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NEXT GENERATION VUT: Zvyšování kvality a relevance vzdělávání na VUT CZ.02.02.XX/00/23_022/0009052

Electricity

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MINISTRY OF EDUCATION, YOUTH AND SPORTS

Contents

Electricity

The name electricity originated in ancient times from the word amber (ήλεκτρον in the Greek translation electron), because it was first discovered on it.

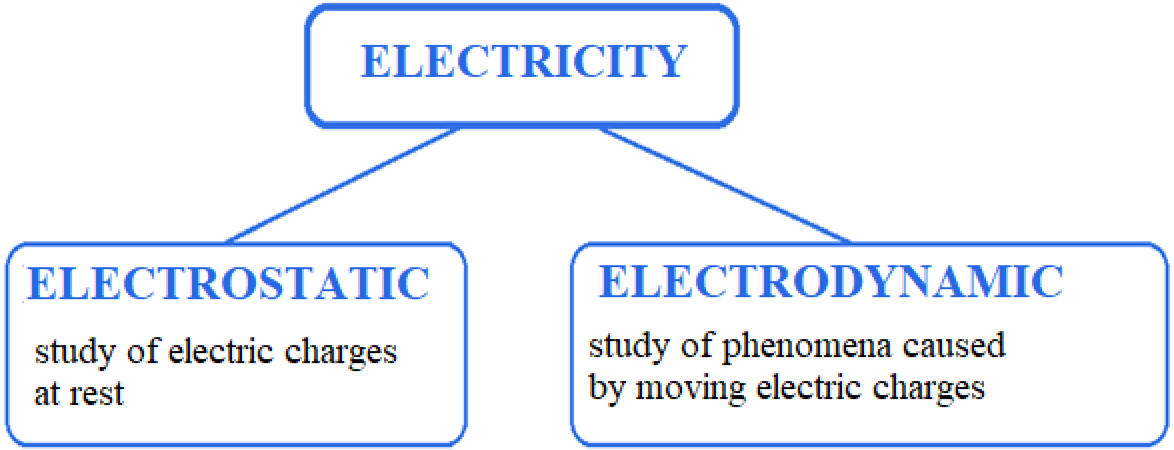
In Greece at that time, amber spools were used for spinning, and it happened that when the yarn was wound, its fibers bristled.

The Greeks also observed that some natural "stones", such as the mineral magnetite, attracted iron.

The first scientific work on electricity is considered to be the work of the English physician William Gilbert "De magnete" from 1600, where the name electricity is also introduced.

Electricity

Electricity is also the name of a branch of physics that focuses on the study of electrical phenomena.



Electricity and magnetism as a science originated at the turn of the 18th and 19th centuries. Luigi Galvani, Alessandro Volta, André Marie Ampére, Charles Auguste de Coulomb, Michael Faraday, James Clerk Maxwell and a number of other important physicists of that time contributed to their development.

Electric charge

The basic electrical property of bodies is electric charge.

Electric charge is a scalar physical quantity that we call Q, and its unit is coulomb $[Q] = \mathrm{C}$.

The smallest electric charge is the elementary charge

$$\boldsymbol{e = 1{,}602 \cdot 10^{-19}\ \mathrm{C}}.$$

Experimentally, it has been found that there are 2 types of charge: positive and negative.

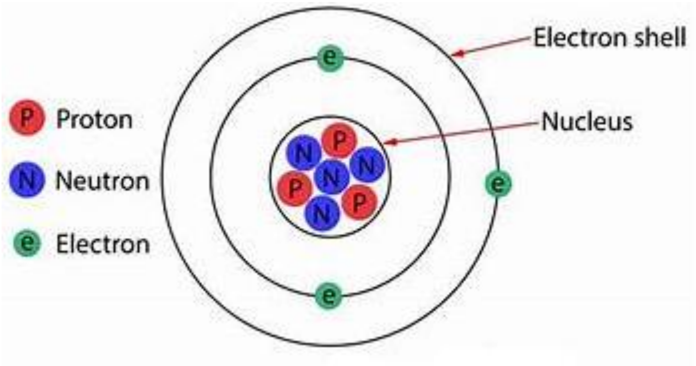
Electric charge cannot exist independently and always has its carrier. A body with an electric charge is called electrically charged.

Electric charge

The electrical properties of substances are related to the structure of atoms, which are composed of atomic nuclei and electron shells.

Nuclei are made up of neutrons and protons. Neutrons are devoid of electric charge. Protons and electrons are carriers of electric charge.

An electron is a particle that carries a negative elementary charge $-e$. A proton is a carrier of positive elementary charge $+e$.



Internal structure of the atom

Electric charge

Under normal conditions, the number of protons with a positive charge in a substance is equal to the number of electrons with a negative charge. This ensures electroneutrality.

External action can disrupt this equilibrium and thus create an electrically charged body that is

- negatively charged if electrons predominate,
- positively charged if protons predominate.

Laws for electric charge

The basic properties of electric charges are described:

- **Law of conservation of electric charge** – the value of the total electric charge in an electrically isolated system is equal to the algebraic sum of all charges in the system and is invariable.

  Electric charges do not arise or disappear, they are only moved from one body to another (i.e. charges are not created by friction, they are only displaced).

  The law can be proved by an experiment with electrical induction (see below).

Laws for electric charge

- **Law of quantization of electric charge –** all charges are multiples of elementary charge.

  Any charge $Q = n \cdot e$, where n is a natural number,

  e is the elementary charge.

  In 1911, Robert Andrews Millikan made a direct attempt to prove this law.

- **Law of invariance of electric charge –** The value of electric charge does not change during motion.

  The magnitude of the charge is independent (invariant) of the speed at which the charge carrier moves.

Electric charges in materials

According to the ability of bodies to keep the accumulated electric charge in one place or to allow this charge to spread throughout the body, materials can be divided into conductors and non-conductors.

Conductors are materials containing free electric charge. Conductors conduct electricity.

A free electric charge can be transferred from one body to another and can also be moved in one body at macroscopic distances.

The free charge is carried by electrons in metals and semiconductors, ions in gases and liquids.

Nonconductors (insulators, dielectrics) are materials that do not contain free electric charge and do not conduct electricity.

Properties of materials with respect to electric charge:

- **Conductors** – some of the charges move quite freely. For example: metal, drinking water, a living organism.
- **Nonconductors** – practically no charge moves freely. For example: glass, ebonite, distilled water, dielectrics.
- **Semiconductors –** between conductors and insulators – differ in their ability to release electrons from atoms.
  For example: silicon, germanium.
- **Superconductors –** zero resistance.

Electric field

An electric field is a force field in which an electric force is applied to bodies or particles with an electric charge.

An electric field exists around every electrically charged body or particle with an electric charge.

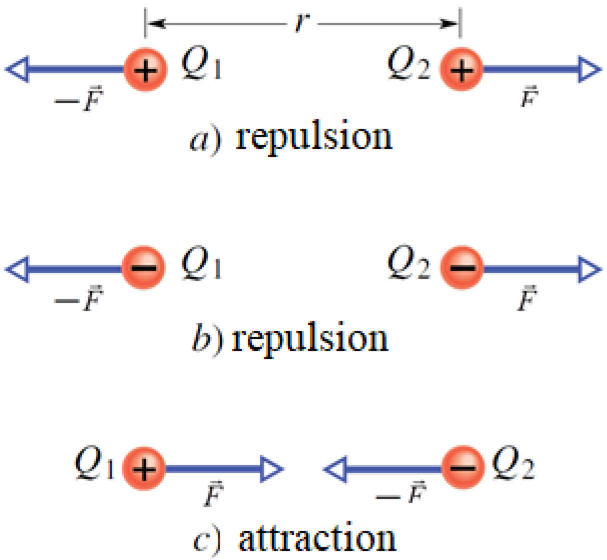
A field is called **electrostatic** if the particles that produce it are at rest with respect to the observation system and **electrodynamic** if the particles move relative to the observation system.

The electrodynamic field in general has two components: electric and magnetic.

Electric field

Electric charges act on each other with an electric force $\vec{F}$, while

- **similar charges,** or similarly charged bodies are **repelled**,
- **opposite charges,** or oppositely charged bodies, **attract** each other.



Point charges are electric charges of bodies or particles whose dimensions are considerably smaller than the distances between them, i.e. their dimensions can be neglected.

Coulomb's law

The electrostatic force action between two point charges can be described using **Coulomb's law**:

The magnitude of the force exerted by two point charges on each other is directly proportional to the product of both charges and inversely proportional to the square of their distance.

$$F = k\frac{|Q_1||Q_2|}{r^2}, \text{ where } k = \frac{1}{4\pi\varepsilon} \text{ N.m}^2\text{.C}^{-2}.$$

Q_1 and Q_2 are the magnitudes of the point charges, r is the distance, ε is the **permittivity of the environment**, which can be expressed by the relation

$$\varepsilon = \varepsilon_0 \cdot \varepsilon_r,$$

where $\varepsilon_0 = 8{,}854 \cdot 10^{-12}$ F.m^{-1} is the **permittivity of the vacuum (permittivity constant)**, ε_r is the **relative permittivity**.

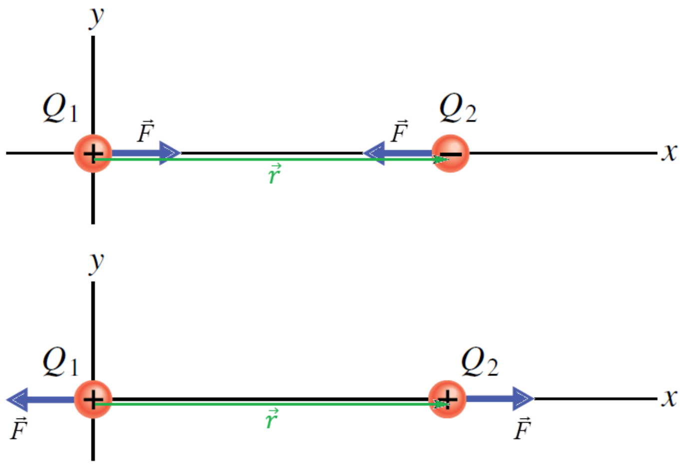
Coulomb's law

The following table shows the relative permittivity values for different environments.

Vacuum	1	Silicone oil	2,7
Air	1,00054	Plexiglas	3,7
Mica	5,4	Paraffin	1,7-2,3
Water (20 °C)	80,4	Condenser paper	3,5

Vector expression of Coulomb's law

Vectors $\vec{F}$ and $\vec{r}$ lie on the same line and the beginning of the system is in one of the charges.



The electrostatic force exerted by point charges on each other has the form:

$$\vec{F} = \frac{1}{4\pi\varepsilon}\frac{Q_1 \cdot Q_2}{r^2}\vec{r}^0 = \frac{1}{4\pi\varepsilon}\frac{Q_1 \cdot Q_2}{r^2}\frac{\vec{r}}{r},$$

i.e. $\vec{F} = \frac{1}{4\pi\varepsilon}\frac{Q_1 \cdot Q_2}{r^3}\vec{r}$.

Here $\vec{r}^0 = \frac{\vec{r}}{r}$ is the unit vector of the vector $\vec{r}$,

r is the magnitude of the vector $\vec{r}$, i.e. the distance between the charges.

The principle of superposition

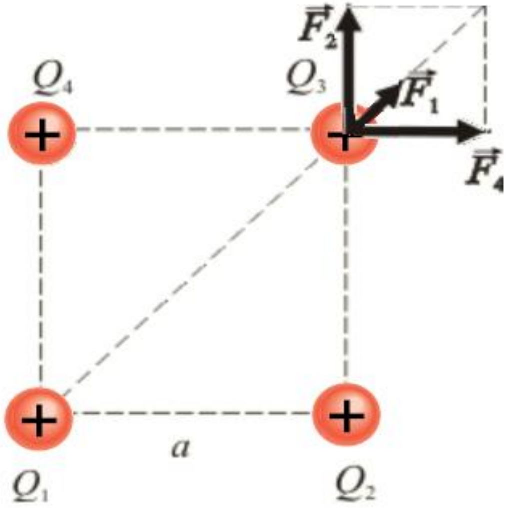
The principle of superposition applies to *n* point charges – the force effects of all charges are **vector-added**.

If there are *n* charged particles in the system, the force acting on any of them is the sum of all the forces acting from the other particles.

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n = \sum_{i=1}^{n} \vec{F}_i$$

For example:

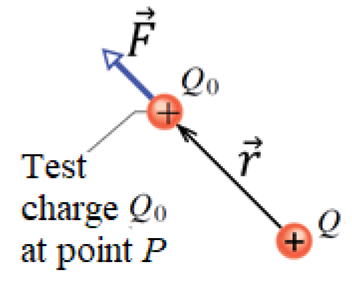
The four charges are located at the vertices of the square. The resultant force acting on the charge Q_3 is shown in Figure.



Electric field

The interaction of charges at a distance is realized through an electric field.

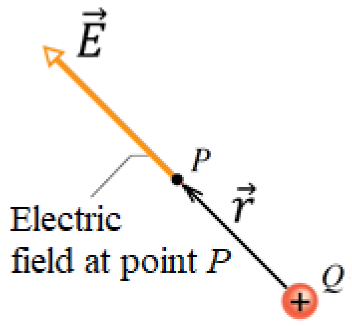
The **electric field** is characterized by a vector quantity, which is defined by the relation



a)

$$\vec{E} = \frac{\vec{F}}{Q_0},$$

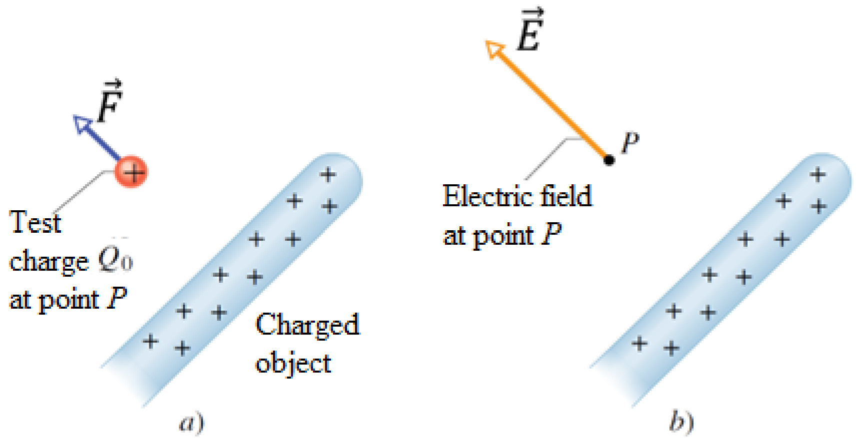
where $\vec{F}$ is the force exerted by the electric field at a given location on a positive point test charge Q_0.



b)

Electric field unit is $[E]$ = N.C^{-1} = V.m^{-1}.

Electric field



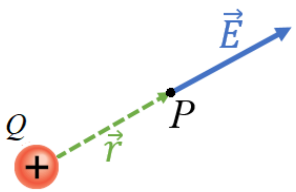
a) Positive test charge Q_0 placed at point P near the charged body. Electrostatic force $\vec{F}$ acts on the test charge.

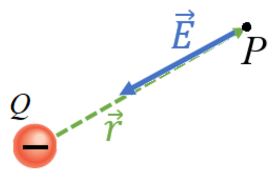
b) Electric field $\vec{E}$ at point P, which is excited by a charged body.

Electric field due to a point charge

The magnitude of the electric field of the point charge at point P is

$$E = \frac{F}{Q_0} = \frac{k\frac{|Q|\cdot Q_0}{r^2}}{Q_0} = k\frac{|Q|}{r^2} \text{ or } E = \frac{1}{4\pi\varepsilon}\frac{|Q|}{r^2}.$$



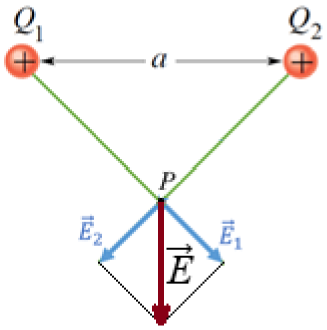


Vector form of the electric field

$$\vec{E} = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}\vec{r}^{\,0} = \frac{1}{4\pi\varepsilon}\frac{Q}{r^3}\vec{r}.$$

If the field is excited by multiple charges, we determine the resulting electric field using the principle of superposition

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n = \sum_{i=1}^{n} \vec{E}_i.$$



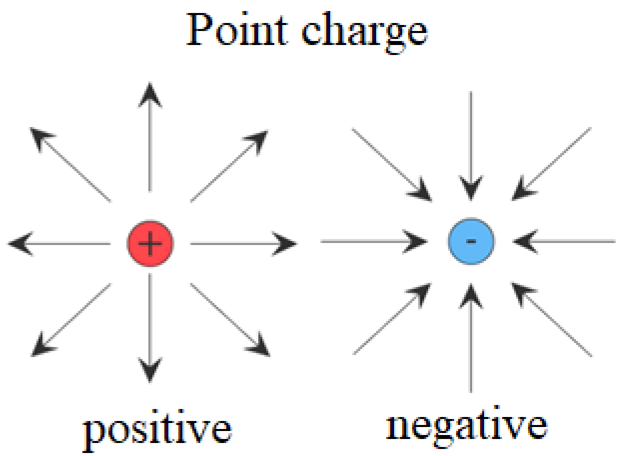
Electric field lines

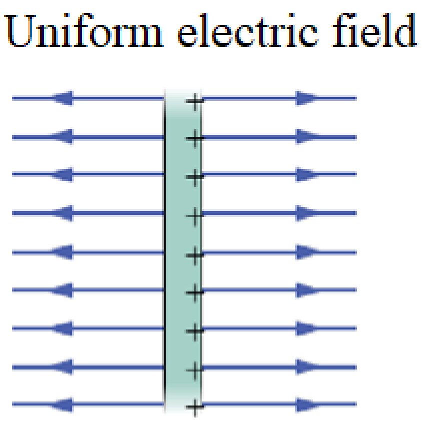
The electrostatic field can be visualized using electric field lines. **Electric field lines** are oriented curves that have the following properties:

- A single field line passes through each place (point) of space (except for its own point charge) – so field lines cannot intersect.
- The oriented tangent of each point of the field line is determined by the direction of the electric field.
- Field lines extend away from positive charge and toward negative charge.
- Field lines can also originate and terminate at infinity.
- The density of field lines is proportional to the magnitude of the electric field.

Electric field lines

If the field lines of the electrostatic field are parallel to each other and the electric field has the same magnitude at all points, the field is called **homogeneous**.



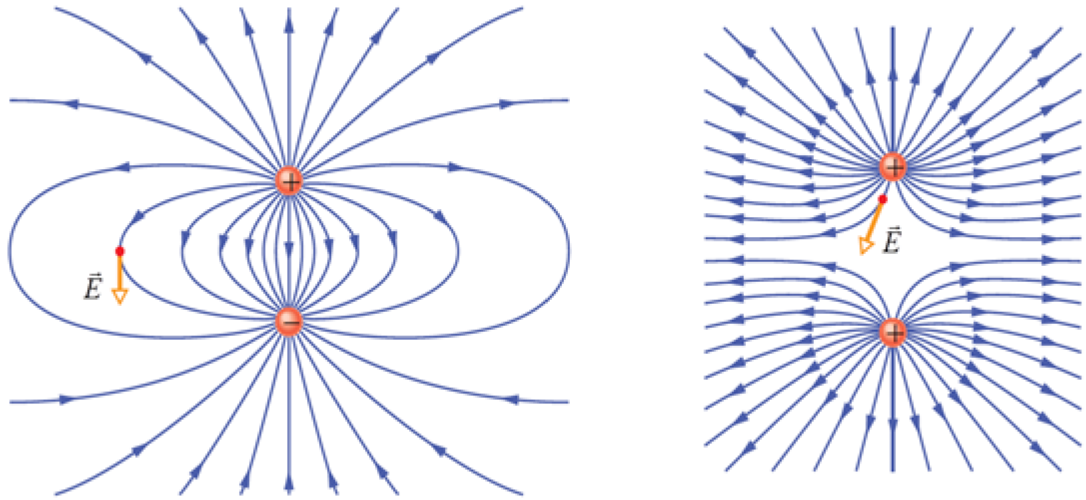


Electrostatic field lines around point charges and near the charged plane.

Electric field lines

The simplest but important case of a system of point charges is a pair of charges equal in magnitude and of opposite sign, whose distance *d* is small compared to the distance of the charges from the points at which we determine the electric field.

2 point charges equal in magnitude



opposite sign

equal sign

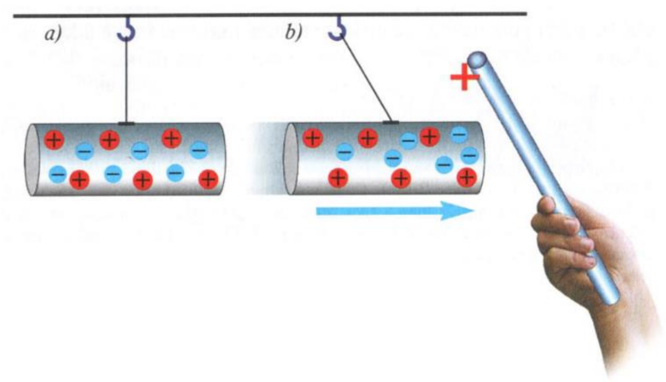
Field lines for two point charges equal in magnitude.

Conductor in an electrostatic field

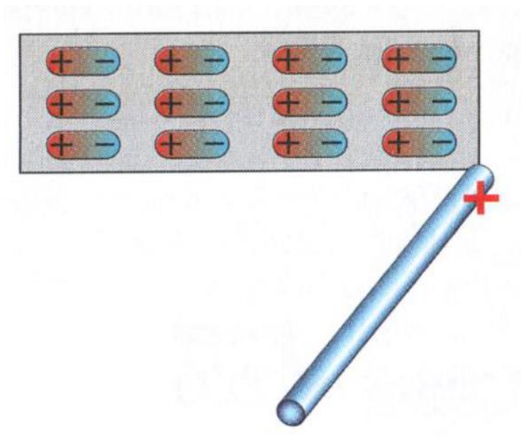
If we insert a conductor into the electric field, the forces caused by the electric field begin to act on the free electrons in it.

The free electrons will move in the conductor in such a way that a negative charge will prevail at one end and a positive charge at the other end.

This phenomenon is called **electrostatic induction**.

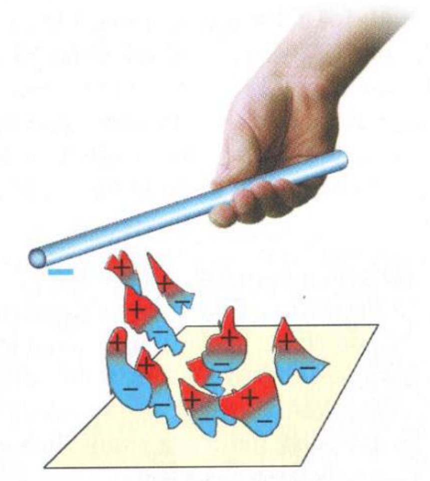


Conductor in an electrostatic field



If we insert a body from an insulator into an electric field, electrically charged particles move inside the atoms in such a way that a positive charge (pole) appears at one end of the body and a negative charge (pole) appears at the opposite end.

This phenomenon is called **dielectric polarization.**



In both of these phenomena, the charge of opposite sign appears on the side of the body that is closer to the electrically charged body.

As a result of these phenomena, an electrically charged body can also attract electrically charged bodies.

Electric potential

The charge Q creates an electric field in the space around it. When displacing a charge Q in an electric field $\vec{E}$, it is necessary to overcome a force of magnitude $F = Q \cdot E$. In doing so, some work is done and the charge gains potential energy.

The electric potential or electric field potential at any point is a scalar quantity defined by the relation

$$\varphi = \frac{E_p}{Q_0},$$

where E_p is the potential energy of the positive test charge Q_0 at a given point.

The unit of potential is volt $[\varphi] = \mathrm{V} = \mathrm{J.C^{-1}}$.

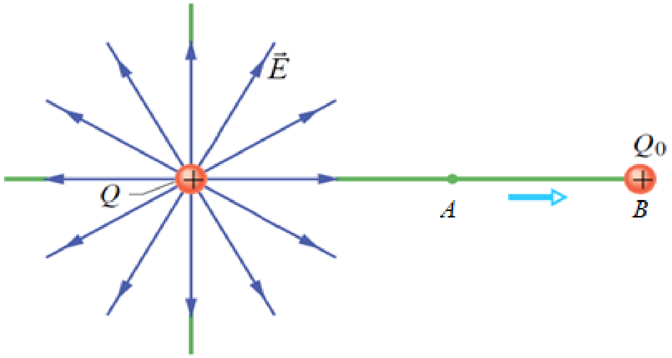
If the electrostatic field is formed by a system of n point charges, the resulting potential can be determined according to the principle of superposition.

Electric voltage

The potential difference between any two points of an electric field is called the **electric voltage** between these points

$$U_{AB} = \Delta\varphi = \varphi_A - \varphi_B = \frac{W_{AB}}{Q_0},$$

where W_{AB} is the work done by the electric force in displacing the positive electric charge Q_0 from point A to point B.



Thus, the **voltage** between two points of the electric field corresponds to the magnitude of the work done by the electrostatic force in displacing a charge of unit size between these points.

There is a voltage of 1 V between two points of the electrostatic field if, when a charge of 1 C is displaced from one point to another, the forces of the field do a work of 1 J.

Electric voltage

The electric voltage can be positive, negative, or zero (depending on the signs of charge Q and work W).

The unit of electrical voltage is volt $[U] = \mathrm{V}$.

A frequently used unit for the energy of electrons or elementary particles is the **electronvolt**.

$$\mathbf{1\ eV} = e\,(1\ \mathrm{V}) = (1{,}6\cdot10^{-19}\ \mathrm{C})\cdot(1\ \mathrm{J.\ C^{-1}}) = 1{,}6\cdot10^{-19}\ \mathrm{J}.$$

Capacitor

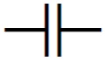
By pulling a bowstring, stretching a spring, compressing a gas, lifting a book, and other similar actions, mechanical energy can be stored as potential energy.

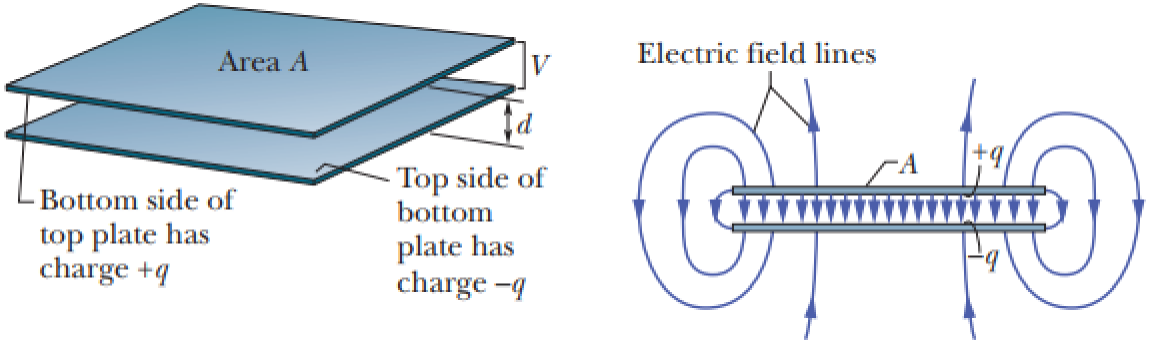
The energy of the electric field can also be stored in capacitors in this way.

A **capacitor** can generally be described as two conductors, called **electrodes**, that are close to each other, but at the same time are electrically isolated (separated) from each other. They are sometimes called „plates", regardless of their actual shape.

Parallel-plate capacitor

A parallel-plate capacitor consists of two parallel plate conductors at a distance of *d*, each with a plate area *A*.

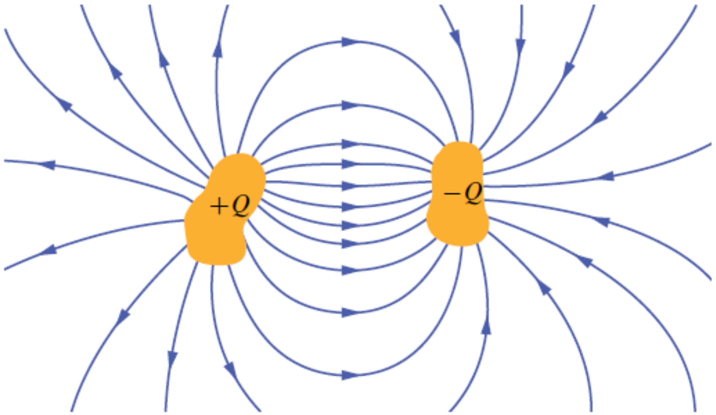
 The symbol used to represent the capacitor in the diagrams is derived from the shape of the plate capacitor (however, it is used for capacitors of all geometric shapes).



(a) (b)

Capacitor

When a capacitor is charged, its electrodes have equal but opposite charges of $+Q$ a $-Q$.



Thus, when we talk about the charge Q of a capacitor, we mean the absolute value of the charge of one of its electrodes, i.e. $|Q|$ and not the total charge, which is equal to zero: $(+Q) + (-Q) = 0$.

There is a potential difference, or voltage, between the two electrodes of a charged capacitor. If we do not know the order of the electrodes, it is natural to take this difference (voltage) positively.

Capacitor capacitance

The charge Q and voltage U of any capacitor are directly proportional to each other. Therefore,

$$Q = CU,$$

where C is a coefficient of proportionality, which is called the **capacitance of the capacitor**

$$C = \frac{Q}{U}.$$

The capacitance is numerically equal to the charge of the capacitor at a voltage of 1 V between its electrodes.

Capacitance is always positive and its unit is farad $[C] = \mathrm{F} = \mathrm{C.V^{-1}}$.

Farad is a unit too large for practice. Therefore, we use smaller units more often, especially microfarad (1 μF = 10^{-6} F), nanofarad (1 nF = 10^{-9} F) and pikofarad (1 pF = 10^{-12} F).

Capacitor capacitance

If we insert a dielectric between the plates of the capacitor, the capacitance increases ε_r-times, so

$$C = \varepsilon_r C_0,$$

where $\varepsilon_r$is the relative permittivity that characterizes a given dielectric and $\varepsilon_r \geq 1$.

In a space filled with a dielectric, all the laws and relations of electrostatics apply if we replace ε_0 by $\varepsilon = \varepsilon_0 \cdot \varepsilon_r$ (ε is the permittivity of the environment).

Consequence. The intensity of the electric field inside the dielectric is ε_r-times smaller than in a vacuum, i.e. the dielectric weakens the external field

$$E = \frac{E_0}{\varepsilon_r}.$$

Capacitor capacitance

There are different types of capacitors: plate, cylindrical, spherical.

For any capacitor, the capacitance is constant and depends only on the geometry of both electrodes of the capacitor (size, shape, distance from each other) and its dielectric.

For example:

The capacity of a plate vacuum capacitor is

$$C = \frac{\varepsilon_0 A}{d}.$$

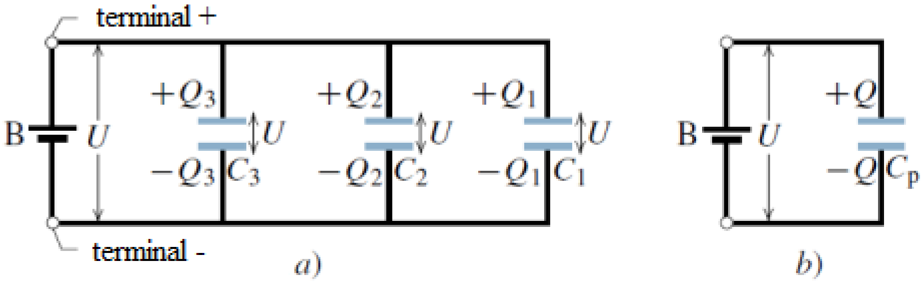
For a plate capacitor with a dielectric, the capacitance is

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}.$$

Combination of capacitors

Capacitors in parallel

When capacitors are connected in parallel (side by side), the voltage on the entire group of capacitors is the same as the voltage on each of them.



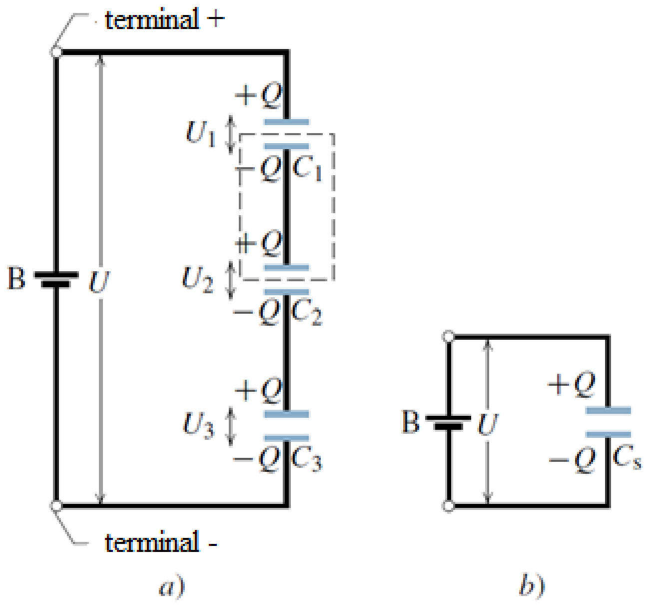
The capacity of the system is $C_p = \frac{Q}{U}$,

$$C_p = \sum_{i=1}^{n} C_i = C_1 + C_2 + \ldots + C_n$$

Combination of capacitors

Capacitors in series

When capacitors are connected in series (consecutively), the voltage on the entire group of capacitors is equal to the sum of the voltages on the individual capacitors.



There is the same charge Q on each capacitor.

The capacity of the system is $C_s = \frac{Q}{U}$,

$$\frac{1}{C_s} = \sum_{i=1}^{n} \frac{1}{C_i} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$$

Electrical energy of a charged capacitor

The energy of a charged capacitor is concentrated in an electric field between its electrodes.

$$E_{\mathrm{el}} = \frac{1}{2} QU = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CU^2$$

The relation holds independently of the geometric shape of the capacitor.

Note.

Electric field $\vec{E}$ is a vector and has a magnitude E (both are without indices).

Energy is a scalar and it always has an index: E_{p}, $E_{\mathrm{p,i}}$, $E_{\mathrm{p,f}}$, E_{el}, $E_{\mathrm{el,i}}$, $E_{\mathrm{el,f}}$ atd.

Electric current

The electric field exerts an electric force on the charged particle. If the particle is free, the force sets it in motion. The motion of particles with an electric charge is called **electric current**.

Electric currents occur all around us, from huge currents when lightning strikes to tiny currents in the nerve fibers that control our muscular activity.

The **direction of the current** is determined by agreement as the **direction of motion of the positive charge carriers**. In fact, the charge carriers are electrons.

Electric current is characterized by a scalar physical quantity of the same name, which we denote I.

Electric current

If the charge ΔQ passes through the cross-section of the conductor in time Δt, then the current is defined by the relation

$$I = \frac{\Delta Q}{\Delta t}.$$

The electric current is numerically equal to the amount of charge that passes through the cross-section of a conductor in 1 s.

The unit of current is the SI basic unit – ampere $[I]$ = A = C.s^{-1}.

Generally, current is a function of time $I = I(t)$. If the magnitude of the current is constant (independent of time), it is a stationary (steady) current.

Then the charge that passes through the cross-section of the conductor during the time interval from 0 to t,

$$Q = I \cdot t.$$

So the current is $I = \dfrac{Q}{t}$.

Electrical resistance

Resistance is a quantity that characterizes the possibility of current passing through a certain material between two points on which a voltage is applied $R = \frac{U}{I}$.

The unit of electrical resistance is ohm $[R] = \Omega = \mathrm{V.\,A}^{-1}$.

Then it also applies: $I = \frac{U}{R}$, $U = I \cdot R$.

A component whose function is to create a certain resistance in an electrical circuit is called a **resistor** and is shown  in the diagrams. The "resistance" quantity is given for a specific component or for a specific arrangement of the material.

Conductivity (conductance) is the reciprocal of resistence $G = \frac{1}{R}$.

The unit is siemens $[G] = \mathrm{S} = \Omega^{-1}$.

Resistivity

Resistivity ρ (for isotropic materials) characterizes a material in terms of its ability to resist current.

It is a local quantity, in non-isotropic materials it can generally be different in different parts of the material.

The unit is $[\rho] = \mathrm{V.A^{-1}.m} = \Omega\mathrm{.m}$.

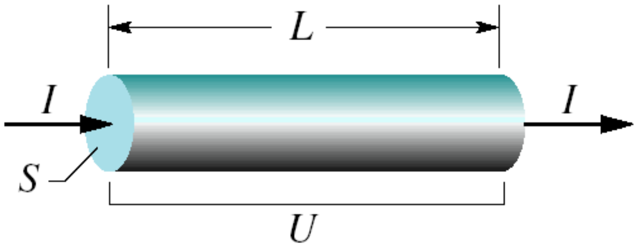
Conductivity is defined as the reciprocal of resistivity $\sigma = \dfrac{1}{\rho}$.

The unit is $[\sigma] = (\Omega\mathrm{.m})^{-1} = \Omega^{-1}\mathrm{.m^{-1}}$.

Calculating resistence from resistivity

Resistance is a property of an object (conductor, resistor).

Resistivity is a property of a material.



The voltage U applied between the ends of the conductor of length L and cross-section S causes the current I to pass through the conductor.

$$R = \rho \frac{L}{S}.$$

The relation can only be used for a homogeneous isotropic conductor with a constant cross-section.

Ohm's Law

At present, the term "law" is too strong. However, at the time it was formulated, it had the nature of a law (only homogeneous conductive materials, most often metal).

Resistance is a property of a component. For a given component, the resistance is constant and does not depend on the magnitude or polarity of the applied voltage. The current passing through the component is directly proportional to the applied voltage.

To describe the situation, following relationships can be used

$$R = \frac{U}{I}, I = \frac{U}{R}, U = I \cdot R.$$

Ohm's law is fulfilled only for components made of homogeneous materials (conductive and semiconductive) and only within certain ranges of applied voltages or passing currents.

For example: a component gets hot with the passing current and the resistance begins to change depending on the temperature.

Work and power of electric current

The voltage U at the battery terminals is the same as the voltage at the terminals of the component (appliance).

Terminal a has a higher potential than terminal b.

The voltage between points a, b is U.

The work done by the forces of the electric field when the charge ΔQ is transferred through the appliance is equal to the decrease in the electric potential energy

$$\Delta W = \Delta E_{\mathrm{p}} = U \cdot \Delta Q = U \cdot I \cdot \Delta t.$$

The power of electric current is defined as the rate of transmission of electrical energy

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta E_{\mathrm{p}}}{\Delta t} = U \cdot I.$$

The unit is watt $[P]$ = W = V.A.

Work and power of electric current

If the appliance is a resistor with resistance R, the work ΔW is converted into **Joule heat** ΔQ_J

$$\Delta Q_J = \Delta W = U \cdot I \cdot \Delta t = I^2 R \Delta t = \frac{U^2}{R} \Delta t.$$

The resistor increases the temperature and thus becomes a source of heat flux. This irreversible process is called **energy dissipation**.

The **dissipated power** is therefore the rate of energy dissipation by the resistor

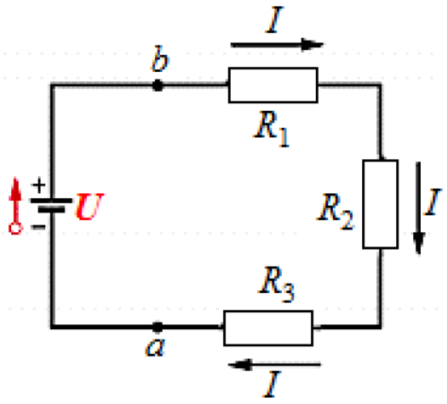
$$P = \frac{\Delta W}{\Delta t} = I^2 R = \frac{U^2}{R}.$$

Note. Q_J here stands for the physical quantity heat, not electric charge.

Combination of resistors

Resistors in series

The resistors have **identical currents**. The sum of the voltages across the individual resistors is equal to the applied voltage.



$$U = IR_1 + IR_2 + IR_3,$$

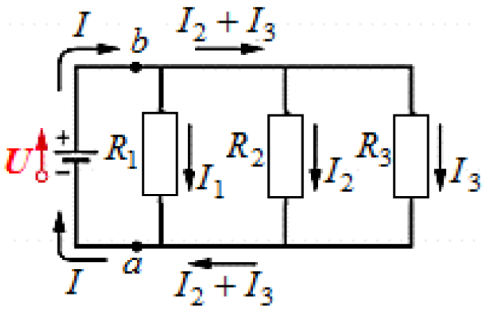
$$I = \frac{U}{R_1 + R_2 + R_3} = \frac{U}{R_s}, \text{ i.e.}$$

$$R_s = \sum_{i=1}^{n} R_i = R_1 + R_2 + \ldots + R_n$$

Combination of resistors

Resistors in parallel

The resistors all have **the same voltage**. The total current is equal to the sum of the currents of the individual resistors.



$$I_1 = \frac{U}{R_1}, I_2 = \frac{U}{R_2}, \ I_3 = \frac{U}{R_3}.$$

$$I = I_1 + I_2 + I_3 = U\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{U}{R_\mathrm{p}},$$

i.e.

$$\frac{1}{R_\mathrm{p}} = \sum_{i=1}^{n} \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

Examples

The following values of physical constants were used in the examples:

Elemental charge $e = 1{,}602 \cdot 10^{-19}$ C,

Permittivity constant $\varepsilon_0 = 8{,}854 \cdot 10^{-12}$ F.m^{-1},

Air relative permittivity $\varepsilon_r \approx 1$.

Examples

1. Two electric charges $Q_1 = 8 \cdot 10^{-8}$ C a $Q_2 = -4 \cdot 10^{-8}$ C are in a vacuum at a distance of 20 cm from each other. Calculate:
a) magnitude and direction of the electric force acting on the charge Q_2,
b) magnitude and direction of electric field at the point of charge Q_1.

Solution:

At the beginning, we need to convert the units to the SI basic units: distance $r = 20\text{ cm} = 20 \cdot 10^{-2}\text{ m} = 0{,}2\text{ m}$.

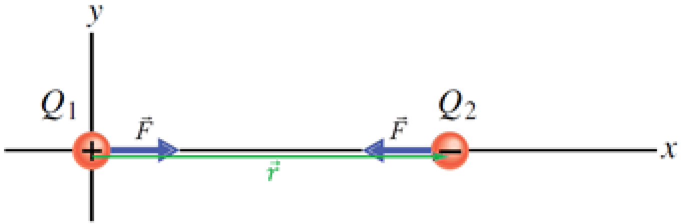
a) To calculate the force, we use Coulomb's law:

$$F = k\frac{|Q_1||Q_2|}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{|Q_1||Q_2|}{r^2}.$$

Examples

Substitute the specified quantities and constants:

$$F = \frac{1}{4\pi \cdot 8{,}854 \cdot 10^{-12}} \frac{|8 \cdot 10^{-8}||-4 \cdot 10^{-8}|}{0{,}2^2} \approx 7{,}19 \cdot 10^{-4}\ \text{(N)}.$$



Charges Q_1 and Q_2 are opposite – positive and negative – therefore, the force acting on the charges is attractive.

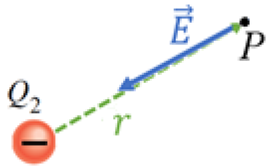
Answer: a) The magnitude of the force exerted by two point charges on each other is $7{,}19 \cdot 10^{-4}$ N.

Force acting on the charge Q_2 has a direction to the charge Q_1 (point charges attract each other).

Examples

b) It is necessary to determine the electric field at the point of charge Q_1, therefore, we consider the charge Q_1 to be a test charge, that is, the magnitude of the charge $Q_0 = |Q_1|$.
According to the definition relation, the magnitude of the electric field is

$$E = \frac{F}{Q_0} = \frac{k\frac{|Q_1||Q_2|}{r^2}}{|Q_1|} = k\frac{|Q_2|}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{|Q_2|}{r^2}.$$



Substitute the specified quantities and constants:

$$E = \frac{1}{4\pi\cdot 8{,}854\cdot 10^{-12}}\frac{|-4\cdot 10^{-8}|}{0{,}2^2} \approx 8987{,}74\ \text{(V/m)}.$$

Answer: b) Magnitude of electric field at the point of charge Q_1 is 8987,74 V.m^{-1}.
Direction of electric field vector excited by negative charge Q_2 at the point P (in the place where the charge Q_1 is located) is towards the charge Q_2.

Examples

2. Two point electric charges at a distance of 11,0 cm from each other exert the same force on each other in a vacuum as in turpentine at a distance of 7,4 cm. Find the relative permittivity of turpentine.

Solution:

We convert the units of the specified quantities into the SI basic units:

$r_1 = 11 \text{ cm} = 11 \cdot 10^{-2} \text{ m} = 0{,}11 \text{ m},$

$r_2 = 7{,}4 \text{ cm} = 7{,}4 \cdot 10^{-2} \text{ m} = 0{,}074 \text{ m}.$

Using Coulomb's law, we write the acting forces:

$F_1 = \frac{1}{4\pi\varepsilon_0} \frac{|Q_1||Q_2|}{r_1^{\,2}}$ for charges in vakuum and

$F_2 = \frac{1}{4\pi\varepsilon_0 \cdot \varepsilon_r} \frac{|Q_1||Q_2|}{r_2^{\,2}}$ for charges in turpentine.

Examples

According to the assignment of the task, the charges act on each other with equal force in vacuum and in turpentine $F_1 = F_2$, i.e.

$$\frac{1}{4\pi\varepsilon_0}\frac{|Q_1||Q_2|}{{r_1}^2} = \frac{1}{4\pi\varepsilon_0 \cdot \varepsilon_r}\frac{|Q_1||Q_2|}{{r_2}^2}.$$

After mathematical adjustment, we get the equation $\frac{1}{{r_1}^2} = \frac{1}{\varepsilon_r}\frac{1}{{r_2}^2}$

and we express the permittivity of turpentine from it

$$\varepsilon_r = \frac{{r_1}^2}{{r_2}^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{0{,}11}{0{,}074}\right)^2 = \left(\frac{55}{37}\right)^2 \approx 2{,}21.$$

Relative permittivity is a dimensionless quantity.

Answer: The relative permittivity of turpentine is 2,21.

Unsolved examples

3. The electric field in a vacuum has the value of $4 \cdot 10^{-5}$ V.m^{-1} at a distance of 10 cm from the point charge. Calculate:

a) charge magnitude,

b) the value of the relative permittivity of an oil in which the same electric charge produces at a distance of 10 cm an electric field of $2 \cdot 10^{-6}$ V.m^{-1}.

[a) $Q = 4{,}45 \cdot 10^{-17}$ C, b) $\varepsilon_r = 20$]

4. At a distance of 10 cm from charge Q_1 , a force of $1 \cdot 10^{-2}$ N acts on charge $Q_2 = 1 \cdot 10^{-8}$ C. Calculate:

a) magnitude of the electric field at this point,

b) magnitude of charge Q_1, who created this field.

[a) $E = 1 \cdot 10^{6}$ V.m^{-1}, b) $Q_1 = 1{,}11 \cdot 10^{-6}$ C]

Unsolved examples

5. Find the distance between two point charges of 10 μC, which exert a force of 10 N on each other in a vacuum.

[30 cm]

6. Calculate the magnitude of the point charge Q_1 that acts on the point charge Q_2 = 1 μC at a distance of 3 cm by an electric force of magnitude 1 N. The charges are

a) in vacuum,

b) in kerosene about relative permittivity $\varepsilon_r = 2$.

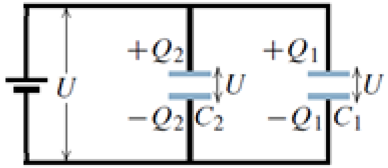
[a) Q_1 = 0,1 μC , b) Q_1 = 0,2 μC]

Examples

7. Capacitors with capacitance C_1 = 2 μF and C_2 = 3 μF are connected in parallel. On a capacitor with capacitance C_1, there is a charge Q_1= 6 μC. Calculate:

a) the resulting capacitance of the capacitors,

b) voltage and charge on the second capacitor.



Solution:

a) When capacitors are connected in parallel (side by side), the voltage on the entire group of capacitors is equal to the voltage on each of them.

Thus, the resulting capacity is

$$C_\mathrm{p} = \sum_{i=1}^{n} C_i = C_1 + C_2 = 2\cdot10^{-6} + 3\cdot10^{-6} = 5\cdot10^{-6}\ \mathrm{(F)}.$$

Examples

b) For the charge on any capacitor, the following relation applies $Q = CU$, i.e. $Q_1 = C_1U_1$ a $Q_2 = C_2U_2$.

Then the voltage on the first capacitor is

$$U_1 = \frac{Q_1}{C_1} = \frac{6\cdot 10^{-6}}{2\cdot 10^{-6}} = 3\ \text{(V)}.$$

Then the $U_1 = U_2 = 3$ V.

The charge on the second capacitor can be calculated

$$Q_2 = C_2U_2 = 3\cdot 10^{-6}\cdot 3 = 9\cdot 10^{-6}\ \text{(C)}.$$

Answer: the resulting capacitance of the capacitors in parallel is 5 μF, the voltage on the second capacitor is 3 V, the charge on the second capacitor is 9 μC.

Examples

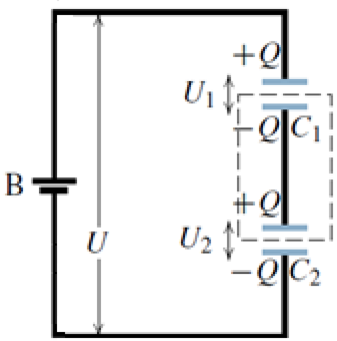
8. Two capacitors with capacitances of 12 μF and 24 μF are connected in series to a 30 V DC voltage source. Calculate:

a) resulting capacitance,

b) charges on capacitor plates,

c) voltage ratio on individual capacitors,

d) the total energy of electric fields of the capacitors.



Solution:

a) When capacitors are connected in series, the voltage on the entire group of capacitors is equal to the sum of the voltages on the individual capacitors.
Thus, the resulting capacity is

$$\frac{1}{C_s} = \sum_{i=1}^{n} \frac{1}{C_i} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_2 + C_1}{C_1 \cdot C_2}.$$

Examples

Then $C_s = \frac{C_1 \cdot C_2}{C_1 + C_2} = \frac{12 \cdot 10^{-6} \cdot 24 \cdot 10^{-6}}{12 \cdot 10^{-6} + 24 \cdot 10^{-6}} = 8 \cdot 10^{-6}$ (F).

b) There is the same charge Q on each capacitor, so we determine the charges on the capacitor plates by substituting the known values into the relation:

$$Q = C_s U = 8 \cdot 10^{-6} \cdot 30 = 240 \cdot 10^{-6} \text{ (C)}.$$

c) Voltage on individual capacitors:

$$U_1 = \frac{Q}{C_1} = \frac{240 \cdot 10^{-6}}{12 \cdot 10^{-6}} = 20 \text{ (V)},$$

$$U_2 = \frac{Q}{C_2} = \frac{240 \cdot 10^{-6}}{24 \cdot 10^{-6}} = 10 \text{ (V)}.$$

Then $\frac{U_1}{U_2} = \frac{20 \text{ V}}{10 \text{ V}} = 2:1.$

Examples

d) The total energy of the electric fields of the capacitors is

$$E_{\mathrm{el}} = \frac{1}{2}QU = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CU^2.$$

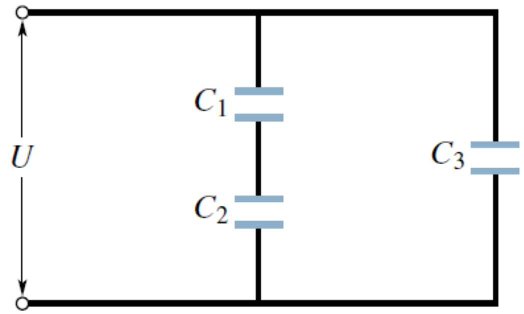
We substitute the specified quantities

$$E_{\mathrm{el}} = \frac{1}{2}\cdot 240\cdot 10^{-6}\cdot 30 = 3{,}6\cdot 10^{-3}\ \text{(J)}.$$

Answer: a) the resulting capacitance of the capacitors in series is 8 μF, b) the charges on the capacitor plates are 240 μC, c) the voltage ratio on the individual capacitors is 2:1, d) the total energy of the electric fields of the capacitors is 3,6 mJ.

Examples

9. Capacitors with capacitances of 6 μF and 4 μF are connected in series and a capacitor with a capacitance of 2 μF is connected in parallel to them. Calculate their resulting capacitance.



Solution:

For capacitors in series,

$$\frac{1}{C_s} = \sum_{i=1}^{n} \frac{1}{C_i} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_2+C_1}{C_1 \cdot C_2},\ \text{i.e.}$$

$$C_s = \frac{C_1 \cdot C_2}{C_1 + C_2} = \frac{6 \cdot 10^{-6} \cdot 4 \cdot 10^{-6}}{6 \cdot 10^{-6} + 4 \cdot 10^{-6}} = 2{,}4 \cdot 10^{-6}\ \text{(F)}.$$

For capacitors in parallel,

$$C_p = \sum_{i=1}^{n} C_i = C_s + C_3 = 2{,}4 \cdot 10^{-6} + 2 \cdot 10^{-6} = 4{,}4 \cdot 10^{-6}\ \text{(F)}.$$

Answer: the resulting capacitance of capacitors is 4,4 μF.

Examples

10. Five identical bulbs are connected to the 230 V consumer network. Specify:

a) what voltage we measure on each of them,

b) what is the total resistance of the bulbs if the resistance of one bulb is 24 Ω,

c) how much current passes through this electrical circuit.

Solution:

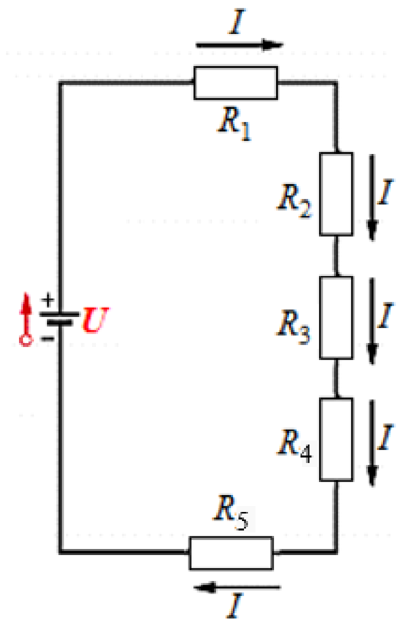
a) We consider light bulbs as resistors, i.e. we use the relation for the resistors in series. At the same time, equal current passes through all resistors and the total voltage is equal to the sum of the voltages on the individual resistors. That is, we measure the voltage on each of the bulbs

$$U^* = \frac{U}{n} = \frac{230}{5} = 46 \text{ (V)}.$$

Examples

b) The total resistance of the bulbs in series is

$$R_s = \sum_{i=1}^{n=5} R_i = R_1 + R_2 + \ldots + R_5,$$



The bulbs are the same, i.e., they have the same resistance, so

$$R_s = n \cdot R = 5 \cdot 24 = 120\ (\Omega).$$

c) Using Ohm's law, we calculate the current that passes through an electrical circuit:

$$I = \frac{U}{R_s} = \frac{230}{120} \approx 1{,}92\ (\text{A}).$$

Answer: a) the voltage on each bulb is 46 V, b) the total resistance of the bulbs connected in series is 120 Ω, c) the current in the circuit is 1,92 A.

Unsolved examples

11. Two capacitors with capacitances of 2,0 μF and 4,0 μF are connected in parallel to a voltage source of 300 V. Find the total energy of the electric fields of both capacitors.
[0,27 J]

12. Four resistors with a resistance of 18,0 Ω are connected in parallel to the ideal 25,0 V battery. How much current is passing through the battery?
[5,56 A]

13. A 54 C charge passed through a conductor with a resistance of 7,5 Ω in 1,5 minutes. Find the voltage of the source to which the wire was connected.
[4,5 V]

Unsolved examples

14. Two capacitors with capacitances of 0,2 F and 0,5 F are connected to a 1 kV voltage source a) in series, b) in parallel. Find the energy of the electric field of the capacitor systems at these connections.

[a) 71,4 mJ , b) 0,35 J]

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Thank you for your attention

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