LINEAR MODEL PREDICTIVE CONTROL OF INDUCTION MACHINE

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Abstract: This article presents new control algorithm for induction machine based on linear model predictive control (MPC). Controller works in similar manners as field oriented control (FOC), but control is performed in stator coordinates. This reduces computational demands as Park's transformation is absent and induction machine mathematical model in stator coordinates contains less nonlinear elements. Another aim of proposed controller was to achieve fast torque response.

Keywords: Linear model predictive control, MPC, induction machine, ACIM

1 INTRODUCTION

Induction machine (ACIM) is the most widespread type of AC machine today. Thanks to its simple construction lacking any expensive permanent magnets or brushes, the ACIM has the advantage of high reliability and low price. Most common control strategies of AC machines are the scalar control and field oriented control (FOC). More and more attention is however being paid to predictive control, especially the model predictive control (MPC) algorithms. The MPC has several advantages over the classical PID control. It can be simply implemented to systems with multiple inputs and outputs and it enables to introduce constraints of system inputs, outputs and states in a simple way. This advantages however come with relatively high computational demands of MPC, which restricts the use of these algorithms only to systems with slower dynamics. That's why MPC is mostly used in chemical industry. The MPC algorithms share characteristics like use of explicitly given model of controlled system to predict future values of outputs and states, optimisation of the control sequence through minimizing cost function and use of receding horizon control (RHC). The RHC means that in each control step, the entire control sequence over prediction horizon of constant length N is being calculated in order to minimize value of cost function J_N. However only the first value of this sequence is applied to the system and new sequence is calculated in following step. This ensures closed-loop control. [2, 6, 4],

The algorithm described in this paper belongs to linear MPC class of algorithms. The linear MPC algorithms combine the equations of the state space model of system, the cost function and the constraints defined as linear combination of system states, inputs and outputs to obtain the quadratic programming optimisation problem. This problem can be solved using fast solvers, which (especially considering possibility of use of explicit controller) allows to employ the algorithms for systems with very fast dynamics. Following chapters describe the mathematical model of induction machine, functional principle of proposed algorithm and results of simulation in MATLAB-Simulink. [3]

2 MATHEMATICAL MODEL OF INDUCTION MACHINE

The most common mechanical construction of induction machine consists of three-phase, harmonically distributed stator windings and the squirrel cage rotor. Unlike other types of AC machines, the rotor magnetic field is created subsequently as the effect of rotating stator magnetic field and is not mechanically aligned with rotor, which complicates mathematical model. The discrete-time model obtained using Euler's discretisation method in α, β stator coordinates can be expressed as

$$i_{s\alpha}(k+1) = (1 - T_s\gamma)i_{s\alpha}(k) + T_s\beta\eta\Psi_{r\alpha}(k) + T_s\beta P_p\omega_m(k)\Psi_{r\beta}(k) + \frac{T_s}{\sigma L_s}u_{s\alpha}(k), \tag{1}$$

$$i_{s\beta}(k+1) = (1 - T_s\gamma)i_{s\beta}(k) + T_s\beta\eta\Psi_{r\beta}(k) - T_s\beta P_p\omega_m(k)\Psi_{r\alpha}(k) + \frac{T_s}{\sigma L_s}u_{s\beta}(k), \tag{2}$$

$$\Psi_{r\alpha}(k+1) = (1 - T_s\eta)\Psi_{r\alpha}(k) - T_s P_p\omega_m(k)\Psi_{r\beta}(k) + T_s\eta L_m i_{s\alpha}(k), \tag{3}$$

$$\Psi_{r\beta}(k+1) = (1 - T_s\eta)\Psi_{r\beta}(k) + T_s P_p\omega_m(k)\Psi_{r\alpha}(k) + T_s\eta L_m i_{s\beta}(k), \tag{4}$$

where $\vec{i}_{s\alpha\beta} = [i_{s\alpha} \;\; i_{s\beta}]^{\mathrm{T}}$ is stator current vector, $\vec{\Psi}_{r\alpha\beta} = [\Psi_{r\alpha} \;\; \Psi_{r\beta}]^{\mathrm{T}}$ is rotor magnetic flux vector, $\vec{u}_{s\alpha\beta} = [u_{s\alpha} \;\; u_{s\beta}]^{\mathrm{T}}$ is stator voltage vector, ω_m is mechanical speed, $\eta = R_r/L_r$ is inverse rotor time constant, $\gamma = \left(R_s + R_r L_h^2/L_r^2\right)/(\sigma L_s)$ is inverse stator time constant, $\sigma = 1 - L_h^2/(L_s L_r)$ is leakage coefficient, $\beta = L_h/(\sigma L_s L_r)$ is coupling factor, R_s and R_r is stator and rotor resistance, L_s, L_r and L_m is stator, rotor and mutual inductance, P_p is number of machine pole pairs, T_s is sampling period and k denotes discrete time step. Equations (1) and (2) are called stator model while equations (3) and (4) are rotor model. Speed ω_m, stator current $\vec{i}_{s\alpha\beta}$ and voltage $\vec{u}_{s\alpha\beta}$ are all known, only the rotor flux $\vec{\Psi}_{r\alpha\beta}$ has to be obtained using observer. The equation

$$T_m = \frac{3}{2}P_p\frac{L_h}{L_r}\left(\Psi_{r\alpha}i_{s\beta} - \Psi_{r\beta}i_{s\alpha}\right). \tag{5}$$

defines mechanical torque of induction machine. [6]

3 PROPOSED CONTROL ALGORITHM

Block diagram of proposed algorithm is shown in figure 1. The control is conducted in α, β stator coordinates. This, compared to FOC, eliminates the need of Park's transformation and with it the calculation of sine and cosine of angle of rotor flux vector, which is usually performed using division and square root functions. This significantly reduces computational demands. Similarly to FOC, there are two outer control loops, both of them using PI controller. First control loop keeps the square value of rotor magnetic flux amplitude $|\vec{\Psi}_{r\alpha\beta}|^2 = \Psi_{r\alpha}^2 + \Psi_{r\beta}^2$ at the reference value $|\vec{\Psi}^*_{r\alpha\beta}|^2$. Just like in case of FOC, the amplitude of rotor flux is kept constant, unless the field weakening is required. It was chosen to control squared values because it eliminate need for calculation of square root function. The output of rotor flux controller h_Ψ is then used for calculation of stator current reference. Second PI controller sets the mechanical torque reference T_m^* so that the mechanical speed ω_m follows its reference ω_m^*. Both PI controllers output h_Ψ and T_m^*, speed ω_m, stator current $\vec{i}_{s\alpha\beta}$ and rotor $\vec{\Psi}_{r\alpha\beta}$ are used in system state calculation (SSC) block to prepare input for linear MPC controller of stator currents. The inner loop linear MPC controller generates the stator voltages $\vec{u}_{s\alpha\beta}$. These are modulated using standard vector modulation (SVM) technique and then applied to stator. [2, 5]

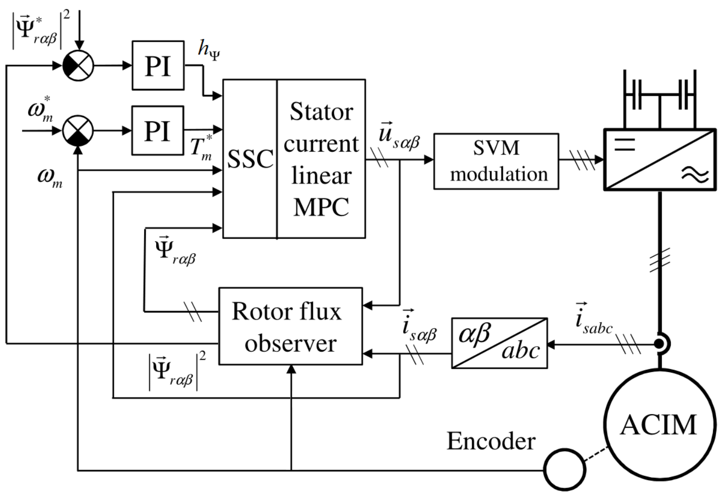

Figure 1: Block diagram of proposed control algorithm of ACIM

The task of linear MPC current controller is to ensure, that stator current $\vec{i}_{s\alpha\beta}$ follows its reference $\vec{i}^*_{s\alpha\beta} = [i^*_{s\alpha} \;\; i^*_{s\beta}]^{\mathrm{T}}$. The current reference is calculated in SSC block in a way that emulates FOC. It means that the part of the stator current $\vec{i}^*_{s\alpha\beta T} = [i^*_{s\alpha T} \;\; i^*_{s\beta T}]^{\mathrm{T}}$ is perpendicular to $\vec{\Psi}_{r\alpha\beta}$ and is responsible for torque, while part $\vec{i}^*_{s\alpha\beta\Psi} = [i^*_{s\alpha\Psi} \;\; i^*_{s\beta\Psi}]^{\mathrm{T}}$ is parallel with $\vec{\Psi}_{r\alpha\beta}$ and maintains the rotor flux. Final current reference can then be calculated as $\vec{i}^*_{s\alpha\beta} = \vec{i}^*_{s\alpha\beta T} + \vec{i}^*_{s\alpha\beta\Psi}$.

To calculate torque-creating current reference $\vec{i}^*_{s\alpha\beta T}$ we first consider equation (5). If the current parts are set to its references $i^*_{s\alpha T} = -h_T \Psi_{r\beta}$ and $i^*_{s\beta T} = h_T \Psi_{r\alpha}$, where h_T is arbitrary scalar, the torque reference will be

$$T^*_m = \frac{3}{2} P_p \frac{L_h}{L_r} \left(\Psi_{r\alpha} i^*_{s\beta T} - \Psi_{r\beta} i^*_{s\alpha T} \right) = \frac{3}{2} P_p \frac{L_h}{L_r} \left(h_T \Psi^2_{r\alpha} - h_T \Psi^2_{r\beta} \right) = \frac{3}{2} P_p \frac{L_h}{L_r} |\vec{\Psi}_{r\alpha\beta}|^2 h_T. \tag{6}$$

If $|\vec{\Psi}_{r\alpha\beta}|^2$ is replaced with its reference, the torque-producing current reference is then going to be

$$\vec{i}^*_{s\alpha\beta T} = \frac{2L_r}{3P_p L_h |\vec{\Psi}^*_{r\alpha\beta}|^2} \left[-\Psi_{r\beta} \;\; \Psi_{r\alpha} \right]^{\mathrm{T}} T^*_m = const \left[-\Psi_{r\beta} \;\; \Psi_{r\alpha} \right]^{\mathrm{T}} T^*_m. \tag{7}$$

The flux-producing stator current reference $\vec{i}^*_{s\alpha\beta\Psi}$ can be obtained from rotor model equations (3) and (4). When considering steady state $\vec{\Psi}_{r\alpha\beta}(k+1) = \vec{\Psi}_{r\alpha\beta}(k)$ and speed $\omega_m = 0$ it can be derived that $\vec{i}^*_{s\alpha\beta\Psi} = \frac{1}{L_m} \left[\Psi_{r\alpha} \;\; \Psi_{r\beta} \right]^{\mathrm{T}}$. However these simplifications cannot be met during nonzero speed. Rotor flux-controlling PI controller is therefore introduced to correct any disturbances. Final reference then can be obtained as

$$\vec{i}^*_{s\alpha\beta\Psi} = \frac{1}{L_m} \left[\Psi_{r\alpha} \;\; \Psi_{r\beta} \right]^{\mathrm{T}} h_\Psi = const \left[\Psi_{r\alpha} \;\; \Psi_{r\beta} \right]^{\mathrm{T}} h_\Psi, \tag{8}$$

where h_Ψ is output value of rotor flux PI controller. This way of rotor flux control however cannot startup the induction machine because at the beginning $\vec{\Psi}_{r\alpha\beta} = \vec{0}$. It is therefore necessary to apply small but sufficiently high voltage vector $\vec{u}_{s\alpha\beta} \neq \vec{0}$ before the minimal rotor flux $\vec{\Psi}_{r\alpha\beta}$ is reached and the proposed algorithm can be used. It should be noted that only mathematical operation used in equations (7) and (8) is multiplication, which makes them easy to compute. [5]

Mathematical model of induction machine was described with equations (1) to (4). The model unfortunately contains nonlinear operation of multiplication of two states. This prevents it from being expressed using state space representation $\vec{x}(k+1) = \mathbf{A}\vec{x}(k) + \mathbf{B}\vec{u}(k)$, where $\vec{x}$ is state vector, $\vec{u}$ is system input vector, $\mathbf{A}$ is system matrix and $\mathbf{B}$ is system input matrix. Usual practice is to declare nonlinear elements as measured disturbances, constant over prediction horizon. From this point of view the ACIM model in α, β coordinates is more beneficial then the model in d, q coordinates as it contains less nonlinear operations. In our case we introduce new state variables $\widehat{\omega_m \Psi_{r\alpha}}$ and $\widehat{\omega_m \Psi_{r\beta}}$. Another states we have to add are stator current references $\vec{i}^*_{s\alpha\beta}$ so we can penalise difference between stator currents and its references in cost function. [1] This leads us to extended state vector

$$\vec{x} = \left[i_{s\alpha} \quad i_{s\beta} \quad \Psi_{r\alpha} \quad \Psi_{r\beta} \quad \widehat{\omega_m \Psi_{r\alpha}} \quad \widehat{\omega_m \Psi_{r\beta}} \quad i^*_{s\alpha} \quad i^*_{s\beta} \right]^{\mathrm{T}}. \tag{9}$$

Considering equations (1) to (4) and state vector (9), the system matrix will take form

$$\mathbf{A} = \begin{bmatrix}
1 - T_s\gamma & 0 & T_s\beta\eta & 0 & 0 & T_s\beta P_p & 0 & 0 \\
0 & 1 - T_s\gamma & 0 & T_s\beta\eta & -T_s\beta P_p & 0 & 0 & 0 \\
T_s\eta L_m & 0 & 1 - T_s\eta & 0 & 0 & -T_s P_p & 0 & 0 \\
0 & T_s\eta L_m & 0 & 1 - T_s\eta & T_s P_p & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}, \tag{10}$$

$$\mathbf{B}=\begin{bmatrix}\frac{T_s}{\sigma L_s} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \frac{T_s}{\sigma L_s} & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}. \quad (11)$$

The cost function in form of $\mathfrak{L}_2$-norm can be in most simple case defined as

$$J_N(k)=\frac{1}{2}\sum_{i=1}^{N-1}[\vec{x}(k+i)^{\mathrm{T}}\mathbf{Q}\vec{x}(k+i)+\vec{u}(k+i-1)^{\mathrm{T}}\mathbf{R}\vec{u}(k+i-1)], \quad (12)$$

where N is length of prediction horizon, $\mathbf{Q}\succeq 0$ is state cost matrix and $\mathbf{R}\succ 0$ is system input cost matrix. The prediction horizon can be only few steps long as measured disturbances $\widehat{\omega_m\Psi_{r\alpha}}$ and $\widehat{\omega_m\Psi_{r\beta}}$ were defined with no dynamics, while in reality they change rapidly at higher speed. Only goal of proposed linear MPC controller is that stator current vector $\vec{i}_{s\alpha\beta}$ follows its reference $\vec{i}^{*}_{s\alpha\beta}$ as close as possible. This leads us to

$$\mathbf{Q}=\begin{bmatrix}C_i & 0 & 0 & 0 & 0 & 0 & -C_i & 0\\ 0 & C_i & 0 & 0 & 0 & 0 & 0 & -C_i\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ -C_i & 0 & 0 & 0 & 0 & 0 & C_i & 0\\ 0 & -C_i & 0 & 0 & 0 & 0 & 0 & C_i\end{bmatrix}, \quad \mathbf{R}=\begin{bmatrix}C_u & 0\\ 0 & C_u\end{bmatrix}, \quad (13)$$

where coefficient $C_i>0$ penalizes difference between stator current and its reference and $C_u>0$ penalizes system input (only to ensure positive definiteness of matrix $\mathbf{R}$). Constraints in linear MPC are defined as affine function of system states, inputs and outputs. In our case the stator voltage and current constraints need to be implemented. If the PWM modulation technique was used, the nonlinear equations $\sqrt{i_{s\alpha}^2+i_{s\beta}^2}\leq I_{\max}$ and $\sqrt{u_{s\alpha}^2+u_{s\beta}^2}\leq U_{\max}$, where $I_{\max}$ and $U_{\max}$ are respective amplitude limits, would have to be approximated. But since the control problem is defined in stator coordinates the SVM technique offers more benefits as it can generate stator voltage with hexagon border with $\frac{\sqrt{3}}{2}U_{\max}$ outer radius. Current and voltage constraints can be therefore defined as $\mathbf{M}\vec{i}_{s\alpha\beta}\leq\vec{b}\frac{\sqrt{3}}{2}I_{\max}$ and $\mathbf{M}\vec{u}_{s\alpha\beta}\leq\vec{b}\frac{\sqrt{3}}{2}U_{\max}$, where

$$\mathbf{M}=\begin{bmatrix}-1 & -1 & 0 & 0 & 1 & 1\\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} & -\frac{2}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{3}}\end{bmatrix}^{\mathrm{T}}, \quad \vec{b}=\begin{bmatrix}1 & 1 & 1 & 1 & 1 & 1\end{bmatrix}^{\mathrm{T}}. \quad (14)$$

4 RESULTS OF SIMULATION

Proposed algorithm was simulated in MATLAB-Simulink version 8.0.0.783 using the MPT toolbox version 3.0.20 and Gurobi version 6.0 as a solver. Parameters of simulation are stated in table 1. Due to nonlinearity of linear MPC controller, the PI controllers had to be tuned experimentally. Proportional gain was set to $K_\Psi=20$ and $K_{\omega_m}=2$ and integral time constant $T_{i\Psi}=0,06\ s$ and $T_{i\omega_m}=0,02\ s$ for rotor flux and speed controller. Prediction horizon was set to 3 steps.

Parameter	Unit	Value	Parameter	Unit	Value
R_s	$[\Omega]$	2,66	J	$[\mathrm{kgm}^2]$	0,01
R_r	$[\Omega]$	2,27	$I_{\max}$	[A]	7
L_s	[mH]	255	$U_{\max}$	[V]	150
L_r	[mH]	255	T_s	[ms]	0,2
L_h	[mH]	266	C_i	[−]	100
P_p	[−]	1	C_u	[−]	$1\cdot 10^{-3}$

Table 1: Parameters used in simulation [6]

The resulting charts can be seen in figure 2. We can notice that both torque and rotor flux begins to mildly oscillate when maximal torque or speed is achieved. It is caused by hexagon-shaped constraints of current and voltage. Generally very fast torque response can be observed, especially when 1 Nm load torque is applied in 1,5 s. Due to the absence of integrator, the linear MPC controller acts as proportional controller and the reference T_m^* is not followed exactly, but the difference is negligible.

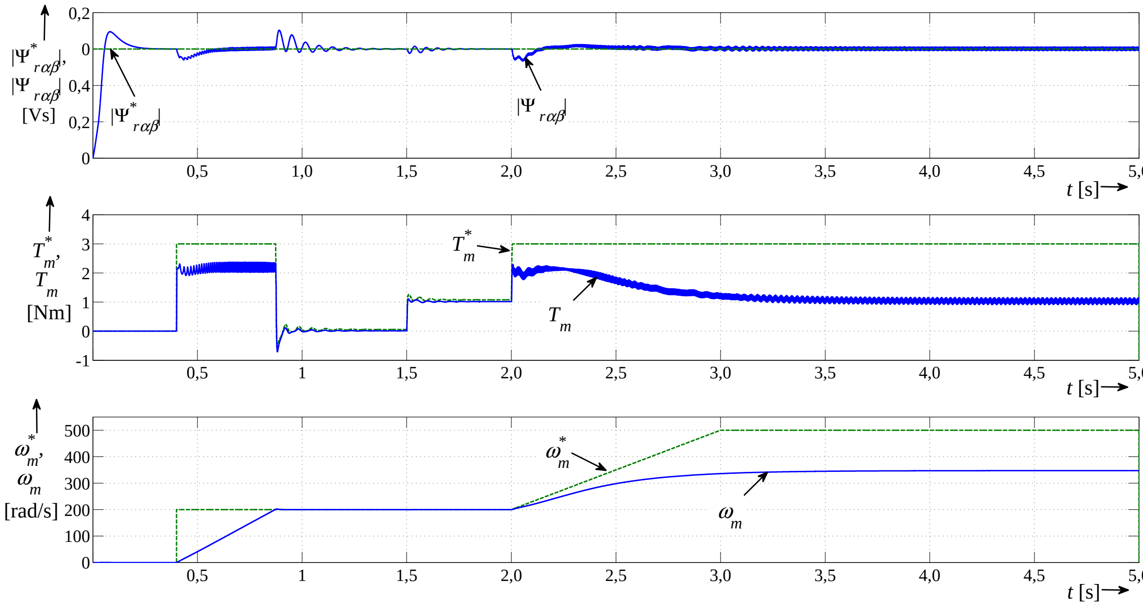

Figure 2: Simulated rotor flux amplitude, torque and speed response

5 CONCLUSION

New control algorithm for induction machine based on linear MPC was proposed in this paper. Algorithm emulates behavior of FOC without necessity of using the Park's transformation. Stator voltage and current limitation was set to utilize maximal possibilities of SVM modulation, which would be difficult to implement with classic PI controllers in stator coordinates. The functional principle of proposed algorithm was described in third section. Fourth section was dedicated to results of simulation in MATLAB-Simulink. Figure 2 shows very fast torque response of linear MPC controller. Future research will be focused on controller robustness and further optimisation to achieve low computational demands and possible future use on real motor.

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