120$  mm, cross-sectional area  $S = 4$  cm<sup>2</sup>.



$$l_f = \pi D_s = 0,377 \text{ m}$$

The coil is a source of magnetic voltage:  $U_{mn} = NI = 200$  A

which is distributed along the field line in the magnetic circuit.

Since the cross-sectional area of the circuit is constant along the entire length of the field line, the intensity is:

$$H_f = \frac{U_{mn}}{l_f} = \frac{200}{0,377} = 530 \text{ A/m}$$

From the material's magnetization curve, we determine the induction value for  $H_f = 530$  A/m:

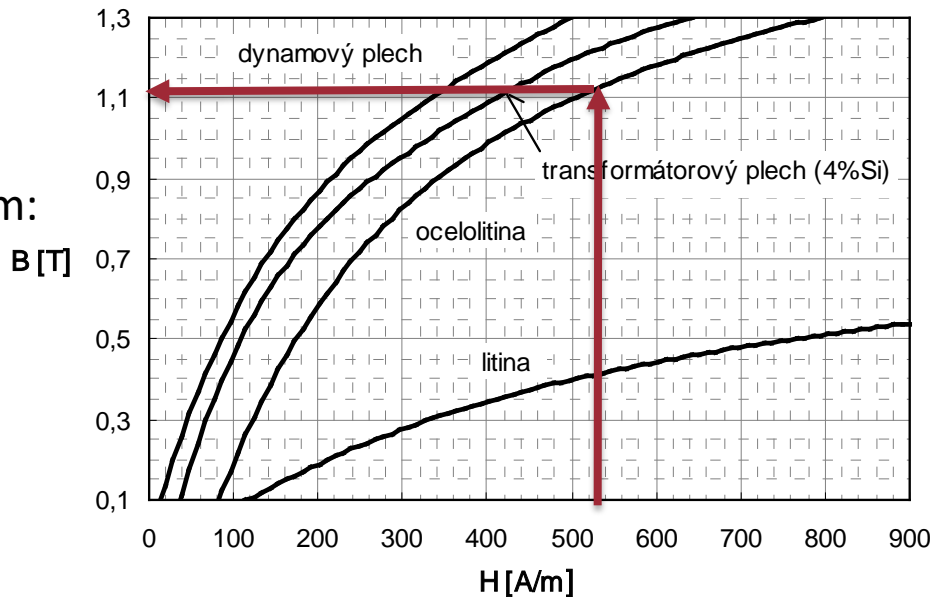
$$B_f \doteq 1,12 \text{ T}$$

The magnetic flux through the circuit is:

$$\Phi = B_f \cdot S = 1,12 \cdot 4 \cdot 10^{-4} = \underline{\underline{448 \cdot 10^{-6} \text{ Wb}}}$$

The inductance of this coil is:

$$L = \frac{N\Phi}{I} = \frac{200 \cdot 448 \cdot 10^{-6}}{1} = \underline{\underline{89,6 \text{ mH}}}$$

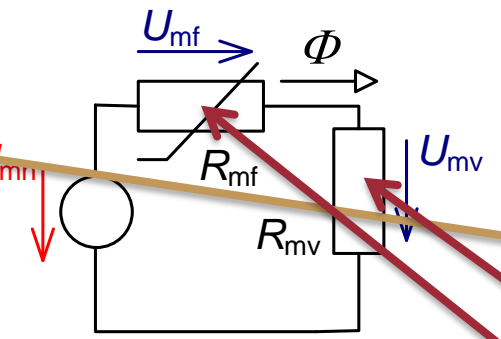
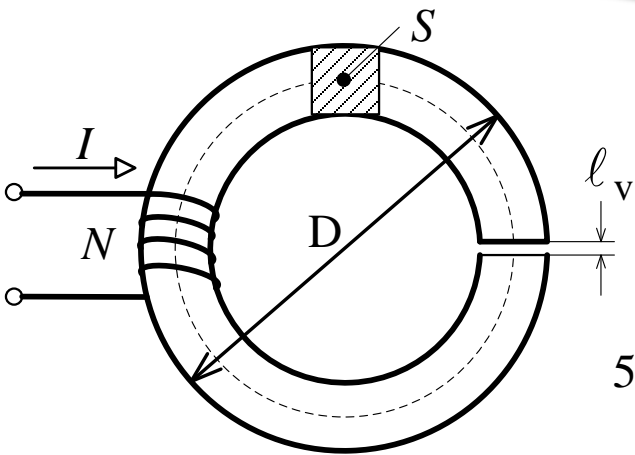




# MAGNETIC CIRCUIT

A ferromagnetic ring has a diameter  $D = 90$  mm, and the cross-sectional area of the core is  $S = 10 \times 10$  mm. In the air gap  $\ell_v = 1$  mm, we require an induction  $B_v = 0.5$  T. Calculate the required number of turns in the excitation coil for a current  $I = 5$  A and the inductance  $L$  of the coil for this current. Neglect stray fluxes.

$B_f$ (T)	0.3	0.5	0.7	0.9
$H_f$ (A/m)	66	109	167	262



$$U_{mn} = NI = U_{mf} + U_{mv}$$

$$\Phi = B_f \cdot S = B_v \cdot S \Rightarrow B_f = B_v$$

$$H_f (B_f = 0,5 \text{ T}) = 109 \text{ A/m}$$

$$NI = H_f \ell_f + H_v \ell_v = H_f \ell_f + \frac{B_v}{\mu_0} \ell_v$$

$$5 \cdot N = 109 \cdot 0,2503 + \frac{0,5}{4\pi \cdot 10^{-7}} \cdot 1 \cdot 10^{-3} = 27,28 + 397,9 = 425,2 \text{ A}$$

$$N = 85,03 \doteq \underline{\underline{85}}$$

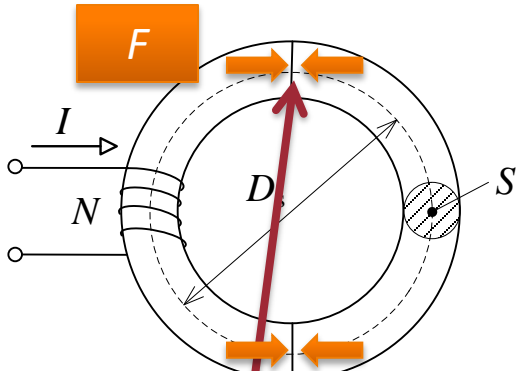
$$L = \frac{N \cdot \Phi}{I} = \frac{N \cdot B_v \cdot S}{I} = \frac{85 \cdot 0,5 \cdot (10 \cdot 10^{-3})^2}{5} = \underline{\underline{850 \mu\text{H}}}$$

$$D_s = D - 10 = 90 - 10 = 80 \text{ mm}$$

$$\ell_f = \pi D_s - \ell_v = 0,2503 \text{ m}$$



# CALCULATION OF ELECTROMAGNET FORCE



The toroidal core of a coil made of E11 electrical steel consists of two parts. **What is the force  $F$  holding the two parts together** if the cross-sectional area of the ring is  $S = 4 \text{ cm}^2$ , its mean diameter  $D_s = 0.177 \text{ m}$ , and a current  $I = 0.8 \text{ A}$  flows through the coil with  $N = 210$  turns?

**Solution:**

$$U_{\text{mn}} = NI = H_f \ell_f \Rightarrow H_f = \frac{NI}{\ell_f}$$

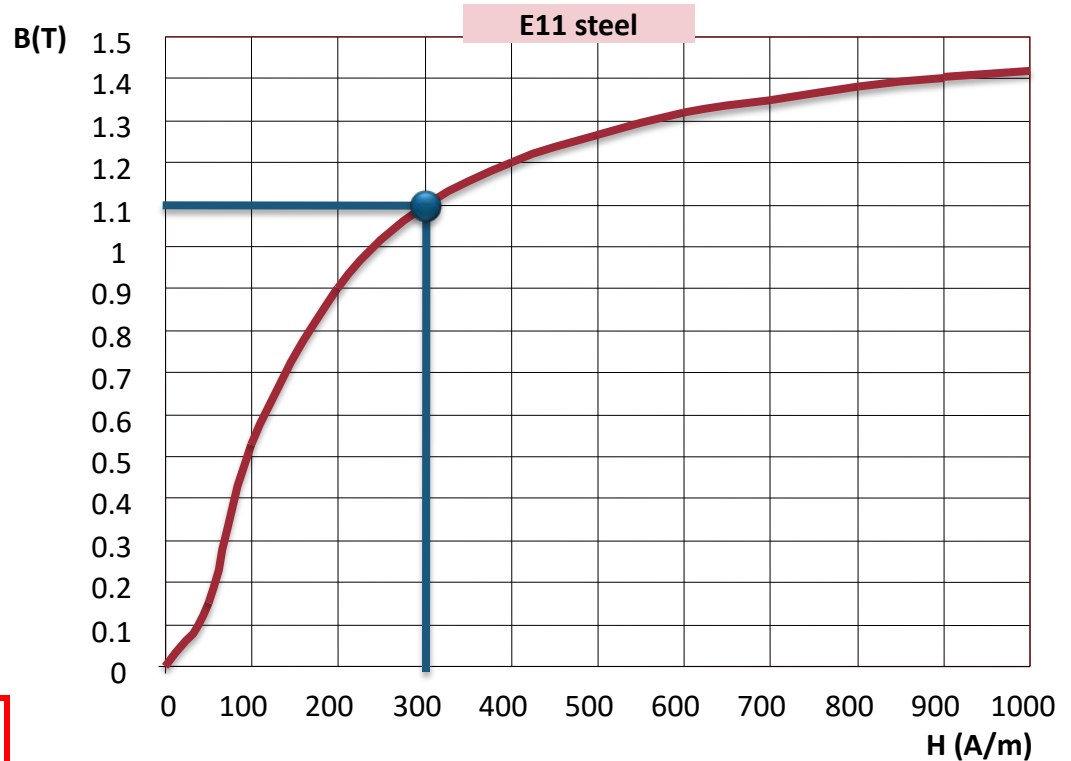
$$\ell_f = \pi D_s = \pi \cdot 0,177 \doteq 0,565 \text{ m}$$

$$H_f = \frac{210 \cdot 0,8}{0,556} = 302 \text{ A/m}$$

z grafu pro  $H_f = 302 \text{ A/m}$

odečteme  $B_f \doteq 1,1 \text{ T}$

$$F = 2 \frac{B^2 S}{2 \mu_0} = 2 \frac{1,1^2 \cdot 4 \cdot 10^{-4}}{2 \cdot 4\pi \cdot 10^{-7}} \doteq 385 \text{ N}$$



# Quiz

Click the **Quiz** button to edit this object

## Quiz 6 Magnetic Circuits

- Read each question carefully
- To submit your answers, click the **Submit** button
- To pass, you must answer at least 75% of the questions correctly



DONE!

- Congratulations, you have successfully completed the entire "Introduction to Electrical Engineering" course material
- **Based on this, you may be credited with the BPA-ELS course**

**Any suggestions or comments:**

[Miloslav Steinbauer](#)

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