REVIEW

by the official opponent on dissertation work by
Ganna PIDDUBNA "LINEAR MATRIX DIFFERENTIAL EQUATION WITH DELAY" for the degree of Ph.D.:
Brno: Vysoké učení technické v Brně.
Fakulta elektrotechniky a komunikačních technologii. Ústav matematiky, 2014

Dissertation work by Ganna Piddubna "Linear Matrix Differential Equations with Delay" is devoted to important problems of applied mathematic, specifically to the research of the systems of differential equations with aftereffect delayed type, specifically of getting solutions view of the linear stationar systems with one delay, research of their stability and controllability.

Systems of this type are actively used in wide spread of the study fields of processes in ecology, population dynamics and control theory. In the population dynamics the delay makes sense as time of puberty, in control theory it is a time of a control signal passing, in the economy it is a time of making decision and its implementation. Thus, the dissertation is devoted to **current** topics and has plenty of great practical value.

The dissertation consists of the six chapters, the conclusion and of the list of used literature (References).

In the chapter one the historical development and current state of the researches on the theory of dynamical systems with aftereffect is analyzed. The problems of stability of dynamical systems, especially the problems of stability of systems with delay, systems with aftereffect of neutral type are considered. In separate paragraph the problems of optimal control are analyzed. At the end of the chapter illustrative examples with dynamics described by the equations with delay are given.

The second chapter is in a review style. In the first section the basic definitions and principles of the theory of motion control are presented. Definitions of controllability of linear systems are given. The linear stationary systems are considered. Auxiliary facts on the commutative of matrices are given. In the second section the basic definitions of the theory of stability of motion are presented, the first and second Lyapunov's theorem on stability and asymptotic stability are formulated.

The third chapter deals with the representation problems of linear stationary systems with a constant delay solutions. In Theorem 3.1.1 the general solution of the homogeneous system with a constant delay and arbitrary matrix with the coordinate delay are recorded in the form of an iterative procedure. On the basis of this result a view of the fundamental matrix of solutions of the equations is presented. In Example 3.1.1 an illustration of the theoretical results for the three-dimensional systems with the same matrix is given. Next in Theorem 3.1.6 autho

provides representation for the solution of the Cauchy problem for the non-homogeneous system (analogue of the Cauchy problem for systems of ordinary differential equations). An illustrative Example 3.1.2 is given.

Special, but quite interesting case is the case of commutative matrices with phase coordinate without delay and with delay. It is considered in section 3.2. Results solutions view are recorded in Theorem 3.2.2.

The fourth chapter deals with the problems of asymptotic stability of linear stable systems with constant matrices. Upon receipt of the stability conditions the method of Lyapunov quadratic functions with the B.S. Razumihin condition is used.

The fifth chapter deals with the problems of systems with delay controllability. The conditions of control of the system with pure delay, with commutative matrices and matrices of generaly form are presented. The theoretical results are supported by corresponding examples.

The sixth chapter examines questions of the structural control constrution. Systems with pure delay, with commutative matrices, systems of general form are considered. The corresponding examples are given.

In my opinion, the most interesting and important scientific results obtained in the dissertation work are following:

1. The view of the fundamental matrix of solutions of linear homogeneous differential equations with arbitrary matrices is considered (Theorem 3.3.2, Example 3.3.1).

2. The conditions for controllability of systems with arbitrary matrices and an algorithm for constructing the control function are obtained.

All results obtained by the author proposed to the dissertation defence are new. Especially, in my opinion, theorems about control of the systems with commutative matrices should be noted.

Plausibility of the results is based on a rigorous mathematical proof of consistency and previously known facts.

Results obtained in the Ganna Piddubna dissertation work have both theoretical and practical importance. They represent a further development of the theory of control systems with aftereffect.

The main results are published in the correspondent articles and were reported at conferences [97-119]. The thesis passed good approbation. Author's abstract of thesis is fully and adequately reflects the content dissertation work.

It should be noted that formalization part is good in this dissertation work. Presentation of the results carried out briefly and clearly. All results are proved rigorously and convincingly. Pre-known results and provided links to them are presented. References are fairly complete and well display the status of the problem.

Based on the dissertation work analysis, I believe that Ganna Piddubna scientific qualifications corresponds of doctor Ph.D. in Physical and Mathematical Sciences.

Comments on the dissertation work are following:

1. Theorem 3.1.2 is proved for arbitrary matrices, the right-hand side of the differential equation. At the same time, an illustrative Example 3.1.1 considered coinciding matrix. A similar remark concerns the commutative matrices (Examples 3.2.1, 3.2.2).
2. When recording a fundamental matrix of solutions in Theorem 3.3.2 the designation $f_n(t)$ is used, which is inherent in the linear inhomogeneous system. Another notation should be used there.
3. Results presented in chapter 4, are not develop to the problems of controllability and optimization. They are separate and they can be omitted.
4. Conditions on the conditions of Theorem 5.1.1 with the same coinciding matrices $A_1 = A_2 = A$, Theorem 5.2.1 systems with commutative matrices, Theorem 5.3.2 with arbitrary matrices should be compared with the known (R. Gabasov, F.M. Kirilova).

Based on the above I think that:

- Dissertation work "Linear Matrix Differential Equations with Delay" is a work prepared independently by candidate for a degree;
- candidate for a degree obtained significant results in the field of control theory of systems with aftereffect.

Believe that the dissertation work "Linear Matrix Differential Equations with Delay" in terms of research, relevance, scientific level, quantity and quality of publications meets all the requirements, and its author Ganna Piddubna deserves conferment degree of doctor Ph.D. Physical and Mathematical Sciences.

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