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SOLUTION METHODS OF COMPOSITE BEAMS

SOLUTION METHODS OF COMPOSITE BEAMS

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Zásady pro vypracování

Solutions of composite beams encounter some specific problems, such as shrinkage and creep of concrete, cracking of concrete and plasticity of steel, partial interaction of elements or history of erection and loading process. These factors and others affect the distribution of internal forces along the beam, the distribution of stresses along the cross-section and also stiffness and deflection of beams.
The goal is to describe and compare methods for analysis of composite steel and concrete beams by more simplified approaches allowed by Eurocode with more advanced techniques. The studies will be carried out on the simply supported and continuous beams designed with respect to Eurocode 1, 2, 3 and 4.

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Vedoucí diplomové práce

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Solutions of composite beams encounter some specific problems, such as shrinkage and creep of concrete, cracking of concrete and plasticity of steel, partial interaction of elements or history of erection and loading process. These factors and others affect the distribution of internal forces along the beam, the distribution of stresses along the cross-section and also stiffness and deflection of beams.
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Prohlašuji, že jsem diplomovou práci zpracoval(a) samostatně, a že jsem uvedl(a) všechny použité, informační zdroje.

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podpis autora

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Prohlašuji, že elektronická forma odevzdané práce je shodná s odevzdanou listinnou formou.

V Brně dne 12.1.2012

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List of symbols

In this book, commonly-used symbols are listed in the format they appeared.

Latin letters

a	thickness of concrete slab
a_{ext}	quantity of external supports
a_{gap}	distance between concrete slab and the steel beam
a_{int}	the first end of the span (the start) in a numerical integral
a_s	concrete cover of the reinforcement
A	the whole area of IPE steel section
A_c	area of effective concrete slab section
A_{ci}	converted concrete slab cross-sectional area
A_i	area of the whole composite cross-section (with converted parts)
A_{si}	cross-sectional area of reinforcement
$A_{s,max}$	maximal reinforcement cross-section
$A_{s,min}$	minimal reinforcement cross-section
b	width of IPE flange
b_0	the distance between outer studs (in our case $b_0 = 0$ since we do not focus on studs effect).
b_{ci}	the converted effective width in the ideal cross-section
b_{ei}	the effective width of concrete slab on each side of the longitudinal axis of the beam.
b_{eff}	total effective width of the concrete slab.
b_{int}	the other end of the span (the very end) in a numerical integral
b_{si}	converted width of reinforcement according to n factor
C_1	constant that is determined by the boundary conditions
C_2	constant that is determined by the boundary conditions
d_{si}	the representative height of reinforcement
E	Steel modulus of elasticity

$E_{c,eff}$	reduction of Young modulus due to creep
E_{cm}	Concrete modulus of elasticity
E_d	design value of internal forces
EI_{iy}	stiffness of the whole composite section
$EI_{i,k+1}$	stiffness value of *i* hundredth (element) at *k+1* step
$E(t_i)$	modulus of elasticity of i time
$EI(x)$	Beam siffness at *x* point
f_{ck}	characteristic tensile strength of concrete
f_{ctm}	the mean value of the tensile strength of the concrete
f_{sk}	Reinforcement characteristic tensile strength
f_y	steel yield strength
h	the length of one hundredth of the span
h_0	the notional size of the member
h_a	distance from the middle of IPE section to upper concrete slab face.
h_{a1}	distance from the middle of steel cross-section above the neutral axis to the upper face of concrete slab
h_{a2}	distance from the middle of steel cross-section under the neutral axis to the upper face of concrete slab
h_{all}	overall depth of the whole section
h_i	the length of the web
h_{IPE}	the half of IPE section length
i	number of dividing elements
I_{iy}	the moment of inertia of the whole composite section along *y* axis
I_y	the moment of inertia of the steel section along *y* axis
l_{ab}	ab span length
L_e	is the equivalent length of the focused span
$M(x)$	Bending moment at *x* point in the simply supported beam
$\bar{M}(x)$	Fictional bending moment on the beam at *x* point

M_{Ed}	design bending moment
M_{cr}	critical moment of the whole cross-section
$M_{pl,Rd}$	resistance plastic bending moment
M_y	bending moment on y axis
n	the conversion factor, for which: $n = E / E_{cm}$
n_0	ratio Young modulus values of E_a/E_{cm} for a short-term loading
n_s	static indeterminacy level of a beam structure
N_a	force of the steel part of the composite cross-section
N_{a1}	force of the steel cross-section part above the neutral axis
N_{a2}	force of the steel cross-section part under the neutral axis
N_c	is the force of the concrete part of the cross-section
N_s	force of reinforcement in the section
q	continuous load along the beam
r	radius of curvature
r_{all}	centre of gravity of the whole section (Concrete + Steel + Reinforcement)
r_c	the centre of gravity of concrete section
r_{ci}	distance between the centre of gravity of concrete part to the centre of gravity of the whole cross-section
r_s	the centre of gravity of steel section
r_{as}	distance from the centre of gravity of reinforcement to the centre of gravity of the whole cross-section (Concrete + Steel + Reinforcement)
R_d	resistance value
t_f	thickness of IPE flange
t_w	thickness of IPE web
u	perimeter of a concrete member in contact with the atmosphere
w	deflection at x point
w_0	a helping value for deflection at x point
x	positioning of neutral axis from the upper face of concrete slab
x_i	the positioning of every hundredth of the span according to x axis

z	distance between the middle of IPE section and the upper face of it
z_c	distance from the centre of the whole (Concrete+Steel) cross-section to the upper face of concrete

Greek letters

α, β	basic deformation rotations of a simple beam
β_c	coefficient describing the development of creep with time after loading
γ_a	safety factor of steel
γ_s	concrete safety factor of reinforcement steel
ε_{uk}	the average characteristic strain of reinforcement under a maximal load
ρ	curvature
$\Delta\sigma(t_i)$	change if stress in i-time
σ_c	mean stress of the concrete acting on the part of the section
φ	cross-section deformation
φ_0	a helping value for rotation at x point
φ_{0creep}	the notional creep coefficient and is dependant on a coefficient related to the effect of the relative humidity
φt	creep coefficient, which is equivalent to $\varphi_{(t,t0)}$ defining creep between times t and t_0
$\varphi(t_i ,t_2)$	creep coefficient of time interval (t_i ,t_2)
Φ_{ba}	rotation of the whole beam cross-section at support b
ψ_L	factor of creep coefficient, which is dependant on load type. For permanent load it is equal to *1,1*

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Introduction

Composite beams have long been considered as most advantegeous structural members in construction field for its using the special features of the materials used which give many benefits from the statical, economical and many other factors that must be considered which eliminate the disadvantages the beams built up with each material alone can have.

First benefit and most important of all is using concrete to take most or all of the compression in time the steel takes all the tension, which in turn neglects the consideration of concrete’s low tensile strength. Another advantage that composite beams have is the rigidity joining the two parts together which makes the resulting system stronger than the sum of concrete and steel rigidities split apart.

The uniform reinforced concrete slabs make the spans to be economical once their thickness reach the sufficiency to resist the loads applied. For spans of more than few meters, it becomes cheaper to support the slabs by concrete beams. Once the spans are more than 10m, then steel beams become cheaper than concrete ones as it is in bridges and factories. For this the composite beams become challenging all these types of buildings with different slabs.

From the point of constructional realization view, reinforced concrete structures always need bedding for creating the shape the structural elements are supposed to have. Which needs lots of preparations and maintenance. In time the composite structures do not need a good care for bedding since a trapezoidal plate is set up for making the concrete slab connected to the steel beam, which takes the most of bedding job.

Once concrete is supported by a steel beam along the whole beam, then problems of obtaining possible cracks in the span are eliminated. A reason considering the composite beams stonger and reinforced concrete ones.

After all these factors written before make the composite construction competitive for all kind of structures with their different spans making no better combination for low cost, high strength and resistance.

2. Aims of the thesis

Structure‘s design is made up with the assumption of fulfilling safety and effectivity along their validity life. The two phenomenons of safety and effectivity could both reach the maximum with respect to each other by simply over-safe design, which in tern cannot be perfect in terms of economical view. To respect the three phenomenons for an optimal design, calculations of structures must include defects that must be faces.

In case of composite beams these defects must be related to physical and rheological properties of materials used. These defects must appear mainly over the internal supports of continuous composite beams specifically, where cracks occur on concrete part under the negative bending moments, and by this effect, another behaviour appears concerning the steel bar being plasticized at those supports.

All this leads to a change of bending moment distribution, which means a different behaviour of the beam. This redistribution is included in national codes, where its solutions are simply general and fixed for all cases. An approach that cannot be precise for every case specifically, that could be reached by more advanced mathematical methods of solution based on principles of elasticity and statics.

Working on such a problem is because of non-effective behaviour of all cases usually used. Where in simple composite beams the maximal bending moment occurs in the middle of the span that needs a higher grade of steel section that cannot be necessarily economic for the rest pars of the beam. In time the highest bending moment shows up over the internal support in the continuous beams, which is a massive disadvantage for concrete having cracks. The reason why an optimal bending moment distribution which is between the two previous cases must be reached to minimize all disadvantages possible.

Same proceed would be with the discussion of creep effect, where we evaluate the difference between the simple methods of national codes with more advanced ones using software ASTERES that is based on theory of finite element method and time discretization method.

By reaching the previous points we would get into economical design of continuous composite beams, highly safe and effective. Where in comparing with national code, these results obtained give us more precise bending moment redistribution allowed reaching the maximal values of safety, effectiveness and economical safe altogether.

3. Rules of design through Eurocode

There are two classes of limit states:

- Ultimate (denoted ULS), which are connected to the structural failure, whether by crushing, fatigue or overturning, and
- Serviceability (SLS), such as exaggerating deformation, vibration, or width of cracks in concrete.

3.1. Structure analysis

The special feature of composite elements is being built up by two materials, where the two connected parts to one another are supposed to act as a whole cross-section altogether. And to get into the perfect merged acting of the cross-section, there must be a need to respect a physical behaviour that appears once the steel-concrete composition is loaded.

This behaviour is so called the frictional deformation of concrete slab, which makes an influence of non-constant distribution of normal forces along the beam. For making the calculations more precise by respecting the mentioned effect, there will be a need to modify the effective width of concrete slab with reinforcement included in the composite beam since its material capacity influences the normal forces along its beam.

The effective width is obtained by the following relation (Studnička, J.):

$$b_{eff} = b_0 + \Sigma b_{ei} \qquad (2.1)$$

Where: b_0 the distance between outer studs (in our case $b_0 = 0$ since we do not focus on studs effect)

b_{ei} the effective width of concrete slab on each side of the longitudinal axis of the beam, where $b_{ei} = L_e/8$, *where* L_e *is the equivalent length of the focused span.*

For our case that we focus on two-span and three-span continuous beams, we will deal with side spans (2-span and 3-span beams), and also with a middle span (3-span beams). These cases are having equivalent lengths that are quite different from the real ones.

According to ČSN EN 1994-1-1 these lengths are obtained by these assumptions:

- The side spans have equivalent length of $L_e = 0{,}85.L_1$
- The middle spans have an equivalent length of $L_e = 0{,}7.L_2$

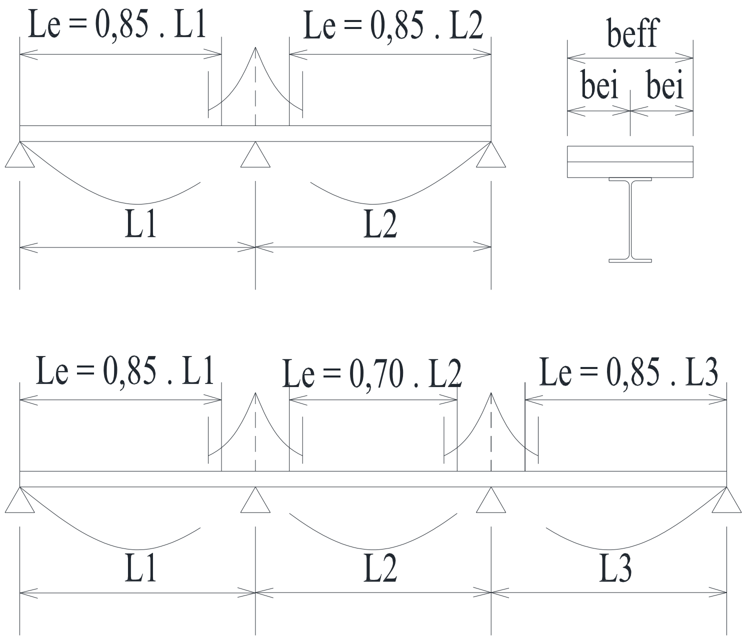


Figure 2.1: *Definition of the substituted length and its effective width*

Since we are counting with the effective width only because of the full effect of concrete slab in the composite beam, then we do not have to count with it over the supports where the negative bending moments show up and concrete is not taken into account. And the only effective material in the upper part of cross-section over the internal support is reinforcement, the reason for simplifying the calculations conditions, a fixed number of reinforcing bars is installed fulfilling only the constructional conditions of reinforcing due to *ČSN EN 1992-1-1*.

Composite beams, not as likely as their special material construct, do not get into behaviour perfection unless its material defects are included into account. One of the most obvious defects the composite beams face is the low concrete tensile strength, the time it concerns the continuous beams. As we know that concrete behaviour in tension is linear accompanied by its equivalent strain, where concrete over the supports still holds and the whole continuous beam has a constant cross-section for all its points, a situation of a beam that makes bending moments dependant only on applied loads. All this is valid until it gets to a point, where a critical stress that is equal to the mean value of axial tensile strength of concrete f_{ctm}.

Once this value is exceeded, then cracks occur and most expectedly over the internal supports. A behaviour that makes a part of the cross-section effective no more, the reason why the cross-section looses some of its stiffness. Which implicates that many parts of the whole beam will differ in their stiffness value according to whether parts of cross-section are defected or not.

Losses of stiffness values mean weakening of bearing features of the whole cross-section, which in tern changes the distribution of internal forces not only at the defected point, but on the whole beam as well. The more defected points along the beam defected, the more rechanges of internal forces distribution is obtained.

To reach the exact amount of defected points of the beam, an iterational process is used by a cycle that can be programmed with the base of principles of elasticity. An approach that makes the first iterations in assuming phase until they meet at a closest point by later iterations.

3.2. ULS

Starting with calculation process of loads and material characteristics we take into account the design values, that are obtained by design factors taken from *EN ČSN 1992-1-1* which is focusing on concrete structures and *EN ČSN 1993-1-1* which is focusing on steel ones.

The values starting the whole calculation are: the design values of permanent load (γ_q = 1,35), and variable load (γ_g = 1,5).

Same as it is with safety factors (γ_a = 1,0) for steel, and (γ_c = 1,5) for concrete.

After calculating the load effect on the beam and getting its reults, we get into a review that is always based on the following inequality (Studnička, J.):

$$E_d \leq R_d \tag{2.2}$$

Where:

E_d design value of internal forces

R_d resistance value

For our case that, where bending is the focused effect on our beam due to the applied loads and no other effect of normal or shear forces discussed, then our review must be directly related to bending effect. This makes the previous inequality to be mentioned as:

$$M_{Ed} \leq M_{pl,Rd} \tag{2.3}$$

Where:

M_{Ed} design bending moment

$M_{pl,Rd}$ resistance plastic bending moment

This inequality is set only with the assumption of total plasticization of cross-section in bending. A condition that is fulfilled for beams of class 1 and 2. These mentioned classes allow us to design the composite beam according to theory of plasticity, where the plastic neutral axis is obtained on the cross-section by the maximum usage of concrete and steel. This use must be obtained by the stress of concrete multiplied by the area under compression and by *0,85*. And to get into a safer design we take the worst case of combination of loads applied on the structure. We apply the previous checking on the composite beams by reviewing the critical bending moments, which are the maximal bending moments in the spans and over the internal supports that obtain a sudden change of cross-section.

3.3. SLS

For the serviceability limit state we consider deflection the base property, by which we evaluate the effects of concrete cracks over the supports, plasticization of steel bar and effects of creep and shrinkage. For calculating the deflection, principles of theory of elasticity are used.

As we mentioned before about redistribution of internal forces along the continuous beam due to concrete crack effects and steel plasticization, then deflections are also influenced by the previous effects. This behaviour is because of having different points along the beam with different stiffnesses. In our case we see this difference between points on the span and the ones ofer the supports, where the internal supports become points of weaker cross-sections in comparing with all other points of the structure.

For our examples we assume the shear connection is prefectly set so both parts (concrete and steel) act as one united cross-section altogether.

Another advantage of deflections is measuring the changes that could be made by changes of material properties. Since composite beams have concrete as a part building them up, then all what affects concrete properties, affects the composition. For this we use this advantage in measuring the effect of creep of concrete and shrinkage.

4. Utilized methods

4.1. Cross-section Resistance – Excel software programming

The concrete slab is connected to the steel beams by shear studs which give the concrete-steel complex the ability to work as one whole beam altogether. The point that the stress distribution along beam cross-section goes regarding the moment distribution on the beam and physical properties (strength) of each member in the composite beam. So the concrete slab is always in compression zone in the simple beams, while it must be in tension zone over the supports in the continuous beams where concrete is not taken into account and the longitudinal reinforcemen takes place instead.

According to the ultimate limit state, the structure must be designed such that will no collapse when the design load is subjected for which the basic review is the condition $E_d \leq R_d$ that must be fulfilled.

Where:

E_d is the design value of the internal force.

R_d is the resistance value of the internal force.

While calculating the internal forces all options of design values for load combinations are taken into account additional to the reduction factors of members and load safety factors.

The plastic analysis is parctically used in most cases once the steel cross-section is having the required rotation capacity to form the plastic hinge as it is with Class 1; or on cross-sections that even they have a limited rotation capacity, they can develop their plastic moment resistance which applies on steel cross-sections of Class 2. The two cross-section classes that form the plastic hinges using the sufficient rotation capacity to enable the development of redistribution of bending moments.

The following calculation explains the way Eurocode 3 defines the cross-section classification that depends on proportion of its height to its thickness (Where we use the cross-section of IPE type).

Where:

$$\varepsilon = (235 / f_y)\ 0{,}5 \qquad (3.1)$$

$$f_y = 235\ MPa \quad S235 \qquad (3.2)$$

$$\varepsilon = (235 / 235)0{,}5 = 1 \qquad (3.3)$$

$$c = h - 3\ .\ t \qquad (3.4)$$

$c / t \leq 72\ \varepsilon$ Element of Class 1 subjected to bending *(3.5)*

Table of appendix 6.1. leads us to a result that all IPE cross-sections included belong to CLASS 1 once affected by bending.

The plastic analysis approach allow us to reach the composite beam design starting from the force distribution along its cross-section regarding the conditions fulfillment of the required redistribution of bending moments to develop.

For choosing the optimal (most economical, most effective and safest) design, the following steps are applied on all IPE steel cross-sections mentioned in the table of (appendix 6.1.). Where later on the various positioning of the neutral axis that change the proportion of areas in tension and compression will be shown.

The very first condition to fulfill the stability in every cross-section must be the equality of forces obtained from stresses that are divided into a tension zone and a compression zone along the cross-section.

Here we get two different cases to fulfill this assumption. The first case is when the cross-section is located in the middle of the span which is making up positive bending moments for which the bottom steel part takes all tension in time concrete takes all compression. The second case is for the fibers located over the internal supports in which the distribution of stresses go differently than in the previous case since the bending moments over the internal supports are negative and make the upper concrete part to be under tension. The situation which is not the most favourable for concrete because of its low tensile strength (f_{ctk}) the reason why concrete is not taken into account but the used reinforcement is instead. The reinforcement has a big influence for taking all tension occured while it has a tiny effect in compression, the influence that is not counted.

4.1.1. Positive bending moment

For the first case where concrete is fully under compression. And to fulfill the equilibrium of distribution of internal forces, then we assume that: $F_c = N_a$ *(3.6)*

Where:

F_c is the force of the concrete part of the cross-section

N_a is the force of the steel part of the cross-section

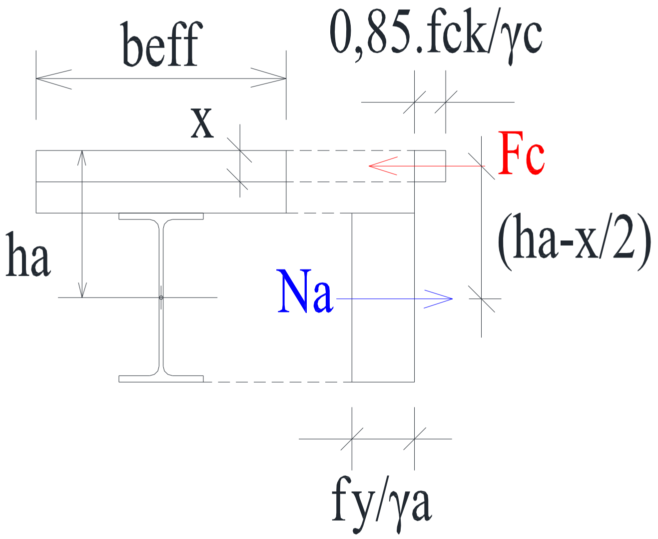


Figure 3.1: *Positive bending moment. The neutral axis crosses the concrete slab*

The total force of the steel cross-section is calculated by the multiplication of the characteristic tensile steel strength with the cross-sectional area respecting the safety factor:

$$N_a = f_y \,.\, A \,/\, \gamma_a \tag{3.7}$$

This force will be a helpful value for obtaining the real steel force values that divide N_a according to the positioning of neutral axis *(x)* which in tern divides the areas of tension and compression zones in the steel cross-section and it is obtained by this equation:

$$(A_a \,.\, f_y / \gamma_a) \quad = \quad (0{,}85 \,.\, f_{ck} \,.\, x \,.\, b_{eff} / \gamma_c) \tag{3.8}$$

Where: $(A_a \,.\, f_y \,/\, \gamma_a) = N_a$ and $(0{,}85 \,.\, f_{ck} \,.\, x \,.\, b_{eff} \,/\, \gamma_c) = F_c$ (3.9)

b_{eff} the effective width of the concrete slab.

But we have to note that $F_c = N_a$ is the equality where bigger steel cross-section needs more concrete area to keep the force equilibrium.

The last force equilibrium is valid only in case we want to get all the concrete section under compression in time steel is taking all the tension. The case where the neutral axis must be in the edge dividing concrete from steel or most likely it has to go through the gap where the trapezoidal steel plate is. A case that is not practical at some point especially once we need massive steel beams and thin concrete slabs where a part of the steel cross-section will be under compression.

To make the task more realistic to include more cases the value of the concrete slab thickness in the composite cross-section will be fixed by *a* . Where a along with N_a will be considered the start to calculate F_c .

We are going to count with every possible positioning of the neutral axis in the case of a cross-section that is in the middle of the span by having a fixed concrete slab thickness and maximizing the steel cross-section.

To include all the assumptions mentioned before there is a need to write a logical formula for calculating starting the use of formula

$$(A_a \,.\, fy / \gamma_a) = (0{,}85 \,.\, f_{ck} \,.\, x \,.\, b_{eff} / \gamma_c) \qquad (3.10)$$

where in our case $(A_a \,.\, f_y / \gamma_a)$ is the progressively increasing part and each of f_{ck} , b_{eff} and γ_c values will be fixed. Then only the value x will increase along with the force of the steel section.

By this we could assume that the neutral axis will start going through the concrete part with smaller cross-sectional steel areas and then its positioning moves lower with every bigger cross-sectional steel area we install than the previous ones until it crosses the steel web with the biggest steel sections used.

We can set the previous formula to get the increasing neutral axis x which is going to be compared with *a* (the concrete section detail) to the aim of knowing whether the whole concrete section will be under compression or not.

$$(N_a \,.\, \gamma_c) / (0{,}85 \,.\, f_{ck} \,.\, b_{eff}) \leq a \qquad (3.11)$$

must be the condition we start from and is equivalent to $x \leq a$ m . If this sentence was true, then $F_c = N_a$. Where N_a is not big enough to make a tension force covering the whole concrete compressive area, but is enough to cover just a part of the concrete section which is under compression. If the sentence was not true, it means the neutral axis crosses the steel section which in tern is divided to a part under tension and the other in the compression zone with the concrete section. At this time the whole concrete section will be under compression that is equal to

$$F_c = 0{,}85 \,.\, f_{ck} \,.\, b_{eff} \,.\, a / \gamma_c \qquad (3.12)$$

After we knew that the concrete section is divided accordnig to the position of the neutral axis, we see the steel section will be diveded for the same reason as well.

Here we will use a condition according to the forces obtained from the concrete section

$$F_c \geq 0{,}85 \,.\, b_{eff} \,.\, a \,.\, f_{ck} / \gamma_c \qquad (3.13)$$

which is a comparison of the obtained concrete force to the one representing the whole concrete section. In time it is true, the neutral axis is crossing the steel part and the tensile steel force is

$$N_{a2} = (N_a - F_c) \,.\, 0{,}5 + F_c \qquad (3.14)$$

Where $(N_a - F_c)$ is the difference between the steel tensile force of its whole section and concrete compressive force in the composite cross-section. This difference is split to two halves and each of them is added to one part of the cross-section. In case the inequality is not true, then the neutral axis is not crossing the steel section and it is taking the whole tensile force then $N_{a2} = N_a$.

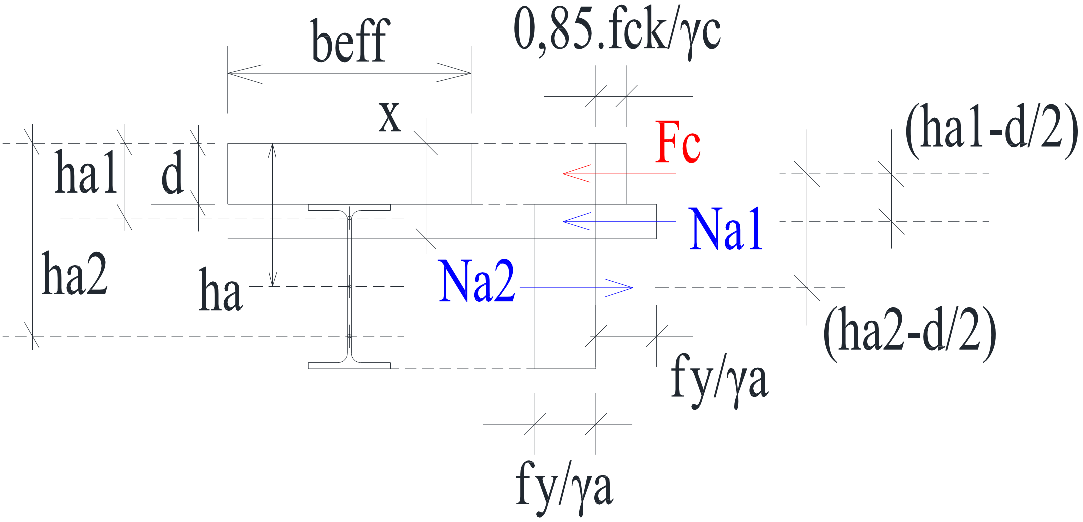


Figure 3.2: *Positive bending moment, for which the neutral axis crosses steel section*

For the rest of the steel cross-section will be under compression and this must be the difference of what is left from forces in the composite cross-section that can be expressed by: $N_{a1} = N_{a2} - F_c$ where N_{a1} would equal to zero in case the concrete part is taking the whole compression in time it will have a value once the neutral axis crosses the flange or the web of the steel part.

After setting the force distribution along the cross-section, the situation of the neutral axis can be defined. And by going back to the conditions we went through before the use of the force equality must be always fulfilled. For the case where concrete takes the whole compression and steel takes the whole tension $(N_a / F_c) = 1$ means the neutral axis goes through the edge of conrete part that

$$x = F_c / (0{,}85 \,.\, b_{eff} \,.\, f_{ck} / \gamma_c) \qquad (3.15)$$

In time if $(N_a / F_c) = 1$ is not true, then we know the neutral axis crosses the steel section. For making the positioning calculation more precise, defining areas of the flange and the web of the steel section on which the cross-sectional force is distributed must be taken into account. A condition is set for this purpose

$$A - [(N_{a2} / N_a) \,.\, A] \leq t_f \,.\, b \qquad (3.16)$$

Where

$t_f \cdot b$ area of the flange of IPE section

A the whole area of the steel section and

$(N_{a2} / N_a) \cdot A$ must be the steel area that holds tension and is under the neutral axis which makes the difference

$A - [(N_{a2} / N_a) \cdot A]$ steel area under compression that is a part of IPE section.

The condition $A - [(N_{a2} / N_a) \cdot A] \leq t_f \cdot b$ will be used for the comparison. In case it is true then the neutral axis goes through the flange will be

$$x = \{[A - (N_{a2} / N_a) \cdot A] / b\} + a + a_{gap} \qquad (3.17)$$

in time the condition will not be fulfilled, then the neutral axis goes through the web which means:

$$x = \{A - [(N_{a2} / N_a) \cdot A + (t_f \cdot b)]\} / t_w + t_f + a + a_{gap} \qquad (3.18)$$

For getting resistance bending moments the lengths needed were obtained for every material force of the cross-section. And as we knew from the values obtained before, resistance moments will differ in value according to the location of the neutral axis that has two possibilities which must be programmed as follows:

- For cross-sections of lower IPE designations make the neutral axis go through the concrete part as expected. Which implicates the needed force for making the resistance moment comes from the whole IPE section or the equivalent force from the concrete slab. The two equivalent forces for which we need along with the arm (their distance to each other). This evidence can be calculated by:

  $h_a = a_{gap} + a + h_{IPE} \cdot 0,5$ distance from the middle of the tensed area (whole IPE section for this case) to upper concrete slab face.

  $h_a – x/2$ the arm

- For cross-sections of greater IPE designations make the neutral axis get through the steel part to fulfill the equality of internal forces of the composite cross-section. Which divides the steel section to two parts (one in tension that has the greatest force N_{a2} and the other in compression of N_{a1} which is added to compressive concrete force N_c from the slab for making the force equality) that leads to take the two steel forces into account for calculating the resistance bending moment. And for this two arms are needed.

First, we will explain the way of obtaining the distance of steel forces (N_{a1} and N_{a2}) to the concrete face:

- For the distance of the compressive force in the steel section N_{a1} to the concrete upper face h_{a1} we would give conditions since different location of the neutral axis will affect the calculations. In case the neutral axis goes through the upper flange of IPE section according which we write the two logical sentences:

$$x > a_{gap} + a \quad \text{and} \quad x \leq a_{gap} + a + t_f \tag{3.19}$$

Then for fulfilling these conditions the following equation is true:

$$h_{a1} = a_{gap} + a + 0{,}5 \,.\, [x-(a_{gap}+a)] \tag{3.20}$$

Where:

$x - (a_{gap} + a)$ the thickness of the compressed steel area that is a part of the flange.

In time that the neutral axis could go even through the web, the following condition must be fullfilled:

$$x > a_{gap} + a + t_f \tag{3.21}$$

It is obvious here that x must include all the flanges area and h_{a1} must equal the following:

$$h_{a1}= t_f \,.\, b*(t_f \,.\, 0{,}5 + a_{gap} + a) + \{t_w \,.\, [x - (a_{gap} + a + t_f)] \,.\, [a_{gap} + a + t_f + (x - (a_{gap} + a + t_f)) \,.\, 0{,}5]\} \,/\, \{t_f \,.\, b + t_w \,.\, [x - (a_{gap} + a + t_f)]\} \tag{3.22}$$

For the purpose of getting the distance between the exact middle of the steel compressed area and the upper concrete face we have to consider the fact that the compressed part became the flange with an additional piece of the web where we are must apply the following:

*[(Area of the Flange * centre of the flange to the concrete face) + (Area of the add. compressed web part * centre of the web part to the concrete face)] / (area of the flange + area of the web part)*

As we can see:

$t_f \,.\, b$	Area of the Flange
$t_f \,.\, 0{,}5 + a_{gap} + a$	Centre of the flange to the concrete face
$t_w \,.\, [x - (a_{gap} + a + t_f)]$	Area of the add. compressed web part
$a_{gap} + a + tf + [x - (agap + a + tf)] \,.\, 0{,}5$	Centre of the web part to the concrete face

- In the tensile part of IPE section the distance of the resulting force to the concrete face will be calculated according to the same conditions we used for h_{a1}. The conditions come from the difference of neutral axis location in IPE section whether it is on the flange or on the web.

For the neutral axis locating on the flange we will use the conditions

$$x > a_{gap} + a \qquad \text{and} \qquad x \leq a_{gap} + a + t_f \qquad (3.19)$$

Due to these conditions we will obtain h_{a2} by defining the distance from the upper face of concrete slab to the centre of an IPE cross-section that has a whole bottom flange, a web and a part of the upper flange that is calculated

$$h_{a2} = \{[t_f \,.\, b \,.\, (a_{gap} + a + h_{IPE} - 0{,}5 \,.\, t_f)] + [t_w \,.\, h_i \,.\, (h_i \,.\, 0{,}5 + a_{gap} + a + t_f)] + [(a_{gap} + a + t_f - x) \,.\, b \,.\, ((a_{gap} + a + t_f - x) \,.\, 0{,}5 + x)]\} / \{t_f \,.\, b + t_w \,.\, h_i + [a_{gap} + a + t_f - x] \,.\, b\} \qquad (3.23)$$

Where:

$t_f \,.\, b$ Area of the bottom flange

$(a_{gap} + a + h_{IPE} - 0{,}5 \,.\, t_f)$ Distance of the centre of the bottom flange to the upper face of concrete slab.

$t_w \,.\, h_i$ Area of the web

$h_i \,.\, 0{,}5 + a_{gap} + a + t_f$ Distance between the centre of the flange to the upper face of concrete slab.

$(a_{gap} + a + t_f - x) \,.\, b$ Area of the part of the upper flange that is under tension

$(a_{gap} + a + t_f - x) \,.\, 0{,}5 + x$ Distance between the centre of the tensed part of the upper flange and the concrete slab face.

After obtaining all the parameters we need for getting the resistance bending moment of all cases possible to appear according the positioning of the neutral axis we start calculating the resistance bending moment by a condition:

$$x \leq a \qquad (3.24)$$

Which makes the neutral axis go through the concrete slab by which we get:

$$M_{pl,rd} = N_{a2} \,.\, [h_a - x \,.\, 0{,}5] \qquad (3.25)$$

Where the whole steel section owns a tensile force N_{a2} that is equal to the compressive force of the concrete slab. But this appears only in composite cross-sections that have lower IPE designations in time it looks different with the sections of higher IPE designations where we find the resistance bending moment by:

$$M_{pl,rd} = -N_{a1} \,.\, [h_{a1} - a \,.\, 0{,}5] + N_{a2} \,.\, [h_{a2} - a \,.\, 0{,}5] \qquad (3.26)$$

Where:

N_{a1} Compressive force of IPE steel section

$h_{a1} - a \,.\, 0{,}5$ The compressive force arm

N_{a2} Tensile force of IPE steel section

h_{a2} - a . $0{,}5$ The tensile force arm

In this formula we see that the equality of internal forces along the composite cross-section is applied since there are two parts of the IPE section that are in compression and tension. Where we see $N_{a1} \cdot (h_{a1} - a \cdot 0{,}5)$ is the moment of the compressive steel part in time $N_{a2} \cdot (h_{a2} - a \cdot 0{,}5)$ is the tensile moment of the same section. The fact shows that the rest of what the subtraction gives represents the bending moment what the composite cross-section can hold.

4.1.2. Negative bending moment

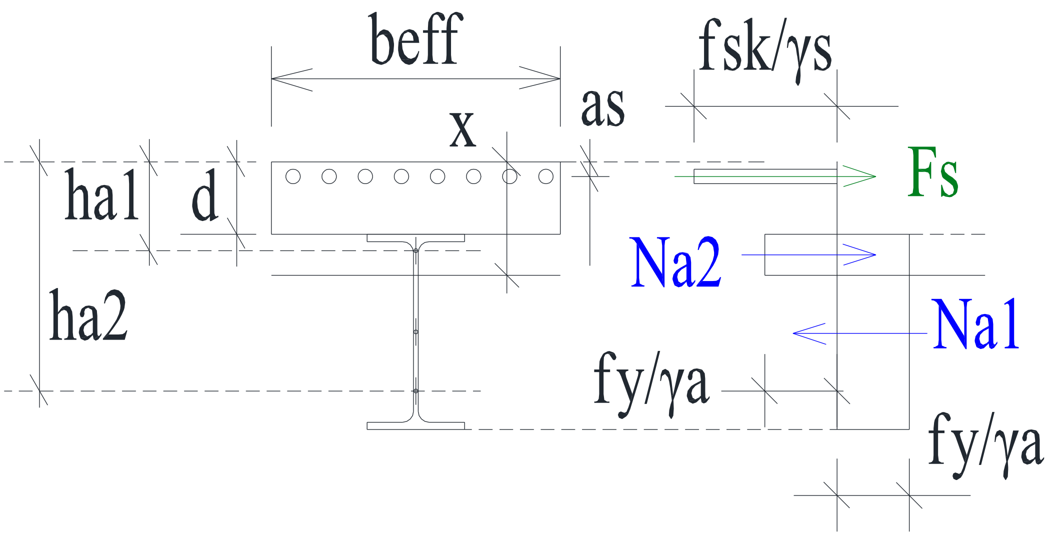


Figure 3.3: *Negative bending moment, for which the neutral axis crosses steel section*

The previous explanation was to define the resistance needed in the span where the bending moments are positive due to loading. So how about the parts of continuous load that are located over the internal supports.

For this we have to remember once the continuous beam is subjected by a continuous load, then we get negative bending moments over the internal supports. Which means the upper fibers where the concrete slab is under tension and the bottom ones (the steel beam) are under compression. A situation that might be critical to the concrete slab since it does not need a big deal of tension to crack. The reason we would not count with concrete tensile strength but with reinforcement bars tensile strength instead.

According to the basis of what is written before we do the following:

We calcutale the cross-sectional area of concrete slab that is a part of the composite beam by:

$$A_c = a \cdot b_{eff} \qquad (3.27)$$

Where:

A thikcness of the concrete slab

b_{eff} the effective width of the concrete slab

And to define the reinforcement needed we should know the maximum and the minimum limits of reinforcement available for the composite beam. Where we use the the formulas take from Eurocode 1992.

For the maximum reinforcement we write: $A_{s,max} = 0{,}04 * A_c$ (3.28)

For the minumum reinforcement we write: $A_{s,min} = (A_c \,.\, f_{ctm} \,.\, k \,.\, k_c) / f_{sk}$ (3.29)

Where:

f_{sk} Reinforcement characteristic tensile strength

A_c Area of concrete under the tension zone. Which is the part of the section that is calculated to be in tension before the formation of the first crack.

f_{ctm} the mean value of the tensile strength of the concrete

$k_c = 0{,}4 \,.\, [1 + \sigma_c / \{k_1 \,.\, (h_{all} / h^*) \,.\, f_{ctm}\}] \leq 1$ is used for bending

Where:

σ_c mean stress of the concrete acting on the part of the section under consideration (σ_c < 0 for compression force): $\sigma_c = N_{Ed} / b_h$

h_{all} overall depth of the section

h^* $h^* = h_{all}$ for $h_{all} < 1{,}0\ m$

$h^* = 1{,}0\ m$ for $h_{all} \geq 1{,}0\ m$

k_1 a coefficient considering the effects of axial forces on the stress distribution:

$k_1 = 1{,}5$ if N_{Ed} is a compressive force

$k_1 = 2h^*/3h$ if N_{Ed} is a tensile force

After getting the maximum and the minimum limit of the cross-sectional reinforcement area available to be installed in the effective concrete slab we choose A_s that is a value fulfilling the constructional reinforcement area.

For obtaining the distribution of the internal forces in the composite cross-section that is unknown especially in the steel section that is dependent on the reinforcing force from which we will find it out. And we know in advance we can use tha fact the reinforcement is in tension zone while the major part of IPE steel section is under compression.

The start can be from the first known force in tension zone that comes from the chosen reinforcement which is calculated by following:

$$N_s = A_s \, . \, f_{ck} \, / \, \gamma_s \qquad (3.30)$$

Where:

A_s the chosen reinforcement cross-sectional area

f_{ck} characteristic tensile strength of concrete

γ_s concrete safety factor of reinforcement steel, $\gamma s = 1{,}15$

Another known value can be used along with the obtained previous force is the nominal yield force of whole IPE section from:

$$N_a = A_a \, . \, f_y \, / \, \gamma_a \qquad (3.31)$$

Where:

A_a the used IPE section in the composite beam.

f_y steel yield strength

γ_a safety factor of steel, $\gamma a = 1{,}0$

But this force value is divided to tension and compression zones, and it is directly dependent on how much reinforcement we have in the composite cross-section.

From the equality of the internal forces in the cross-section we know we must obtain the fact that all compression forces must equal the tension ones. For this we have:

$$N_{a2} = N_{a1} + N_s \qquad (3.32)$$

Where:

N_s reinforcement tensile force

N_{a1} resulting force coming from the part of IPE section that is under compression

N_{a2} resulting force from the rest of IPE section that is under tension

From the base of previous forces equation and its conservation in the cross-section we have to define how the total steel force N_a is divided. We could reach the right division if we look at the picture of the cross-section then we see that the reinforcement is not the whole force taking the compression which is in need for more from the steel section to make equilibrium. In time the tensile force of the steel does not cover its whole section which shows us the fact it must be reduced to a point it will fulfill the equality. Where it proves the equality of internal forces in the composite section might be reached by every increase of the tensile force to the reinforcement that is taken from the total compressive force of the steel section that is decreased. Then by which the neutral axis positioning will be defined.

This compression force increase that equals the tension force decrease in the same section should be coming from the difference of the total material forces of what the composite beam is made up that is expressed by:

$$\Delta N = N_a - N_s \tag{3.33}$$

Where:

N_s reinforcement tensile force

N_a total resulting force of IPE steel section

$$N_a = N_{a1} - N_{a2} \tag{3.34}$$

Since we have two stress cases that will always appear making the force equality in the same cross-section with whichever materials it is made up, then the force difference of the materials involved must be halved and every half in turn is added to the rest of compression and tension forces.

A solution that can be easily applied later on for getting the final tesile force in the cross-section of composite beam by:

$$N_{a2} = (N_a - N_s) \,.\, 0{,}5 + N_s \tag{3.35}$$

Where:

$(N_a - N_s) \,.\, 0{,}5$ is the additional compression force that is added from the force difference between IPE steel section and the reinforcement to make the internal forces equality.

N_s reinforcement tensile force which in this formula must equal the part of steel section in compression without the additional force coming from the difference.

The rest of what the steel section can express as a force might be calculated by:

$$N_{a1} = N_{a2} - N_s \tag{3.36}$$

$$N_{a1} = N_a - (N_a - N_s) \,.\, 0{,}5 \tag{3.37}$$

After getting the plastic distribution of the internal forces along the cross-section, then we will be obviously having the point where the internal forces jump from the tension zone to compression one which is the positioning of the neutral axis.

For the accuracy of defining the neutral axis positioning we need to differ the entry of calculation according to the steel tension and compression forces covering its cross-section area. This definition difference must be a condition expressed by:

$$A - [(N_{a2} / N_a) . A] \leq t_f . b \qquad (3.38)$$

Where:

$(N_{a2} / N_a) . A$ the tensile area of IPE section

$A - [(N_{a2} / N_a) . A]$ the compressive area of IPE section

$t_f . b$ area of the upper flange of IPE section

If the previous inequality is true, then the compressed area is smaller than the upper flange and the neutral axis is not crossing over its thickness. Which means the neutral axis can be calculated by the following:

$$x = \{[A - ((N_{a2} / N_a) . A)] / b\} + a + a_{gap} \qquad (3.39)$$

Where:

$[A - ((N_{a2} / N_a) . A)] / b$ the thickness of the compressed steel area that is a part of the upper flange.

Once the previous inequality is false, then the compressed area is larger than the area of the upper flange and the neutral axis crosses the web which is calculated by the following:

$$x = \{ [(N_{a1} / N_a) . A - (t_f . b)] / t_w \} + t_f + a + a_{gap} \qquad (3.40)$$

Where:

$(N_{a1} / N_a) . A$ the compressive part of whole IPE section

$(N_{a1} / N_a) . A - (t_f . b)$ the compressive part of the web in IPE section

$[(N_{a1} / N_a) . A - (t_f . b)] / t_w$ the height of the compressive part of IPE section web

After getting the positioning of the neutral axis we can define the centre of tension area of IPE section and the centre of the steel compression area as well. Which both have to face the condition whether the neutral axis crosses the steel flange or the web.

We could define the condition by:

$$x > a + a_{gap} + t_f \qquad (3.41)$$

By this condition we can define the centre of tension and compression steel areas with their varieties, and starting with the centre of the compression steel section we can see once the inequation is true then the neutral axis crosses the web. The fact that the compression zone includes the whole upper flange and a part of the web.

Defining distance from the centre of gravity of this area to the concrete slab face will be expressed by:

h_{a1} = {flange area . dist. from the centre of the flange to the concrete face + area of compressed web part . dist. from the centre of the compressed web part to the concrete face} / {flange area + area of compressed web part}

In symbols:

$h_{a1} = \{t_f . b . (t_f . 0{,}5 + a_{gap} + a) + t_w . [x - (a + a_{gap} + t_f)] . [a + a_{gap} + t_f + (x - (a + a_{gap} + t_f)) . 0{,}5]\} / \{t_f . b + t_w . [x - (a + a_{gap} + t_f)]\}$ (3.42)

Where:

$t_f . b$area of the upper flange of IPE section

$(t_f . 0{,}5 + a_{gap} + a)$ distance from the centre of the upper flange to the face of concrete slab.

$x - (a + a_{gap} + t_f)$ height of the part of the web that is under compression

$t_w . x - (a + a_{gap} + t_f)$ area of the part of the web that is under compression

$a + a_{gap} + t_f + [x - (a + a_{gap} + t_f)]*0{,}5$ distance from the centre of the web part that is under compression to the concrete slab face

And once the previous inequality is false, then the neutral axis crosses the flange which will make the distance of the centre of gravity of compressed area to the concrete slab face will be obtained by:

$$h_{a1} = 0{,}5 . [x - (a + a_{gap})] + a_{gap} + a \quad (3.43)$$

$x - (a + a_{gap})$ height of the upper IPE steel flange under compression

For the distance from the centre of gravity of the tension steel part to the concrete slab face we will use the same inequality:

$$x > a + a_{gap} + t_f \quad (3.44)$$

Once it is true, then we are having two areas to calculate their areas and their centres of gravity distances to the concrete slab face where we express by:

$h_{a2} =\{t_f . b . [a + a_{gap} + h_{IPE} – t_f . 0{,}5] + [a + a_{gap} + h_{IPE} – x - t_f] . t_w . [(a + a_{gap} + h_{IPE} – x - t_f) . 0{,}5 + x]\} / \{t_f . b + [a + a_{gap} + h_{IPE} – x - t_f] . t_w\}$ (3.45)

Where:

$t_f . b$ area of bottom IPE section flange

$a + a_{gap} + h_{IPE} – t_f . 0{,}5$ distance from the centre of gravity of the bottom flange to the concrete slab face.

$(a + a_{gap} + h_{IPE} – x – t_f) . t_w$ area of the tension part of the web

$(a + a_{gap} + h_{IPE} - x - t_f) . 0,5 + x$ distance between the centre of gravity of the tension part of the web to the concrete slab face

And when the inequality is false, then the neutral axis goes through the upper flange. Which means we have to consider the areas of the bottom flange of IPE cross-section, the web and a part of the upper flange and their centres of gravity.

For the areas mentioned we obtain the centre of gravity of tension part of IPE cross-section and its distance from the face of concrete slab by:

$$h_{a2} = (\{t_f . b . [a + a_{gap} + h_{IPE} – t_f . 0,5]\} + \{t_w . h_i . [(0,5 . h_i) + a_{gap} + a]\} + \{[(a + a_{gap} + t_f) - x] . b . [((a + a_{gap} + t_f) - x) . 0,5 + x] \}) / ([(a + a_{gap} + t_f) - x] . b + t_w . h_i + t_f . b) \quad (3.46)$$

$t_f . b$ area of the bottom flange of IPE cross-section

$a + a_{gap} + h_{IPE} – t_f . 0,5$ distance from the centre of gravity of the bottom flange to the concrete slab face.

$t_w . h_i$ area of the web

$(0,5 . h_i) + a_{gap} + a$ distance from the middle of the web to concrete slab face

$[(a + a_{gap} + t_f) - x] . b$ area of the tension part of the upper flange

$[((a + a_{gap} + t_f) - x) . 0,5 + x]$ distance between the middle of tesion part of the upper flange to concrete slab face

After these calculations we have all the forces and distances needed to get the resistance bending moment by:

$$M_{pl,rd} = -N_{a1} . (h_{a1} - a_s) + N_{a2} . (h_{a2} - a_s) \quad (3.47)$$

$h_{a1} - a_s$ the arm of the steel compression force in the composite cross-section

$h_{a2} - a_s$ the arm of the steel tension force in the composite cross-section

4.2. Cross-Section Stiffnesses Investigation – Excel software programming

The continuous beam along which is having different values of bending moments because of its supports and the loading subjected. The fact that makes the material behaviour of its cross-section changing according to the stresses resulting from the bending moments along the beam. This behaviour could be represented by simplified stiffnesses as follows:

4.2.1. CONCRETE+STEEL (1):

This section is the typical cross-section of IPE steel section that is bounded with the concrete slab by shear struts so this composition will behave as whole. A system that perfectly works because of concrete's high compressive strength that makes is highly resistant to compression since it is in the upper side of the section. In time steel has a high tensile strength that resists the high values of bending moments. The reason why this cross-section is considered as an ideal section used into account for the middle of the span.

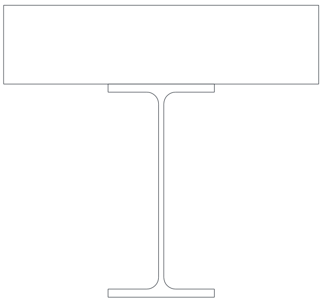


Figure 3.4: *Typical composite cross-section in the span under positive bending moments*

For this we could calculate the stiffness of this section by the following:

We will perform the ideal cross-section by converting the concrete part and make it as steel to simplify the calculations through:

$$b_{ci} = b_{eff} / n \qquad (3.48)$$

Where:

b_{eff} the effective width of concrete slab which is acting as a part of the composite beam

n the conversion factor, for which: $n = E / E_{cm}$

Where:

- E Steel modulus of elasticity
- E_{cm} Concrete modulus of elasticity

Which also changes the concrete area to an ideal area by:

$$A_{ci} = a \,.\, b_{ci} \qquad (3.49)$$

a the thickness of concrete slab

Afterwards we get the whole cross-section area by the sum:

$$A_i = A_{ci} + A \qquad (3.50)$$

A IPE cross-sectional area

To obtain the moment of inertia, we would need to define the centre of gravity of the whole section and its distance from the centres of both steel and concrete sections.

For the distance between IPE steel centre of gravity and the composite one will be calculated as follows:

For the centre of gravity of steel section:

$$r_s = (A_{ci} / A_i) \,.\, (0{,}5 \,.\, h_{IPE} + 0{,}5 \,.\, a + a_{gap}) \tag{3.51}$$

For the centre of gravity of concrete section:

$$r_c = 0{,}5 \,.\, h_{IPE} + 0{,}5 \,.\, a + a_{gap} - r_s \tag{3.52}$$

Then the moment of intertia is defined as follows:

$$I_{iy} = I_y + A \,.\, r_s^2 + (1/12) \,.\, b_{ci} \,.\, a^3 + A_{ci} \,.\, r_c^2 \tag{3.53}$$

After getting the necessary values we only need to multiply the obtained moment of inertia with the steel modulus of elasticity as we can write:

$$EI_{iy} = I_{iy} \,.\, E \tag{3.54}$$

4.2.2. RFCMT+STEEL (3):

The previous stiffness is perfectly fitting the sections that are in the middle of the span, but it can not fit along the whole continuous beam specially over the internal supports. Where we obtain the negative bending moment that makes tension in the upper part of the section where concrete is. A situation that is not the best because of concrete's low tensile strength that becomes the reason of absolute cracking occurence specially after the system exceeds the critical bending moment of the section. Which in turn there will be a need to count with reinforcement to stand the tension while we eliminate concrete's effect.

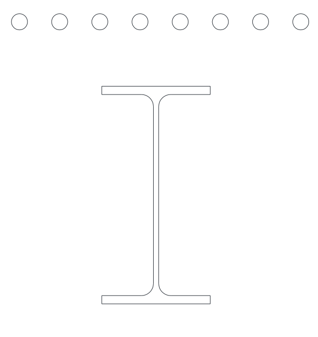


Figure 3.5: *Typical composite cross-section over the support exceeding M_{cr}*

For the area of reinforcement we will tranform it to a rectangular area that will simplify our calculations where we consider the reinforcing bar diameter as thickness of the new reinforcing rectangle. Which means we can get the width by:

$$b_{si} = A_{si} / d_{si} \tag{3.55}$$

d_{si} the representative height of reinforcement

Equation *(3.55)* is added only for more precision of claculations, otherwise there is no need to count with it.

The total cross-sectional area is obtained by the sum:

$$A_i = A_{si} + A \qquad (3.56)$$

A IPE section

The distance between the centre of gravity of tensile part from the centre of gravity of the whole cross-section is:

$$r_s = (A_{si} / A_i) \,.\, [0{,}5 \,.\, h_{IPE} + a + a_{gap} - a_s\,] \qquad (3.57)$$

Where:

$a + a_{gap} - a_s$ distance between the centre of gravity of the reinforcing rectangle

As the distance between the centre of gravity of the compression part from the centre of gravity of the whole cross-section becomes:

$$r_c = 0{,}5 \,.\, h_{IPE} + a + a_{gap} - (r_s + a_s) \qquad (3.58)$$

After the distances obtained we will get the whole moment of inertia of the corresponding section which is:

$$I_{iy} = I_y + A \,.\, r_s^2 + A_{si} \,.\, r_c^2 \qquad (3.59)$$

4.2.3. CONCRETE+RFCMT+STEEL (2):

The last section (Section 3) was for once the beam exceeds the critical negative moment that makes the concrete part uneffective which was eliminated in calculation. In time (Section 1) was the set up for the bending in the span where the bending moments are positive and there was no need for using the reinforcement for concrete that is under compression.

Those two situations are considered to be extreme since the beam could reach a position in between the previous two. When the beam is under the negative bending moment, but still not exceeding the critical moment, a small interval where concrete in the composite beam still holds under tension without getting cracked with the elongated reinforcing bars that surve avoidance the cracking the exceeded critical tension over the internal support.

This section which is suitable for the such a position must include its all materials in calculations as it went with the previous sections.

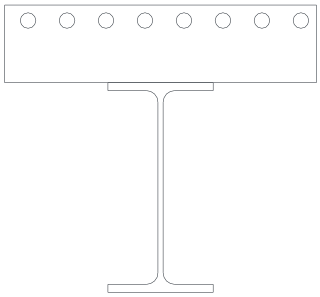


Figure 3.6: *Typical composite cross-section under negative bending moment that is not exceeding M_{cr}*

The cross-sectional area must be the sum of all materials apart which is:

$$A_i = A_{si}(x\text{-section } 3) + A_{ci}(x\text{-section } 1) + A \qquad (3.60)$$

For this cross-section there is an imaginary distance used:

$$r_{all} = \{A.[0,5.h_{IPE}+a+a_{gap}]+A_{ci}(x\text{-section } 1).0,5.a + A_{si}(x\text{-section } 3).(a_s+0,5.d_{si})\} / \{A + A_{ci}(x\text{-section } 1) + A_{si}(x\text{-section } 3)\} \qquad (3.61)$$

Then the distance between the centre of gravity of the tensile area and the centre of gravity of the whole cross-section becomes:

$$r_{si} = h_{IPE} + a + a_{gap} - (r + 0,5 \,.\, h_{IPE}) \qquad (3.62)$$

In time the other distance that concern the compression area is:

$$r_{ci} = \text{Absolute value of } (r - 0,5 \,.\, a) \qquad (3.63)$$

$$r_{as} = r_{all} - (a_s + 0,5 \,.\, d_{si}) \qquad (3.64)$$

$$I_{iy} = I_y + A \,.\, r_s^2 + (1/12) \,.\, b_{ci} \,.\, a^3 + A_{ci} \,.\, r_{ci}^2 + A_{si} \,.\, r_{as}^2 \qquad (3.65)$$

By this we just have obtained three stiffnesses of these states:

1- Concrete + Steel : where the cross-section does not reach the a negative bending moment nor the critical moment.

2- Concrete + Reinforcement + Steel : where the cross-section is under the bending moment effect, but not the critical one.

3- Reinforcement + Steel : where cross-section is under the critical moment and concrete could not be counted in tension.

4.3. Internal Forces Redistribution – Excel software programming

4.3.1. Force method – linear steps:

A continuous beam is a term meant by a beam that is supported on more than two supports where one of them could be fixed support (normal support or a fixed end) and the rest are sliding supports. In our case we would consider the vertically applied load on the beam, its shear forces and bending moments effects as the only effects along the continuous beam regardless the normal forces effect since it has no influence on the bending moments of the vertical axis to it. For which we will have no need to consider the forces along the beam axis which are in this case the normal forces. The point that leads us not to consider supports for the normal forces which makes all the active supports of the continuous beam sliding in time the only normal support on the side of the beam is not active since there is no normal load to hold. But it must be installed there to fulfill the definition conditions of the continuous beam.

- **Static indeterminacy:**

We consider the static indeterminacy by the following equation:

$$n_s = a_{ext} – 3 - p_k \qquad (3.66)$$

Where:

A	number of supports in the statical system
p_k	number of the internal hinges

Since we have no internal hinges in the continuous beam, then we could mention it in the previous equation as $p_k = 0$ which gives us the indeterminacy equation as follows:

$$n_s = a_{ext} – 3 \qquad (3.67)$$

By the given equation we can define the indeterminacy level of the continuous beam we deal with.

For our case we will have two cases:

- A two-span continuous beam, where we will have three sliding supports which means we need three vertical supports that make $a_{ext} = 4$ (3 vertical + 1 horizontal). A state when indeterminacy is $n_s = 4 – 3 =1$
- A three-span continuous beam that is similar to the previous case except having an additional sliding support, for which $a_{ext} = 5$ (4 vertical + 1 horizontal) that makes the indeterminacy as follows: $n_s = 5 – 3 =2$

Three-moment equation method:

This method is considered as force method since the deformation conditions are set up to maintain a group of linear equations as the a step to obtain the unknown forces and bending moments. A step that leads to results described by support reactions and internal forces.

While setting the deformation conditions on a continuous beam with no break we consider that any point with an internal support on the beam fulfills the following:

$$\Phi_{ba} = - \Phi_{bc} \tag{3.68}$$

When:

Φ_{ba} , Φ_{bc} are tangent slopes of the deflected line at the sections of the exact left and exact right of the support *b* and are obtained by (Kadlčák, J; Kytýr, J.):

$$\Phi_{ba} = M_a \,.\, \beta_{ba} + M_b \,.\, \alpha_{ba} + \varphi_{ba} \tag{3.69}$$

$$\Phi_{bc} = M_b \,.\, \alpha_{bc} + M_c \,.\, \beta_{bc} + \varphi_{bc} \tag{3.70}$$

The deformation angles are attached by two indeces and a sign. The first index stands for the first end point of the field for which rotation we try to obtain. The second index stands for the other end point of the field.

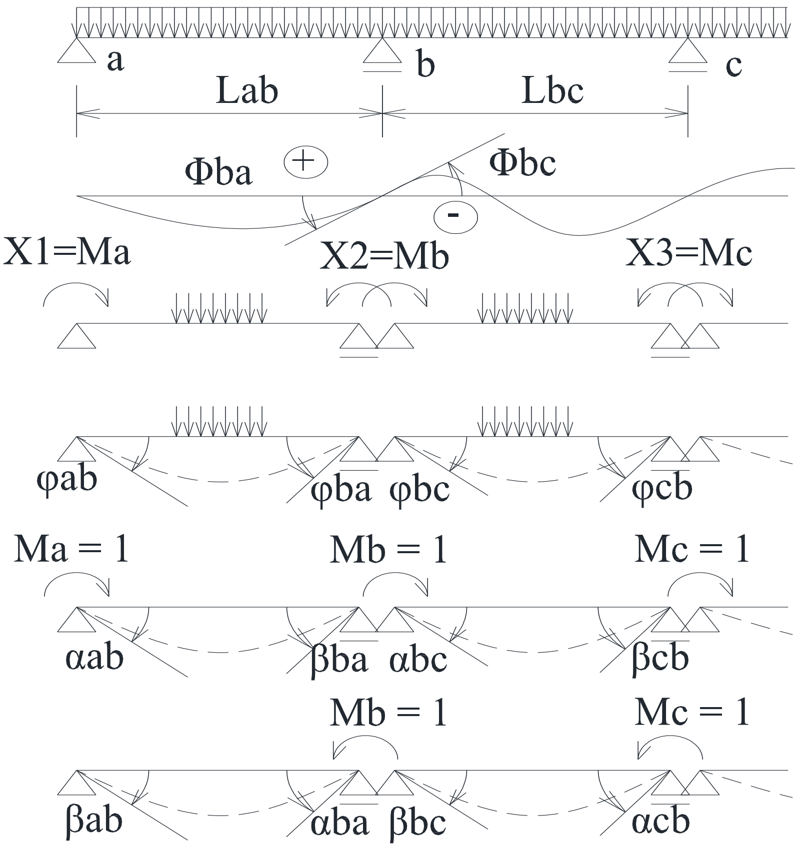


Figure 3.7: *The basic system of determinate struture of a continuous beam*

The rotation of the support section is always positive. If the beam's close areas to the support are deformed downwards, then angles φ, α, β are positive.

From the equations written above we modify them as follows (Kadlčák, J; Kytýr, J. 2004):

$$\Phi_{ba} = - \Phi_{bc}$$

$$\Phi_{ba} = M_a \,.\, \beta_{ba} + M_b \,.\, \alpha_{ba} + \varphi_{ba} \qquad , \qquad \Phi_{bc} = M_b \,.\, \alpha_{bc} + M_c \,.\, \beta_{bc} + \varphi_{bc}$$

$$M_a \,.\, \beta_{ba} + M_b \,.\, \alpha_{ba} + \varphi_{ba} = - M_b \,.\, \alpha_{bc} - M_c \,.\, \beta_{bc} - \varphi_{bc}$$

$$M_a \,.\, \beta_{ba} + M_b \,.\, \alpha_{ba} + M_b \,.\, \alpha_{bc} + M_c \,.\, \beta_{bc} + \varphi_{ba} + \varphi_{bc} = 0$$

Then the final shape of the Three-moment equation applied on the support b looks as follows :

$$M_a \,.\, \beta_{ba} + M_b \,.\, (\alpha_{ba} + \alpha_{bc}) + M_c \,.\, \beta_{bc} + \varphi_{ba} + \varphi_{bc} = 0 \qquad (3.71)$$

The continuous beam of a constant cross-section *(EI = EI₀ = const.)* is having the rotational angles required to the equation usually obtained by:

$$\alpha_{ab} = \alpha_{ba} = l_{ab}/3EI\,, \qquad \alpha_{bc} = \alpha_{cb} = l_{bc}/3EI \qquad (3.72)$$

$$\beta_{ab} = \beta_{ba} = l_{ab}/6EI\,, \qquad \beta_{bc} = \beta_{cb} = l_{bc}/6EI \qquad (3.73)$$

$$\varphi_{ab} = \varphi_{ba} = (1/24).(q.l_{ab}^{3}/EI)\,, \qquad \varphi_{abc} = \varphi_{cb} = (1/24).(q.l_{bc}^{3}/EI) \qquad (3.74)$$

Where α, β are dependant on the length of the span in time φ is dependand on the length of the span with the load upon it.

These rules obtaining the angles must be simplified rules for a sum of effects coming from all points in the span. The effects that originate back from the basics of elasticity where we use the advantage that the only effect we focus is the bending moment where (Šmiřák, S., 2006):

$$\frac{1}{r} = \frac{M_y}{EI_y} \qquad (3.75)$$

According to mathematical analysis of the deflected line here is obtained

$$\frac{1}{r} = -w'' \qquad (3.76)$$

Where the negative sign refers to the opposite direction where the beam bends to the centre of bending. So once the beam bends downwards, then the centre of bending must be above the beam.

After having the previous equation we can do the substitution to obtain:

$$w" = -\frac{M_y}{EI_y} \tag{3.77}$$

Since the deformations are small we can consider that $w' = \varphi << 1$, and the bending bending moment is considered the one along the beam which is here $M_y(x)$, then we can get (Kadlčák, J; Kytýr, J. 2004):

$$\varphi = -\int \frac{M_y(x)}{EI_y} d(x) \tag{3.78}$$

As we can see there is a direct relation between the bending moment and the bending angle where an integration along the beam to the subjected point is performed. But as we can note the obtained angle is the final rotational angle we get for which is different from the angles φ, α, β used in the three moment equation even they might have the same approach to obtain using moments.

For getting the angles numerically, we use unit dummy force method where (Kadlčák, J; Kytýr, J., 2004):

$$\varphi = \int_0^l \frac{M(x).\bar{M}(x)}{EI(x)} dx \tag{3.79}$$

$$\alpha = \int_0^l \frac{\bar{M}(x).\bar{M}(x)}{EI(x)} dx \tag{3.80}$$

$$\beta = \int_0^l \frac{\bar{M}(x).\bar{M}(x)}{EI(x)} dx \tag{3.81}$$

In these equations we have to note again that we are dealing with the indeterminate structures and after splitting indeterminate structure to many determinate ones (a continuous beam into many simply supported beams) then the variables used in the previous integrals stand for as follows:

$M(x)$	Bending moment at x point in the simply supported beam
$\bar{M}(x)$	Fictional bending moment on the beam at x point
$EI(x)$	Beam siffness at x point

This method was applied are every single hundredth of every field and the final result of the angles is calculated by applying an integration for which we used Simpson's rule that goes:

$$\int_a^b f(x)dx \approx \frac{h}{3}\{1.f(x_0 = x_a) + 4.f(x_1) + 2.f(x_2) + 4.f(x_3) + \cdots + 4.f(x_{n-1}) + 1.f(x_n = x_b)\}$$

Where:

a_{int}	the first end of the span (the start)
b_{int}	the other end of the span (the very end)
$i = 0, 1, \ldots, n$	$n \ldots even$
$x_i = a_{int} + i \cdot h$	the exact point for every hundredth of the span
$h = (b_{int} - a_{int})/n$	the length of one hundredth of the span

After calculating the required angles we get the bending moment over the support using this equation:

$$M_b = -(\varphi_{ba}+\varphi_{bc})/(\alpha_{ba}+\alpha_{bc}) \tag{3.82}$$

The previous explanation was about finding out the undefined bending moment on one internal support in a continuous beam of two spans, but we also have the case of a three-span continuous beam where there are two internal supports and the calculation reaching the definition of the bending moment is quite similar to the previous explanation that goes as follows:

Since we have a continuous beam that is having an indeterminacy of $n_s = 2$ as was explained before, we know these indetermined bending moment values are coming from the internal supports when: $M_a = 0, M_d = 0$

Then we write the equations where the following is fulfilled at support *b*:

$$\Phi_{ba} = -\Phi_{bc}$$

And at support *c*:

$$\Phi_{cb} = -\Phi_{cd}$$

Then the tangent angles must equal the following (Kadlčák, J; Kytýr, J., 2004):

$$\Phi_{ba} = M_a \cdot \beta_{ba} + M_b \cdot \alpha_{ba} + \varphi_{ba} \quad , \quad \Phi_{bc} = M_b \cdot \alpha_{bc} + M_c \cdot \beta_{bc} + \varphi_{bc}$$

$$\Phi_{cb} = M_b \cdot \beta_{cb} + M_c \cdot \alpha_{cb} + \varphi_{cb} \quad , \quad \Phi_{cd} = M_c \cdot \alpha_{cd} + M_d \cdot \beta_{cd} + \varphi_{cd}$$

Then:

$$M_a \cdot \beta_{ba} + M_b \cdot \alpha_{ba} + \varphi_{ba} = -(M_b \cdot \alpha_{bc} + M_c \cdot \beta_{bc} + \varphi_{bc})$$

$$M_a \cdot \beta_{ba} + M_b \cdot \alpha_{ba} + \varphi_{ba} + M_b \cdot \alpha_{bc} + M_c \cdot \beta_{bc} + \varphi_{bc} = 0$$

$$M_a \cdot \beta_{ba} + M_b \cdot (\alpha_{ba} + \alpha_{bc}) + M_c \cdot \beta_{bc} + \varphi_{bc} + \varphi_{ba} = 0$$

$$M_b \cdot \beta_{cb} + M_c \cdot \alpha_{cb} + \varphi_{cb} = -(M_c \cdot \alpha_{cd} + M_d \cdot \beta_{cd} + \varphi_{cd})$$

$$M_b \,.\, \beta_{cb} + M_c \,.\, \alpha_{cb} + \varphi_{cb} + M_c \,.\, \alpha_{cd} + M_d \,.\, \beta_{cd} + \varphi_{cd} = 0$$

$$M_b \,.\, \beta_{cb} + M_c \,.\, (\alpha_{cb} + \alpha_{cd}) + M_d \,.\, \beta_{cd} + \varphi_{cb} + \varphi_{cd} = 0$$

After getting the two equations for both internal supports we will define the angles *α, β, φ* at every point of the span, then define the final values of them through a numerical integration of all 100 values we defined along the span.

$$\underbrace{M_a}_{0} \cdot \beta_{ab} + \underbrace{M_b}_{unknown} \cdot (\alpha_{ab} + \alpha_{bc}) + \underbrace{M_c}_{unknown} \cdot \beta_{bc} + \varphi_{ab} + \varphi_{bc} = 0 \tag{3.83}$$

$$\underbrace{M_b}_{unknown} \cdot \beta_{bc} + \underbrace{M_c}_{unknown} \cdot (\alpha_{cb} + \alpha_{cd}) + \underbrace{M_d}_{0} \cdot \beta_{cd} + \varphi_{cb} + \varphi_{cd} = 0 \tag{3.84}$$

As we have two equations with two unknowns we could easily define the undefined values by a mathematical process from which we get M_b and M_c

After getting the bending moments along the whole continuous beam then we can get into simpler values for understanding the behaviour of the continuous beam and one of the most practical values for simply evaluating the various effects on the beam is the deflection.

For deflection we do the calculation starting from the direct relation between the bending moment that we obtained and the second derivation of deflection where:

$$w'' = -\frac{M_y}{EI_y} \tag{3.85}$$

As we notice here that stiffness EI_y is at one point of the beam and M_y is the bending moment at the same point. An advantage to be used since we will focus later on about the changing of stiffness values at the parts of the beam over the supports which will affect the changing of deflection as well.

Another advantage for the previous relation is the fact that getting the deflection needs an integration of the other side to be executed which makes the result of every point of the system dependant on the previous points with their bending moments and stiffnesses, a result which ensures the continuity in the whole system.

Then after executing the integration we get into a step that goes as:

$$\varphi = w' = \underbrace{-\int_{i}^{i+1} \frac{M_y}{EI_y}}_{\varphi_0} + \underbrace{C_1}_{const.} \tag{3.86}$$

Where:

φ rotation at x point

φ_0 a helping value for rotation at x point

C_1 constant that is determined by the boundary conditions

As we note from the previous integration we obtain the rotation of the focused point, the reason why the integration is executed in the length covering every two hundredths that are noted by $i,\ i + 1$.

To obtain the deflection we must execute an integral for the same distance of the two hundredths $i,\ i + 1$ for which we get the following:

$$w = \int_{i}^{i+1} \varphi = -\underbrace{\int_{i}^{i+1} \varphi_0}_{w_0} + C_1 . x + \underbrace{C_2}_{const.} \tag{3.87}$$

Where:

w deflection at x point

w_0 a helping value for deflection at x point

C_2 constant that is determined by the boundary conditions

From the boundary conditions we can note that deflection on the side support is equal to 0 where the equality is as follows:

$$w = -\underbrace{\int_{i}^{i+1} \varphi_0}_{=0} + C_1 . \underbrace{x}_{=0} + \underbrace{C_2}_{const.} = 0 \tag{3.88}$$

We notice that the deflection over the internal support must be $w = 0$ since the support fixes the beam. Let us apply this point on the first span *(ab)* and we make the substitution of $x = l_{ab}$ where we get the following:

$$w = w_0 + C_1 . l_{ab} + \underbrace{C_2}_{=0} = 0 \tag{3.89}$$

$$C_1 = -\frac{w_0}{l_{ab}} \tag{3.90}$$

Then we obtain the rotation that is calculated as:

$$\varphi = \varphi_0 - \frac{\overbrace{w_0}^{last\ point\ of\ ab\ span}}{l_{ab}} \tag{3.91}$$

And deflection is calculated as follows:

$$w = \underbrace{w_0}_{at\ point\ x} - \frac{\overbrace{w_0}^{at\ the\ last\ point\ of\ ab}}{l_{ab}} . x \tag{3.92}$$

For the second span in the continuous two-span beam the whole process of obtaining the deflection is the same, but there is a difference about the usage of boundary conditions for *bc* span where:

$$\varphi = \varphi_0 - \frac{\overbrace{w_0}^{first\ point\ of\ bc\ span}}{l_{bc}} \tag{3.93}$$

And deflection is obtained as follows:

$$w = \underbrace{w_0}_{at\ point\ x} - \frac{\overbrace{w_0}^{at\ the\ first\ point\ of\ bc}}{l_{bc}} . x \tag{3.94}$$

For the 3-span continuous beam we need to change the assumptions of the 2-span continuous beam in calculating since the second span is a middle span where:

$w_b = 0, w_c = 0$ over the supports *b* and *c*

$\varphi_{ba} \approx \varphi_{bc}$ assumption since it is an internal support

$\varphi_0 = 0$ assumption over the support

from the previous two assumptions we can get the constant C_1 that will be a constant for the whole span where:

$C_1 = \varphi_{bc} - \varphi_0 = \varphi_{bc}$ constant over the whole middle span

After getting the necessary values then the integrations are executed to obtain the rotations and deflection as in the previous equations.

Since we were focusing on obtaining values of rotation and deflection for every hundredth of the whole span we have to use a rule that takes the advantage of having this amount of numbers to execute an integration for obtaining the continuity that deflection of the span has. For this aim we better use a numerical integration for the functions shown above and this time we pick up the trapezoidal rule of numerical integration that was applied on the rotations and deflections as follows:

$$\int_{i}^{i+1} f(x)dx \approx \underbrace{\frac{h}{2}\{f(i) + f(i+1)\}}_{result\ at\ (i+1) element} + \underbrace{m}_{result\ obtained\ from\ (i)\ element} \tag{3.95}$$

Where:

$$\overbrace{h}^{length\ of\ one\ element\ of\ the\ span} = \frac{\overbrace{b}^{last\ end\ (support) of\ the\ span} - \overbrace{a}^{first\ end\ (support) of\ the\ span}}{\underbrace{n}_{number\ of\ elements\ on\ the\ whole\ span\ =100}}$$

4.3.2. the entry to the non-linear calculation of the internal forces:

The continuous composite beam will be affected by many factors, because of its boundary conditions, such as the effect of negative bending moments over the internal supports for the upper concrete part that differs according to where those negative bending moments reach on the span. The effect that leads to another one since the only working part of the beam over the support is steel that reaches to plasticity after exceeding the linear behaviour of its stress-strain diagram.

These two effects decrease the stiffness of cross-sections in various points along the beam where the concrete part has no force to calculate with when cracks occur which dramatically drops the stiffness of the whole cross-section of the composite beam at once beyond the crack point on the beam. In time plasticity effect drops the stiffness down gradually according to how much the steel cross-section is plasticized which in tern is directly dependant on the bending moments.

The problems mentioned make different stiffness values at different points along the beam that make different distribution of bending moments and internal forces along the whole beam, which is different from the assumption of the linear calculation where the constant stiffness along the whole beam is considered that in tern gets high bending moments over the supports with no changes on beam materials which might not be real assumption. The reason why the linear calculation does not include any of concrete crack influence and steel plasticity into account should not be taken seriously. For this we include concrete cracks and plasticity effects by adding conditions for both effects to recalculate the distribution of bending moments along the beam that are obtained using iteration steps to obtain the final result.

4.3.3. Calculation varieties

- **Calculation with respect crack effects (2 stiffness values):**

The crack effect must be a big effect on changing cross-sectional stiffness since it is not a gradual influence but it drops at once at the cross-section once the crack occurs keeping steel cross-section alone holding the beam. To express this behaviour there was a need to make a condition related to crack occurence. And since we mainly deal with bending moments, it will be easier for us to connect the bending moment reach to crack occurence by setting a limit describing the peak of concrete holding with no crack.

This limit can be described as the critical moment of the cross-section, which is described by the national code ČSN EN 1994-1-1 that can be calculated by:

$$M_{cr} = (2\, f_{ctm} / E_{cm})\ .\ (EI_i / z_c) \qquad (3.96)$$

Where:

M_{cr} critical moment of the whole cross-section

f_{ctm} mean value of axial tensile strength of concrete

E_{cm} secant modulus of elasticity of normal weight concrete

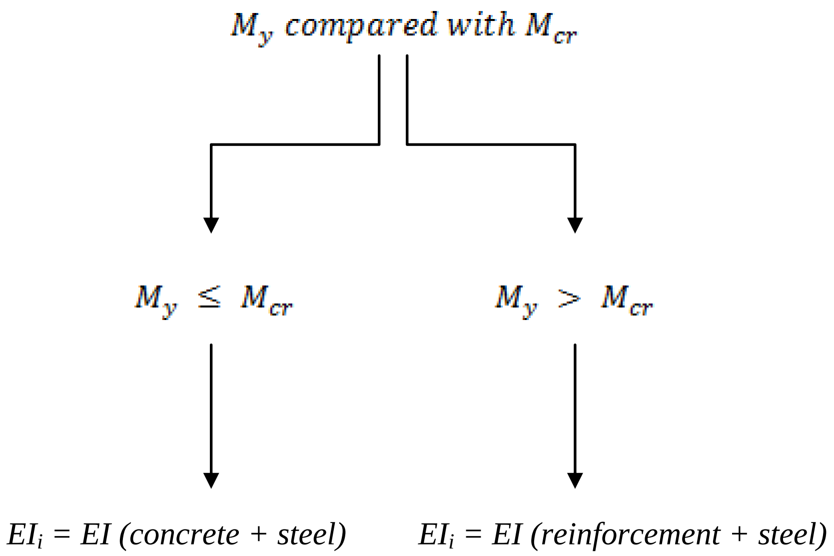
EI_i stiffness of the whole (Concrete+Steel) cross-section

z_c distance from the centre of the whole (Concrete+Steel) cross-section to the upper face of concrete

The critical moment can be considered the limit for which cracks occur once the bending moment exceeds it.

As mentioned before the stiffness value of the cross-section drops at once when a crack accurs. We can assume eliminating the concrete part from calculation to express the crack influence. And since the cross-section is under the negative bending moments for which the upper concrete fibers are under tension then we have another reason not to calculate with concrete and add some reinforcement instead.

For this we will use the two stiffness values we obtained before and the condition will be set as follows:



We have to note that the comparison here is considered according to the negative value of the bending moments since M_{cr} is negative and because concrete cracks once it is affected by tension coming from negative bending moments.

This comparison is applied at every hundredth of the whole span. As must be seen, the elements with positive bending moments will have the stiffness of composite cross-section consisting of constructional IPE steel beam with the concrete slab as it is in the span including the elements with negative bending moments not exceeding the critical bending moment of concrete. In time elements with negative bending moments that are higher than the critical one of concrete will be considered having cracks in concrete slab which stiffness will be expressed for a cross-section consisting of constructional IPE steel bar with reinforcement which is most likely over the internal supports.

But this condition the physical properties are defined along the whole continuous beam by its hundredths according to the distribution of bending moments. This advantage of knowing which hundredths (elements) are having the stiffness of (Concrete + Steel) or (Reinforcement + Steel) enables us to define exactly where the cracks start occuring on the beam.

By the definition of stiffness values of all beam hundredths (elements) we would apply the statical calculation where we will obviously obtain a different bending moment distribution. After this we will repeat the process of applying the stiffness condition with the obtained bending moment distribution of the previous step until we get the optimal distribution of bending moments and cross-section stiffnesses of elements along the whole beam.

- **Calculation with respect crack and additional reinforcement effects (3 stiffness values):**

We are focusing on the detail of the continuous composite beam which is over the internal support where the cracks occur in the upper concrete part, for this it is necessary to add some reinforcement to hold the tension which occurs in that part.

Compared to the condition mentioned before we will make a bit of change to obtain bending moment redistribution including the effect of reinforcement.

We will consider the hundredths (elements) under positive bending moment having the stiffness value of cross-section (Concrete + Steel) since concrete is under compression and no need for reinforcement then. In time elements that are under negative bending moment but not exceeding the critical bending moment M_{cr} are having stiffness of (Concrete + Reinrcement + Steel) since concrete part is under tension in those elements and as know that concrete is not having a big deal of tensile strength to bear, the reason why reinforcement is added. And the rest elements which are under bending moment exceeding the critical one where cracks occur in theirs concrete part must be having the stiffness of (Reinforcement + Steel).

$$M_y \text{ compared to } M_{cr}$$

$$\underbrace{M_y \geq 0}_{positive} \rightarrow EI_i = EI\,(concrete + steel)$$

$$\underbrace{M_{cr} \leq M_y < 0}_{negative,\ but\ inder\ critical} \rightarrow EI_i = EI(concrete + rfcmt + steel)$$

$$\underbrace{M_y < M_{cr}}_{over\ critical} \rightarrow EI_i = EI(rfcmt + steel)$$

4.4. Calculation with respect the effect of steel plasticity:

Theory of plasticity deals with the stresses and strains of bodies made up of ductile materials, non-reversible and permanently deformed by a set of applied forces that exceed the elastic limit. Which makes plasticity the behaviour behind the elastic limit of materials.

In the plastic stage unlike the elastic one, the state of strain does not depend only on the final state of stress. Which means the material can deform more with no increase of stresses, which can be expressed as:

$$\underbrace{\varepsilon_{total}}_{total\ strain} = \underbrace{\varepsilon_e}_{elastic\ strain} + \underbrace{\varepsilon_{pl}}_{plastic\ strain} = \underbrace{\frac{\sigma}{E}}_{Hook's\ law} + \varepsilon_{pl} \qquad (3.97)$$

By this relation we see that the obtained strain due to plastic effect is an additional strain to the elastic one ε_e which has become a constant after exceeding the elastic limit. Since we know that the plastic strain is not related to the elastic strain that becomes a constant in the plastic stage then we could check the the previous properties using Hook's law which says:

$$\underbrace{\varepsilon_{total}}_{increases\ in\ plastic\ stage} = \frac{\overbrace{\sigma}^{constant}}{\underbrace{E}_{decreases}} \qquad (3.98)$$

Then as we notice here that Young modulus of material at any point involved with plasticity decreases. And if we take the assumption that the cross-section does not change by the effect of plasticity, then the stiffness value *EI* must decrease by plasticity effect. Which means there will be a change in internal forces distributions by the effect of plasticity.

Since this effect is applied on ductile materials such as steel then in case of composite beams it affects just a part of the whole cross-section. More specifically, it affects IPE steel beam after cracks occur in the concrete part over the internal supports.

For this we will create the stress-strain diagram fo the composite beam cross-section over the support where the negative bending moments are. For this diagram there must be noted that the vertical axis of stress values will be replaced by equivalent moment values and the horizontal axis of strain values are replaced by equivalent curvature values ρ.

We start by setting the main points of stress-strain diagram (here bending moment-curvatures diagram) by:

I. Starting point where there is no effect and has the following values:

$M_y = 0\ kN.m, \rho = 0$.

II. The elastic limit point after which a crack occurs in the concrete part where critical bending moment is considered that is calculated by:

$$M_{cr} = \frac{2f_{ctm}}{E_{cm}} \frac{EI_i}{z_c}$$

Where: f_{ctm} mean value of axial tensile strength of concrete

E_{cm} secant modulus of elasticity of normal weight concrete

EI_i stiffness of composite beam cross-section

z_c distance between the centre of composite cross-section to upper concrete face

And the curvature equivalent to the obtained bending moment is calculated by the following relation:

$$\rho = \frac{\overbrace{M_y}^{\substack{bending\ moment\ of \\ (i) element}}}{\underbrace{EI_i}_{\substack{stiffness\ of\ composite\ beam \\ cross-section\ at\ (i)\ element}}} \tag{3.99}$$

III. The point which expresses the loss of cross-sectional stiffness at the element where cracks of concrete slab have occured. The drop of stiffness is followed by a drop of bending moment since uneveness ρ is claimed to be similar to the one of the II. point.

$$\underbrace{\rho}_{constant} = \frac{\overbrace{M_y}^{\substack{bending\ moment\ drops \\ because\ of\ EI_i\ decrease}}}{\underbrace{EI_i}_{\substack{Stiffness\ drops \\ because\ of\ concrete\ cracks}}} \tag{3.100}$$

IV. Here a purely dealing with the constructional steel bar has just started since it has just become the bearing part of the whole composite beam after concrete cracks occured, for which we define the yield point.

Yield bending moment is obtained by the following:

$$M = \frac{I_y}{z} f_y \tag{3.101}$$

Where:

I_y moment of inertia of IPE cross-section of steel bar

f_y yield strength of steel

z distance between the middle of IPE section and the upper face of it

V. The last point which is the resistance point of (Reinforcement + Steel) cross-section, for which we use the resistance bending moment $M_{pl,Rd}$ we obtained from the cross-section stiffness programme.

The equivalent curvature is obtained by taking the maximal characteristic strain of reinforcement of type A, where its value equals to: ε_{uk} = 2,5% (according to ČSN EN 1992-1-1).

The following relation was used to calculate the resistance curvature of the section:

$$\rho = \frac{\varepsilon_{uk}}{z} \tag{3.102}$$

Where:

z distance between the centre of (Reinforcement + Steel) cross-section and the upper face of reinforcement.

As we can see from the diagram that the part from I. point till II. is the linear behaviour of the composite beam cross-section, in time a fall of bending moment is noticed after cracks occur that is represented by the part between points II. and III. . The part between points III. and IV. Represent the linear behaviour of steel bar since it has become the part holding the whole beam. The plastic behaviour of steel bar is represented by the line between IV. And V.

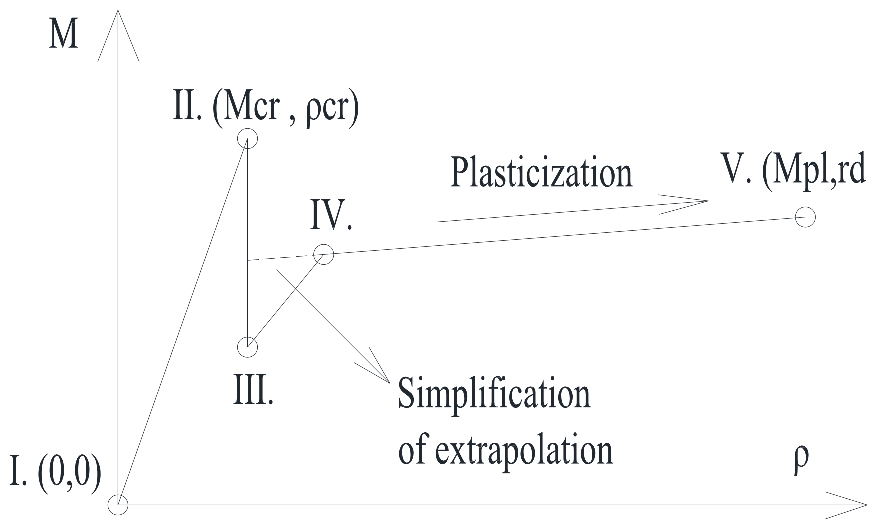


Figure: 3.8: *Generalized programmed (bending moment-curvature) diagram of composite cross-section*

After defining the stress-strain diagram (here bending moment-curvatures diagram), which describes the behaviour of the composite beam sebjected by the gradually increasing loading. We start using the obtained diagram in calculations of plasticity effect on internal forces redistribution by several steps that go as follows:

A. The structure under the continuous loading is solved using the programme made up in Excel that is based on numerical calculation considering this approach as a first step.

   After having this linear solution, we draw the bending moment reach over the internal support on the obtained stress-strain diagram. Which most likely is exceeding the plastic stage and would show an unbeared load according to the digram.

B. The linear calculation, where according to the last point observation exceeds the limit of the drawn stress-strain diagram is obviously not including the effect of plasticity. To this effect, an iterational condition is set up to be added to calculation. Since the stress-strain diagram we obtained has a fall making a gap between the points II. and IV. then we choose Picard's iteration theorem to use.

According to Picard’s theorem, we will find a point on stress-strain diagram that is vertically equivalent to the bending moment point we obtained for every hundredth (element) that has higher bending moment than point III. on the diagram.

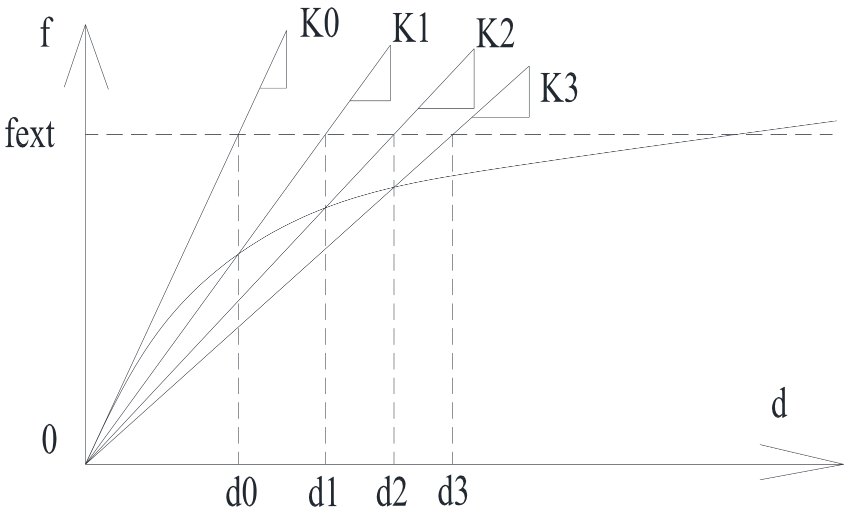


Figure 3.9: *Picard’s theorem principle*

For this we would make another step that has the following condition using a fictional bending moment $M_{i,k}'$ for help:

$$M_{yi} \leq M_{cr} \quad \longrightarrow \quad M'_{i,k} = M_{yi}$$

$$\underbrace{M_{yi} > M_{cr}}_{\text{over critical bending moment of concrete}} \quad \longrightarrow \quad M'_{i,k} = M_{IV} + \left\{\frac{(M_V - M_{IV})(\rho_i - \rho_{IV})}{(\rho_V - \rho_{IV})}\right\}$$

Where:

$M_{i,k}$ bending moment of *i* hundredth (element) at *k* step

M_{yi} bending moment of the first linear step

M_V bending moment at the V. point on stress-strain diagram = $M_{pl,Rd}$

M_{IV} bending moment at the IV. point of stress-strain diagram

ρ_V curvature at the V. point of stress-strain diagram

ρ_{IV} curvature at the IV. point of stress-strain diagram

C. We will get for every hundredth (element) a new stiffness value using the direct relation of following:

$$EI = \frac{M_y}{\rho} \qquad (3.103)$$

Which in our case of connecting two different steps we use the previous relation in obtaining the following:

$$EI_{i,k+1} = \frac{\overbrace{M'_{i,k}}^{\substack{obtained\ from\ the\\ condition}}}{\underbrace{\rho_{i,k}}_{\frac{M_{yi}}{EI_i}}} \qquad (3.104)$$

Where:

$EI_{i,k+1}$ stiffness value of *i* hundredth (element) at *k+1* step

By these steps we can consider one iteration of calculation being done, which in tern was repeated to get to the most precise result of plasticity effect.

4.5. Creep, shrinkage - software ASTERES

The principle of creep effect is the loss of concrete stiffness by the long-term load. When a load presses the water from micro-pores into capillaries from where it evaporates. For this behaviour the load must be so long, so it will manage to affect concrete properties. The reason why creep is calculated by the effect of permanent load.

Creep effect is mostly notices by strain or deflection (as a better property for our case) and it is dependant on many factors such as: - long-term stress in comcrete, - time of loading, - cement properties, - aggregate characteristics, - amount of mixing water, - dimensions of the element, - ambient humidity and temperature.

The reach to acreep result was done by many methods that were used such as calculations coming from *ČSN EN 1994-1-1*, and using Asteres software.

According to *ČSN EN 1994-1-1* code we reach the final result of creep using the ratio n_L that is obtained by the following:

$$n_L = n_0\,(1+\psi_L\,\varphi_t) \qquad (3.105)$$

Where:

n_0 ratio Young modulus values of E_a/E_{cm} for a short-term loading

E_{cm} secant modulus of elasticity of normal weight concrete

φt creep coefficient, which is equivalent to $\varphi_{(t,t0)}$ defining creep between times t and t_0

ψ_L factor of creep coefficient, which is dependant on load type. For permanent load it is equal to *1,1*

Which in tern we must get back to concrete *ČSN EN 1992-1-1* code, where creep coefficient $\varphi_0(t,t_0)$ as follows:

$$\varphi(t,t_0)= \varphi_{0creep}\, \beta_c(t,t_0) \quad (3.106)$$

φ_{0creep} the notional creep coefficient and is dependant on a coefficient related to the effect of the relative humidity and is calculated by:

$$\varphi_{RH} = 1 + (1\text{-}RH/100) / (0,1 \,.\, 3\sqrt{h_0}) \quad \text{for} \quad f_{cm} \leq 35\ MPa \quad (3.107)$$

β_c is a coefficient to describe the development of creep with time after loading, and may be estimated using the following Expression:

$$\beta_c\ (t,t_0) = [\ (t - t_0) / (\beta_H + t - t_0)\ ]^{0,3} \quad (3.108)$$

h_0 is the notional size of the member in mm where:

$$h_0 = 2A_c / u \quad (3.109)$$

A_c is the cross-sectional area

u is the perimeter of the member in contact with the atmosphere

The other method that is used in *ČSN EN 1994-1-1* code, which is considered the simplified method that considers the reduction of Young modulus of concrete $E_{c,eff}$ by the following:

$$E_{c,eff} = E_{cm} / 2 \quad (3.110)$$

While comparing the simplified method with the previous, we can notice that is $\varphi(t,t_0)=1$.

After showing the methods approached throught *ČSN EN 1994-1-1*, a way more advanced method is used through computer calculations of software ASTERES. This software is based on finite element method and time discretization method, where it gets to results through precise calculations.

ASTERES is based on non-linear calculations that are a big advantage for obtaining the fulfillment of cross-sectional equilibrium once changing material properties is proceeded. This behaviour makes changes in positioning the neutral axis in its cross-section, a problem that is solved by setting an element type with a reference axis, according which all material process affecting the shifting the neutral axis to keep the cross-section equilibruim is counted. This element is set up by constitutive equations as follows:

$$\begin{Bmatrix} N \\ V \\ M \end{Bmatrix} = \begin{bmatrix} EA & -ES & 0 \\ -ES & EI & 0 \\ 0 & 0 & GA_\kappa \end{bmatrix} . \begin{Bmatrix} \varepsilon_n \\ \varepsilon_m \\ \varepsilon_v \end{Bmatrix}$$

(3.111)

Where:

E and *G* are material characteristics, *A* is the cross-sectional area, *S* is the first moment of area and *I* is the second moment of area for reference axis, ε_n, ε_m, and ε_v are longitudinal strain of reference axis, bending strain (curvature) and shear strain, respectively.

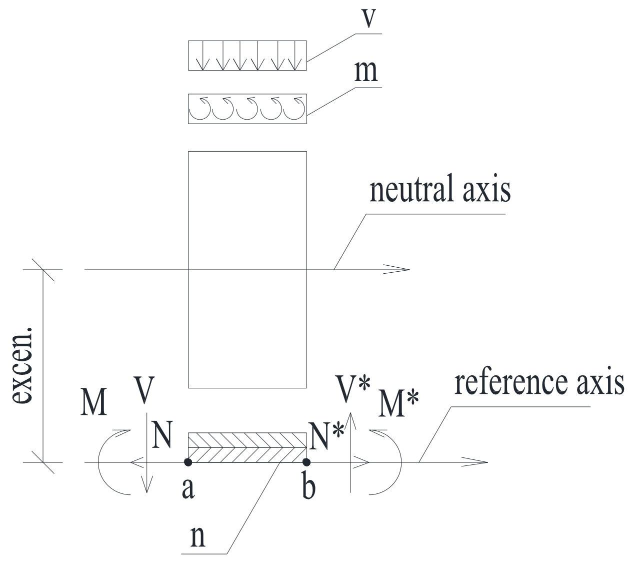


Figure 3.10: *Excentric positioning of the beam*

The use of previous element is fully suitable for analyzing the effects of creep in a composite cross-section since the steel part has no changes in its material properties, in time concrete part obtains a drop in its Young modulus, which changes the positioning of neutral axis to obtain a cross-sectional equilibrium. This affects the stiffness as well, for which the software calculates and it solves the effect on the composite beam.

For the previous elements, to be applied on creep, there is a need to include time dependancy since creep is mostly dependent on time. This was suolved by using time discretization method where the analyzed time of creep is divided to smaller intervals, for which the stresses are constant. And rheological strains are applied as loads making creep strain ε_c for time interval from t_1 to t_i by the following relation:

$$\varepsilon_c(t_2) = \sum_{i=1}^{n} \frac{\Delta\sigma(t_i)}{E(t_i)} \varphi(t_i, t_2)$$

(3.112)

Where:

n is number of intervals of the analyzed time

$\Delta\sigma(t_i)$ change if stress in i-time

$E(t_i)$ modulus of elasticity of i time

$\varphi(t_i ,t_2)$ creep coefficient of time interval (t_i ,t_2)

By the previous methods all concrete elements are under creep effect, which initial strains are changed.

5. Examples

5.1. 2-span continuous beam – redistribution

Problems we usually face are most likely in indeterminate structures as it is in continuous beams for which is considered the most common applied case on IPE beams.

We can consider that a 2-span continuous beam is a practical application for composite slabs that cover two areas of different spans.

The first try of evaluating the redistribution will be on a continuous beam with two spans that are dimensionally equal. A condition which is a better option to compare according to the code ČSN EN 1994-1-1 .

For a 2-span continuous beam of equal span lengths there will be the same effective load. Let us say the span length $l_{ab} = 8\ m$, $l_{bc} = 8\ m$.

In the first span (a side span):

$L_{e1} = 0{,}80 \;.\; L_1 = 0{,}80 \;.\; 8 = 6{,}4\ m$ *coeff. 0,80 for the side spans accord. ČSN EN 1994-1-1*

$b_{eff,1} = L_{e1} / 4 = 6{,}4 / 4 = 1{,}6\ m$

We will choose IPE240 section for the steel bar.

Loads:

Permanent load:

Concrete slab: $\gamma_{concrete} \;.\; a \;.\; b = 25 \;.\; 0{,}05 \;.\; 2{,}0 = 2{,}50\ kN/m$

Concrete filling: $\gamma_{concrete} \;.\; a_{gap}$ *.coeff. of trap plate* $.\; b = 25 \;.\; 0{,}05 \;.\; 0{,}6 \;.\; 2{,}0 = 1{,}50\ kN/m$

Coefficient of trapezoidal plate is an assumption of how much space of the plate concrete is filling.

IPE 270 : *0,361 kN/m*

The trapezoidal plate: *0,05 kN/m*

Floor tiles: *4 kN/m*

$g_k = 2{,}50 + 1{,}50 + 0{,}361 + 0{,}05 + 4 = 8{,}411\ kN/m$

$g_d = g_k\ .\ 1{,}5 = 8{,}411\ .\ 1{,}35 = 11{,}355\ kN/m$

Living load:

According to Eurocode 1991: C4

$q_k = 5{,}0\ kN/m2$

$q_k = 5{,}0\ .\ 2{,}0 = 10{,}0\ kN/m$

$q_d = 10{,}0\ .\ 1{,}5 = 15{,}0\ kN/m$

Total design load = *11,355 + 15,0 = 26,355 kN/m*

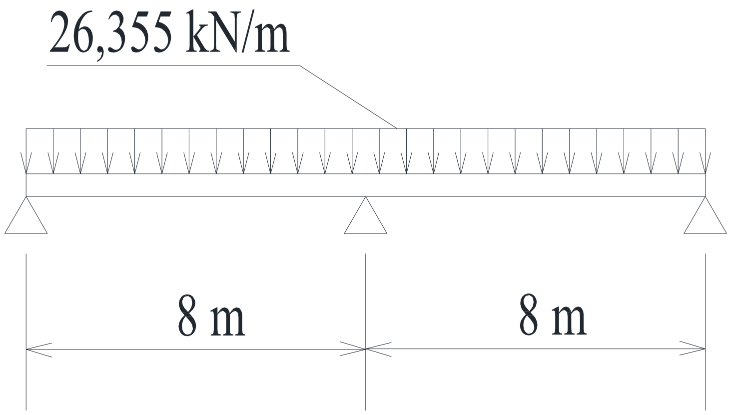


Figure 4.1: *a continuous beam of 2 spans*

Let us calculate the 2-span girder with the mentioned dimensions and loads by 3-moment equation method in a linear way as follows:

$l_{ab} = 8\ m$, $l_{bc} = 8\ m$, $q = 26{,}355\ kN/m$,

Concrete Type: C20/25,

Steel Type: S235 (section: IPE 240)

$M_a\ .\ \beta_{ba} + M_b\ .\ (\alpha_{ba} + \alpha_{bc}) + M_c\ .\ \beta_{bc} + \varphi_{ba} + \varphi_{bc} = 0$

As we know: $M_a = 0\ kN.m$, $M_c = 0\ kN.m$, *then:*

$M_b = -(\varphi_{ba}+\varphi_{bc})/(\alpha_{ba}+\alpha_{bc})$

Where:

$\alpha_{ab}=\alpha_{ba} = l_{ab}/3EI$

$\alpha_{bc}=\alpha_{cb} = l_{bc}/3EI$

$\varphi_{ab}=\varphi_{ba}= (1/24).(q.l_{ab}{}^3/EI)$

$\varphi_{bc}=\varphi_{cb}= (1/24).(q.l_{bc}{}^3/EI)$

We will choose the composite beam with IPE 240 steel section that has the following stiffness values according to the solution of 2 stiffness values:

- For both spans (*ab* and *bc* span):

$EI_{ab1} = 4{,}30 \,.\, 10^{+07}\, N.m^2$ (Concrete slab + Steel bar in the span, no cracks in concrete part)

$EI_{ab2} = 2{,}04 \,.\, 10^{+07}\, N.m^2$ (Steel bar + Reinforcement over the support, cracks occur), where we insert *14* bars of diameter *Φ10 mm.*

Then according to the linear calculation:

$\alpha_{ab}=\alpha_{ba} = \alpha_{bc}=\alpha_{cb} = 8/(3\,.4{,}30\,.10^{+07}) = 8{,}36.10^{-8}$

$\varphi_{ab}=\varphi_{ba}= \varphi_{bc}=\varphi_{cb}= (1/24).\{26{,}355\,.\,8^3/(4{,}30\,.10^{+07})\}= 1{,}31.10^{-05}$

$M_b= -(2\,.\,1{,}31.10^{-05}) / (2\,.\,8{,}36.10^{-8}) = -210{,}84\ kN.m$

According to the beam review we check the resistance moments comparing to design ones are as follows:

	In the span		Over the support *b*	
IPE 270	$M_{pl,Rd}$	Max M_{Ed}	$M_{pl,Rd}$	M_{Ed}
First span (*ab* span)	214,096	118,58	-179,39	-210,84
Second span (*bc* span)	214,096	118,58	-179,39	-210,84

Table 4.1: *the comparison between results of the first (linear) step to resistance ones*

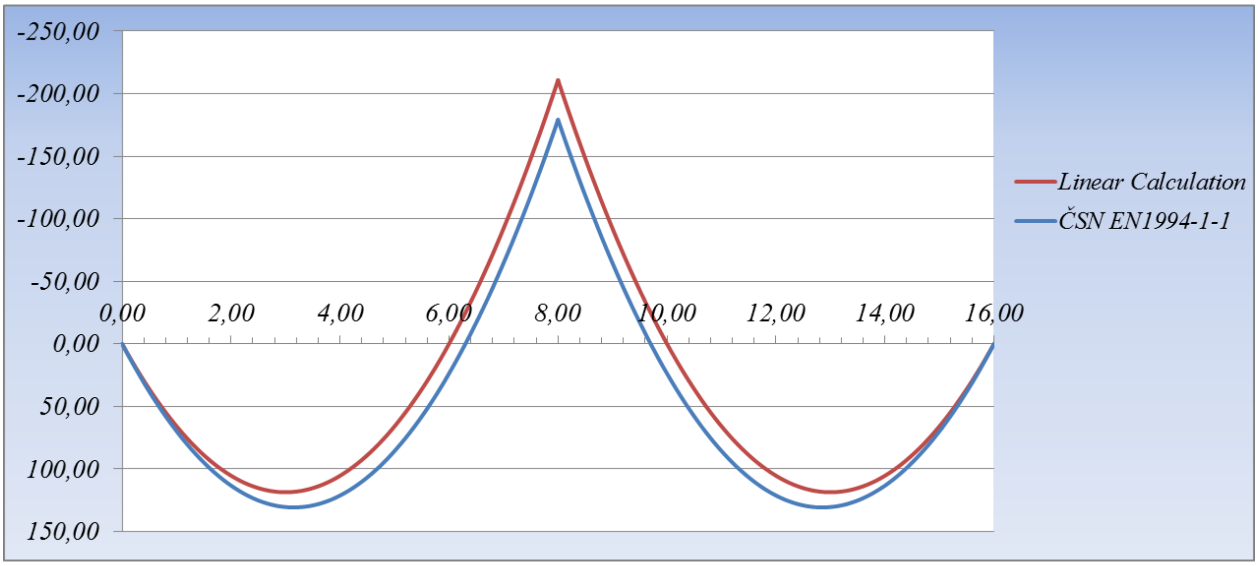
According to the table we find that the ultimate limit state is fulfilled in the span. But over the support *b*, where according to the linear calculation the cross-section is the same as in the middle of the span (Concrete + Steel), then the bending moment is higher than the resistance one of the real cross-section (Reinforcement + Steel). A distribution of bending moments that shows the reason of not taking the linear calculation into account and the necessity of the need of executing the redistribution to fulfill the ultimate limit state.

Then according to ČSN EN 1994-1-1 code the redistribution due to crack of the concrete part over the support which is supposed to drop the bending moment up to **15%**

	M_b [kN.m]	Max M in *ab* and *bc* span [kN.m]
Linear Step	-210,84	214,096
ČSN EN 1994-1-1 Redistribution	-179,21	130,74

Table 4.2: *Bending moment values over the internal support and the maximum positive in the span, with its change due to distribution determined by the national code ČSN EN 1994-1-1*

Which makes the following distibution on the whole continuous beam:



Graph 4.1: *Redistribution of bending moments along the continuous composite beam of IPE270, S235 steel cross-section and C20/25 of (2000 x 50) mm concrete cross-section under the effect of cracks.*

For now we would apply the non-linearity using the two stiffness values way along the whole beam where the stiffness values are:

$EI_{ab1} = 4{,}30.10^{+07}\ N.m^2$

$EI_{ab2} = 2{,}04.10^{+07}\ N.m^2$ (on *ab* and *bc* span)

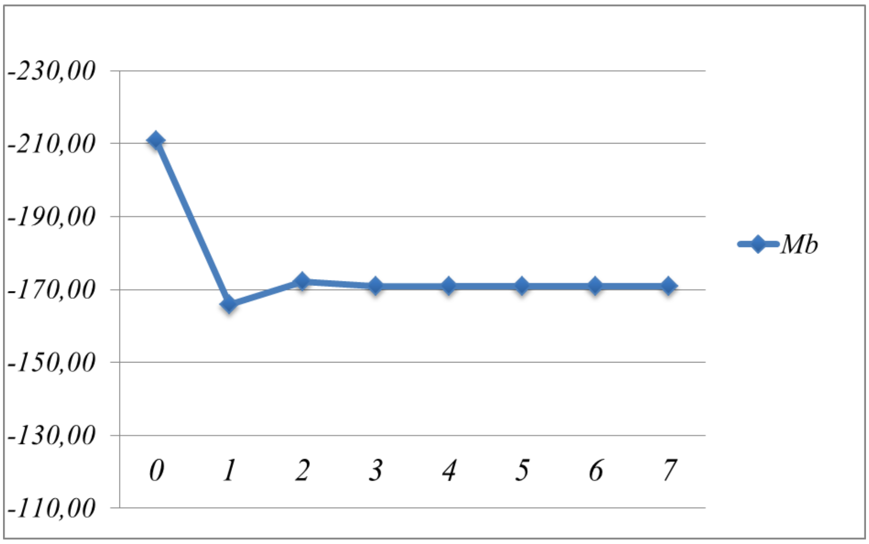
And the critical bending moment that is the main base of the 2-stiffness condition is calculated as follows:

$M_{cr} = (2\ f_{ctm} / E_{cm})\ .\ (EI_i / z_c) = (2\ .\ 2{,}20.10^{+06} / 3{,}00.10^{+10})\ .\ (4{,}30.10^{+07}.\ -8{,}24.10^{-02})$

$M_{cr} = -76{,}57\ kN.m$

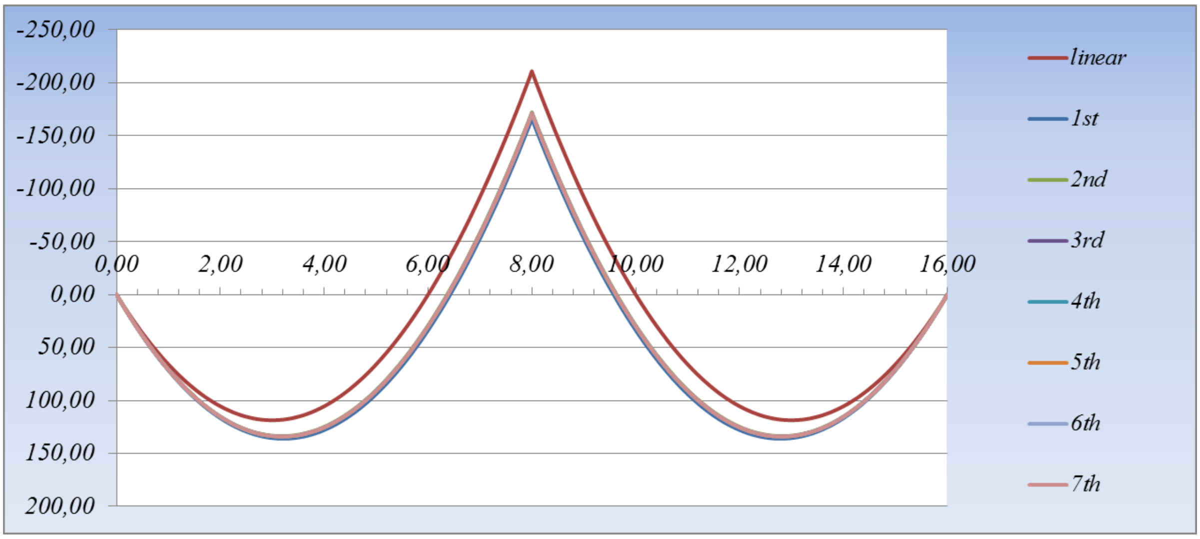
No. of step	No.	M_b
Linear step	0	-210,84
1st step	1	-166
2nd step	2	-172,073
3rd step	3	-170,867
4th step	4	-170,867
5th step	5	-170,867
6th step	6	-170,867
7th step	7	-170,867

Table 4.3: *results of every step after applying the non-linearity using the 2-stiffness value condition*



Graph 4.2: *bending moments obtained for every step of non-linearity using the 2-stiffness values condition*

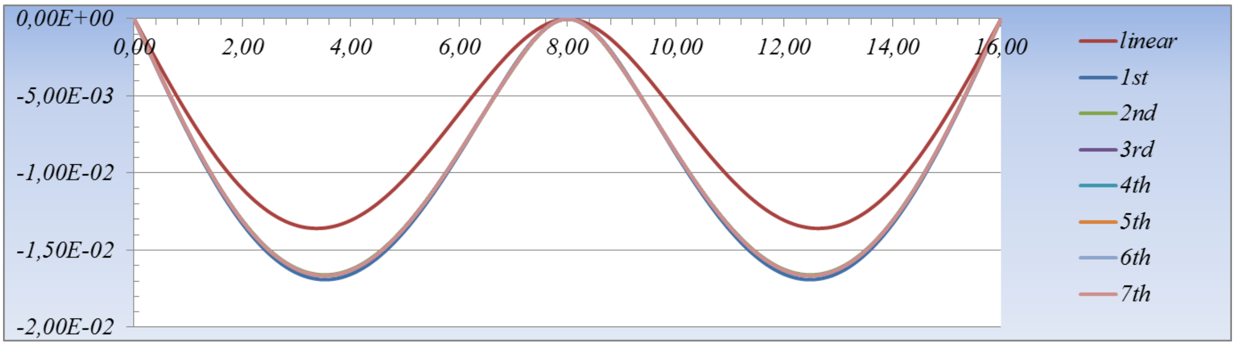
Then the redistribution of bending moments along the whole condinuous beam looks after the 7 iterations as the follows:



Graph 4.3: *redistribution of bending moments in 7 iterations along the 2-span continuous beam using the 2 stiffness values condition*

As we can note from the previous graphs that the bending moment over the internal support dampens till **18,96%** from the original solution of the linear calculation making up a new distribution of bending moments along the whole beam as it is with every non-linear step on the way till the 7th (last) solution.

The deflections go as follows:



Graph 4.4: *Deflections of 2-span continuous composite beams under the effect of 2 stiffness values condition non-linearity*

We repeat the previous calculation using 3 stiffness values condition where:

- Stiffness values used:

$EI_{ab1} = 4,30 \cdot 10^{+07}\ N.m^2$ (Concrete slab + Steel bar in the span, no cracks in concrete part)

$EI_{ab2} = 2,04 \cdot 10^{+07}\ N.m^2$ (Steel bar + Reinforcement over the support, cracks occur)

$EI_{ab3} = 4,38 \cdot 10^{+07}\ N.m^2$ (Concrete slab + Steel bar + Reinforcement in the negative moment area before it gets to cracking)

- Critical moment of concrete slab type C20/25 of thickness $a = 0,05\ m$, which is part of a composite beam having IPE270 steel section:

$M_{cr} = (2\ f_{ctm} / E_{cm}) \cdot (EI_i / z_c) = (2 \cdot 2,20.10^{+06} / 3,00.10^{+10}) \cdot (4,30.10^{+07} \cdot -8,24.10^{-02})$

$M_{cr} = -76,57\ kN.m$

And we get the following results of 7 iterations after applying the previous conditions which go as follows:

No. of step	No.	Mb
Linear step	0	-210,84
1st step	1	-166,012
2nd step	2	-172,225
3rd step	3	-170,995
4th step	4	-170,995
5th step	5	-170,995
6th step	6	-170,995
7th step	7	-170,995

Table 4.4: *results of every step after applying the non-linearity using the 3 stiffness values condition*

As we can notice from the previous table that the redistribution reaches **18,90%** using 3 stiffness values condition.

For now we can try a different example from the previous where we could consider a 2-span continuous beam lifting two rooms with different spans.

By the time we reach calculation of the beam's effective width, then we should not take a fixed value along the whole beam but we have to differ according to the position we are focusing on.

In the first span (a side span):

$L_{e1} = 0,80 \,.\, L_1 = 0,80 \,.\, 8 = 6,4\ m$

$b_{eff,1} = L_{e1} / 4 = 6,4 / 4 = 1,6\ m$

In the second span (a side span):

$L_{e2} = 0,80 \,.\, L_2 = 0,80 \,.\, 4 = 3,2\ m$

$b_{eff,2} = L_{e2} / 4 = 3,2 / 4 = 0,80\ m$

Loads:

Permanent load:

Concrete slab: $\gamma_{concrete} \,.\, a \,.\, b = 25 \,.\, 0,05 \,.\, 2,0 = 2,50\ kN/m$

Concrete filling: $\gamma_{concrete} \,.\, a_{gap}$ *.coeff. of trap plate* $.\, b = 25 \,.\, 0,05 \,.\, 0,6 \,.\, 2,0 = 1,50\ kN/m$

IPE 270 : *0,361 kN/m*

The trapezoidal plate: *0,05 kN/m*

Floor tiles: *4 kN/m*

$g_k = 2,50 + 1,50 + 0,361 + 0,05 + 4 = 8,411\ kN/m$

$g_d = g_k \,.\, 1,5 = 8,411 \,.\, 1,35 = 11,355\ kN/m$

Living load:

According to Eurocode 1991: C4

$q_k = 5,0\ kN/m2$

$q_k = 5,0 \,.\, 2,0 = 10,0\ kN/m$

$q_d = 10,0 \,.\, 1,5 = 15,0\ kN/m$

Total design load = *11,355 + 15,0 = 26,355 kN/m*

Using the composite beam with IPE 270 steel section and the previous effective lengths, we are going to obtain stiffness values that are:

- For the first span (ab span): $l_{ab} = 8\ m$

$EI_{ab} = 4{,}30\ .\ 10^{+07}$ (Concrete slab + Steel bar in the span, no cracks in concrete part)

$EI_{ab} = 2{,}04\ .\ 10^{+07}$ (Steel bar + Reinforcement over the support, cracks occur)

$EI_{ab} = 4{,}38\ .\ 10^{+07}\ N.m^2$ (Concrete slab + Steel bar + Reinforcement in the negative moment area before it gets to cracking)

- For the second span (bc span): $l_{bc} = 4\ m$

$EI_{bc} = 3{,}60\ .\ 10^{+07}$ (Concrete slab + Steel bar in the span, no cracks in concrete part)

$EI_{bc} = 2{,}04\ .\ 10^{+07}$ (Steel bar + Reinforcement over the support, cracks occur)

$EI_{ab} = 3{,}78\ .\ 10^{+07}\ N.m^2$ (Concrete slab + Steel bar + Reinforcement in the negative moment area before it gets to cracking)

In our case we will use the non-linear redistribution by the use of stiffness change along the beam where the made programme on Excel assures the non-linearity in specifying the jump from one stiffness to the other.

After applying seven iterations on these stiffnesses as it was explained in the statical programme part, then we will get the following bending moments over the internal support *b* using 2-stiffness value condition and 3-stiffness value condition as follows:

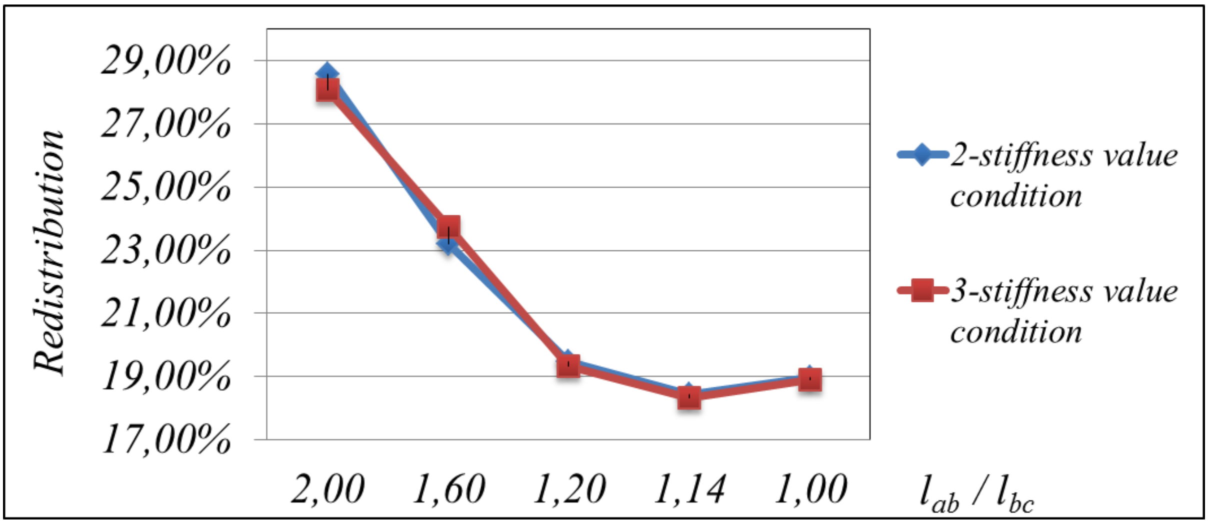
No. of step	No.	M_b	
		2 stiffness values	3 stiffness values
Linear step	0	-151,700	-151,700
1st step	1	-99,217	-99,304
2nd step	2	-111,961	-112,997
3rd step	3	-107,400	-107,610
4th step	4	-108,352	-109,132
5th step	5	-108,352	-109,132
6th step	6	-108,352	-109,132
7th step	7	-108,352	-109,132
Final Redistribution		28,58%	28,06%

Table 4.5: *comparison between 2 stiffness values and 3 stiffness values conditions in their effect of redistributing process*

We repeat the previous calculations on various 2-span continuous composite beams where we will keep one span length fixed in all of them $l_{ab} = 8\ m$ and we will change the other one l_{bc}, where we will keep the same load as before for which we get the following results:

	Redistribution	
l_{ab}/l_{bc}	2 stiffness values	3 stiffness values
2,00	28,58%	28,06%
1,60	23,20%	23,74%
1,33	19,49%	19,34%
1,14	18,43%	18,33%
1,00	18,96%	18,90%

Table 4.6: *comparison between 2 stiffness values and 3 stiffness values conditions in their final redistributions for various span lengths reffered by the ratio* l_{ab}/l_{bc}



Graph 4.5: *Redistribution of a 2-span continuous composite beams having the same applied load of 26,355 kN/m and fixed span length* $l_{ab} = 8$ *m, where the change of span length is described using* l_{ab}/l_{bc} *ratio*

5.2. 2-span continuous beam - plasticity redistribution:

After discussing concrete crack effect on redistributing the internal forces along the whole continuous beam, the structure comes to a point when the only bearing part of the beam is steel which plasticizes because of the overload it gets.

For our case we will keep dealing with our beam of a composite cross-section made up of Steel *S235*, *IPE 270* and concrete slab of *C20/25* with thickness of $a = 0{,}05$ *m* .

First we need to obtain the stress-strain diagram according which we will get to the non-linear calculations of steel plasticity by getting the necessary points as follows:

- For point I. , where there is no load the values of $(\rho, M) = (0, 0)$.

- Point II. is calculated from the critical bending moment of concrete in the composite cross-section which is:

$M_{cr} = (2\ f_{ctm} / E_{cm})\ .\ (EI_i / z_c) = (2\ .\ 2{,}20.10^{+06} / 3{,}00.10^{+10})\ .\ (4{,}30.10^{+07}.\ -8{,}24.10^{-02})$

$M_{cr} = -76{,}57\ kN.m$

Where:

EI_i stiffness of composite cross-section of *IPE 270*, steel *S253* and concrete *C20/25* with thickness $a = 0{,}05\ m$, and effective width of $b_{eff} = 1{,}6\ m$.

z_c distance from the centre of the composite cross-section to upper concrete slab face.

We obtain its curvature as: $\rho = M_{cr} / EI_i = -76{,}57.10^{+03} / 4{,}30.10^{+07} = -1{,}78.10^{-03}\ m^{-1}$

- The III. point is having the same curvature as point II. , but stiffness is dropped since the considered cross-section is for *IPE 270* steel bar and the *14Φ10* reinforcing bars, for which a new bending moment is obtained as:

  $M = \rho\ .\ EI_i = -1{,}78.10^{-03}\ .\ 2{,}04.10^{+07} = -36{,}27\ kN.m$

- Point IV. must be obtained by yield bending moment of steel S235 IPE 270 section only, which is equal to:

  $M = (I_y / z)\ .\ f_y = (5{,}79.10^{-05} / 0{,}135)\ .\ 2{,}35.10^{+08} = -100{,}79\ kN.m$

  And the stiffness here is still considered the one for steel bar and the reinforcement, which we can use to obtain the curvature as follows:

  $\rho = M / EI_i = -100{,}79.10^{+03} / 2{,}04.10^{+07} = -4{,}95.10^{-03}\ m^{-1}$

- The last point which is point V. will be calculated using the resistance bending moment of the section (Steel + Reinforcement) $M_{pl,Rd} = -179{,}386\ kN.m$ and its curvature is obtained as follows:

  Taking $\varepsilon_{uk} = -0{,}025$ *(ČSN EN 1992-1-1)*

  $z = h_{TOTAL} - x_{RFCMT} = 0{,}37 - 0{,}109 = 0{,}2605\ m$

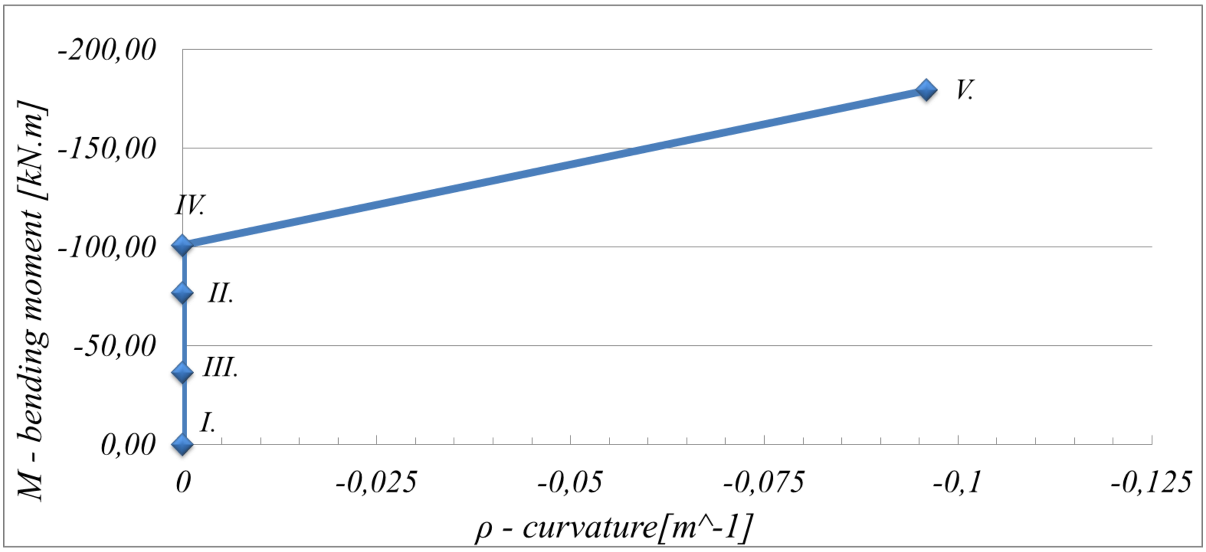
Where:

h_{TOTAL} total height of the whole composite cross-section

x_{RFCMT} distance from the centre of cross-section to reinforcement

$\rho = \varepsilon_{uk} / z = -0{,}025 / 0{,}2605 = -0{,}09596\ m^{-1}$

The diagram of bening moment-curvature (stress-strain) will be determined as follows:



Graph 4.6: *Bending moment-curvature diagram for the composite section*

We start executing the calculation that was explained before on our example remembering again the following:

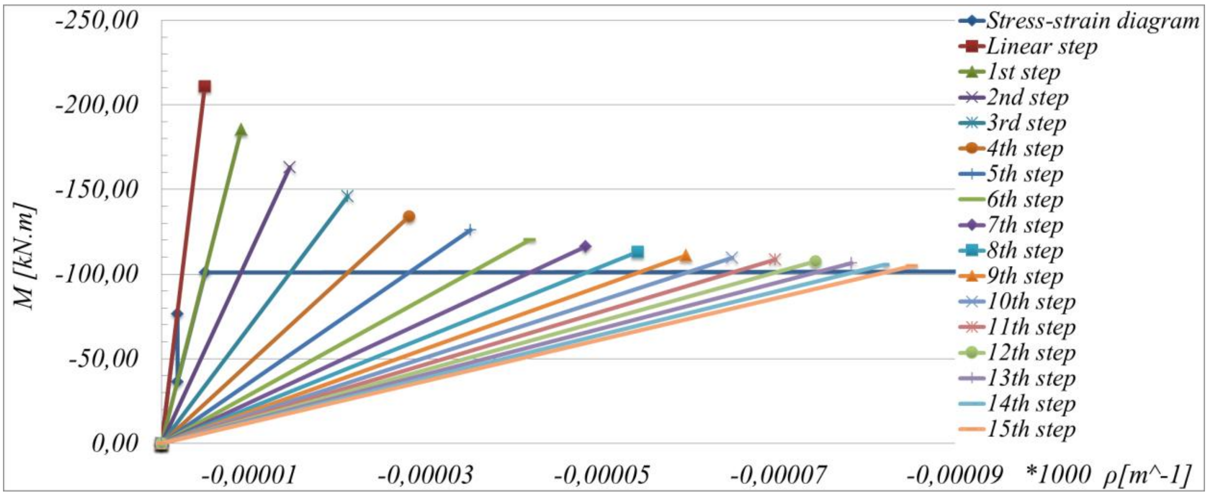
- 2-span continuous beam, where $l_{ab} = 8\ m$ and $l_{bc} = 8\ m$.
- Composite cross-section: *S235, IPE 270, C20/25, a = 0,05 m, beff = 1,6m.*

After calculation executing we obtain the following for the linear step and all the rest results coming from the iteration:

No. of step	No.	Mb
Linear step	0	-210,84
1st step	1	-185,284
2nd step	2	-162,862
3rd step	3	-145,957
4th step	4	-134,063
5th step	5	-125,841
6th step	6	-120,142
7th step	7	-116,132
8th step	8	-113,202
9th step	9	-111,112
10th step	10	-109,642
11th step	11	-108,47
12th step	12	-107,388
13th step	13	-106,407
14th step	14	-105,534
15th step	15	-104,768

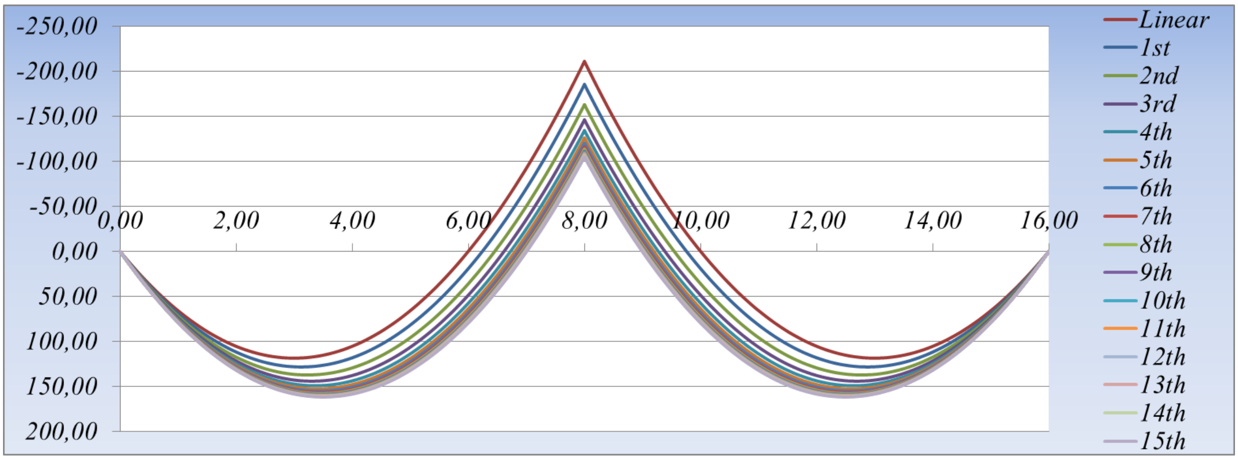
Table 4.7: *redistribution process through plasticity condition on 2-span continuous beam*

The non-linear calculation was using an iterational method taking the obtained bending moment-curvature (stress-strain) diagram as a base for calculation, which was proceeded for our case as follows:



Graph 4.8: *Iterational process of calculating plasticity effect using the bending moment-curvature diagram over the internal support b*

This overview is showing the solution at one cross-section of the internal support *b*, where the plasticity of the steel bar starts and affects the whole structure where the distibution ove internal forces relocates on the whole beam as follows:



Graph 4.8: *redistribution using plasticity condition over the 2-span beam by 15 iterations*

According to the whole process we get to a conclusion of this example as follows:

Type	ČSN EN 1994-1-1	Non-linearity solution after 15 iterations
%	40	50,31

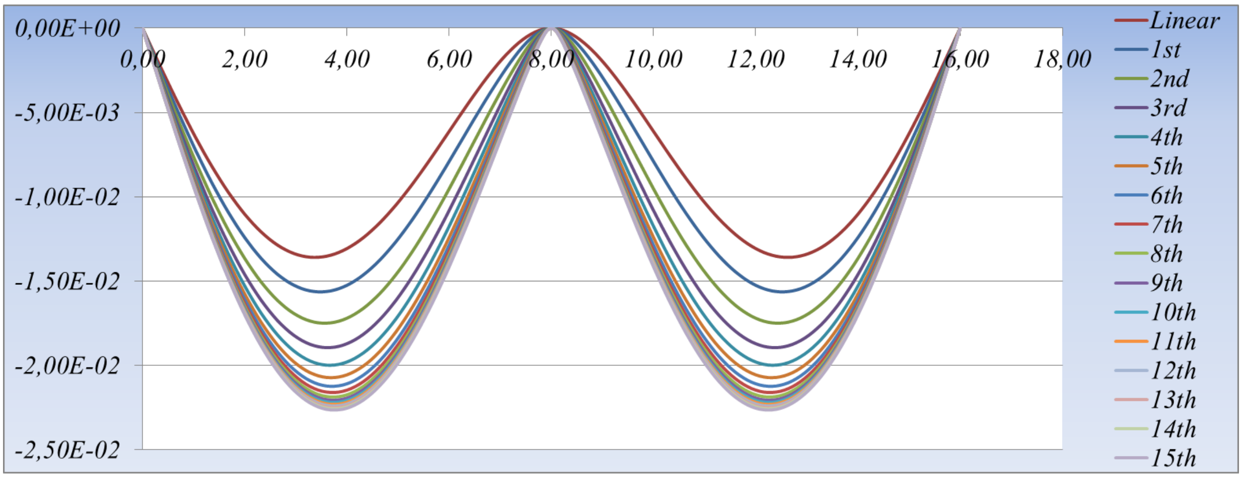
Table 4.8: *Comparison the national code with plasticity condition effect*

The effect of plasticity influences even the deflection of the spans that goes for every iteration as follows:

No. of step	No.	Max w [m]
Linear step	0	-0,013588
1st step	1	-0,015632
2nd step	2	-0,017489
3rd step	3	-0,018938
4th step	4	-0,019983
5th step	5	-0,020718
6th step	6	-0,021231
7th step	7	-0,021595
8th step	8	-0,021864
9th step	9	-0,022056
10th step	10	-0,022191
11th step	11	-0,022298
12th step	12	-0,022398
13th step	13	-0,022488
14th step	14	-0,022568
15th step	15	-0,022638

Table 4.9: *plasticity condition effect on rising deflection by every iteration proceeded*

As we can notice from the results that in the first iteration, the deflection drops dramatically and it dampens with every iteration in forward until it looks so close in the last ones. The way it is shown in the following:

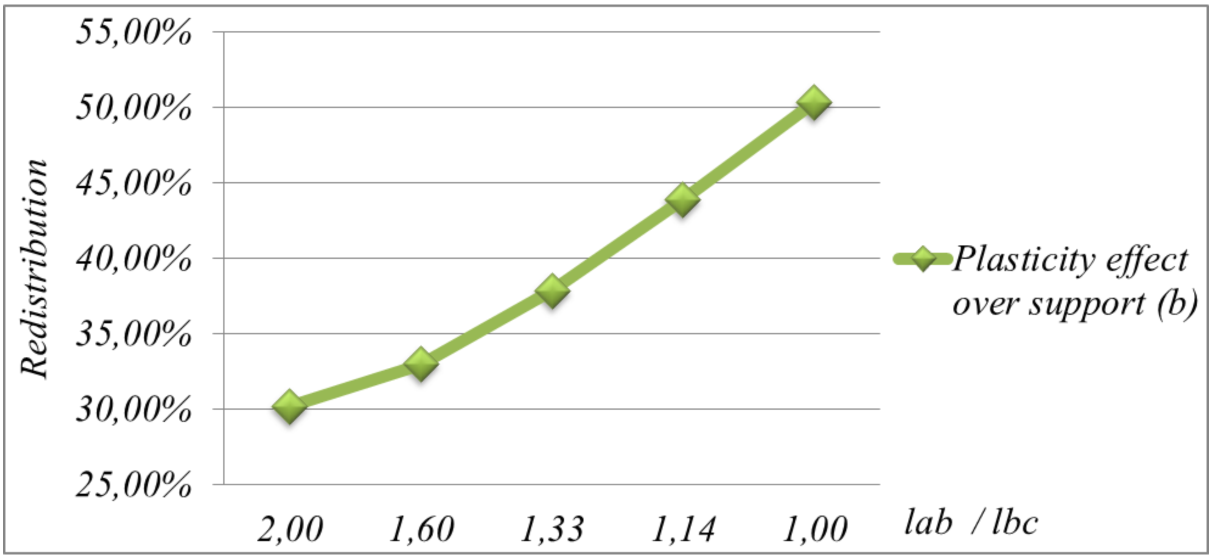


Graph 4.9: *deflection rise on the 2-span continuous beam by plasticity condition effect*

We will try to make the previous analysis on various continuous beams with diffrenet span length ratios keeping the first span $l_{ab} = 8\ m$, and take it as a base for the other span l_{bc} to change. Where redistributions of every beam reach the following:

l_{ab}/l_{bc}	Redistribution over support *b*
2,00	30,16%
1,60	32,95%
1,33	37,80%
1,14	43,85%
1,00	50,31%

Table 4.10: *plasticity condition effect on 2-span continuous beam with various span lengths described by the ratio* l_{ab}/l_{bc} *with* $l_{ab} = 8m$



Graph 4.10: *deflection rise for the 2-span continuous beam of fixed* $l_{ab} = 8m$ *and changing* l_{bc} *according to the ratio* l_{ab}/l_{bc}

5.3. 3-span continuous beam – redistribution

From the interesting indeterminate structures where of the internal forces redistribution due to cracks of concrete must be on the 3-span continuous beam. It is a practical example as well, where it can be used for various structures of multi-purpose spaces.

Let us have one 3-span continuous beam where the side spans belong to fabrication rooms in time the middle span belongs to a corridor that leads to the rooms aside. For this we could define the following:

We take the same used cross-section as in the 2-span beam that is having IPE 270 as the steel part and for the purposes mentioned before we could divide the continuous beam by:

The first span l_{ab} (a side span):

$L_1 = 8\ m$

$L_{e1} = 0{,}80\ .\ L_1 = 0{,}80\ .\ 8 = 6{,}4\ m$

$b_{eff,1} = L_{e1} / 4 = 6{,}8 / 4 = 1{,}6\ m$

The second span l_{bc} (the middle):

$L_2 = 4\ m$

$L_{e2} = 0{,}70\ .\ L_2 = 0{,}70\ .\ 4 = 2{,}8\ m$ *(0,80 for middle spans accord. ČSN EN 1994-1-1)*

$b_{eff,2} = L_{e2} / 4 = 2{,}8 / 4 = 0{,}7\ m$

The third span l_{cd} (a side span):

$L_3 = 8\ m$

$L_{e3} = 0{,}80\ .\ L_3 = 0{,}80\ .\ 8 = 6{,}4\ m$

$b_{eff,3} = L_{e3} / 4 = 6{,}8 / 4 = 1{,}6\ m$

Loads:

Permanent load:

Concrete slab: $\gamma_{concrete}\ .\ a\ .\ b_{eff,1} = 25\ .\ 0{,}05\ .\ 2{,}0 = 2{,}5\ kN/m$

Concrete filling: $\gamma_{concrete}\ .\ a_{gap}$ *.coeff. of trap plate* $.\ b_{eff,1} = 25\ .\ 0{,}05\ .\ 0{,}6\ .\ 2{,}0 = 1{,}5\ kN/m$

IPE 270 : *0,361 kN/m*

The trapezoidal plate: *0,05 kN/m*

Floor tiles: *4 kN/m*

$g_k = 2{,}5 + 1{,}5 + 0{,}361 + 0{,}05 + 4 = 8{,}411\ kN/m$

$g_d = g_k\ .\ 1{,}35 = 8{,}411\ .\ 1{,}35 = 11{,}355\ kN/m$

Living load:

According to Eurocode 1991: C4

$q_k = 5{,}0\ kN/m2$

$q_k = 5{,}0\ .\ 2{,}0 = 10{,}0\ kN/m$

$q_d = 10{,}0\ .\ 1{,}5 = 15{,}0\ kN/m$

Total design load = *11,355 + 15,0 = 26,355 kN/m*

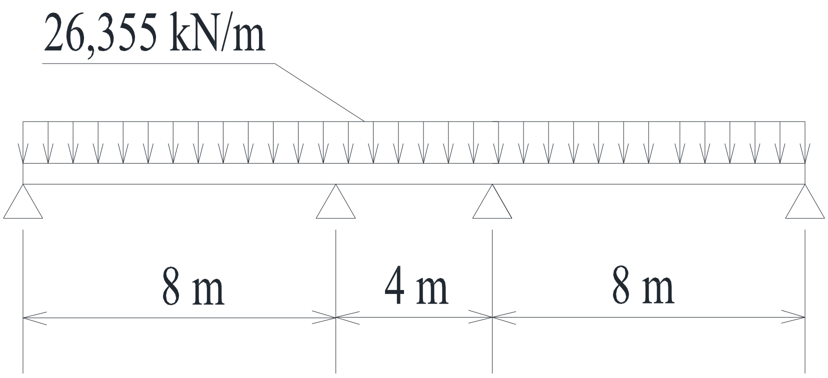


Figure 4.2: *a continuous beam with 3 spans*

As mentioned before choosing the same cross-section as used in the 2-span beam we would get the following stiffnesses using Excel cross-section programme as follows:

- For the first and the third span (*ab* and *cd* span):

$EI_{ab1} = 4,30 \,.\, 10^{+07}$ (Concrete slab + Steel bar in the span, no cracks in concrete part)

$EI_{ab2} = 2,04 \,.\, 10^{+07}$ (Steel bar + Reinforcement over the support, cracks occur)

- For the second span (*bc* span):

$EI_{bc1} = 3,46 \,.\, 10^{+07}$ (Concrete slab + Steel bar in the span, no cracks in concrete part)

$EI_{bc2} = 2,04 \,.\, 10^{+07}$ (Steel bar + Reinforcement over the support, cracks occur)

Now we have everything we need to calculate the beam in a linear way using the values obtained.

Since we have the geometry of the continuous beam where: $l_{ab} = 8\ m,\ l_{bc} = 4\ m,\ l_{cd} = 8\ m,$ then we obtain the following angles:

$\alpha_{ab}=\alpha_{ba} = l_{ab}/3EI_{ab} = 8 / (3 \,.\, 4,30 \,.\, 10^{+07}) = 6,20.10^{-08}$

$\alpha_{bc}=\alpha_{cb} = l_{bc}/3EI_{bc} = 4 / (3 \,.\, 3,46 \,.\, 10^{+07}) = 3,86.10^{-08}$

$\alpha_{cd}=\alpha_{dc} = l_{cd}/3EI_{cd} = 8 / (3 \,.\, 4,30 \,.\, 10^{+07}) = 6,20.10^{-08}$

$\beta_{ab}=\beta_{ba}= l_{ab}/6EI_{ab} = 8 / (6 \,.\, 4,30 \,.\, 10^{+07}) = 3,10.10^{-08}$

$\beta_{bc}=\beta_{cb}= l_{bc}/6EI_{bc} = 4 / (6 \,.\, 3,46 \,.\, 10^{+07}) = 1,93.10^{-08}$

$\beta_{cd}=\beta_{dc}= l_{cd}/6EI_{cd} = 8 / (6 \,.\, 4,30 \,.\, 10^{+07}) = 3,10.10^{-08}$

$\varphi_{ab}=\varphi_{ba}= (1/24) \,.\, (q.l_{ab}^3/EI_{ab}) = (1/24) \,.\, (26,355 \,.\, 8^3/ 4,30 \,.\, 10^{+07}) = 1,31.10^{-05}$

$\varphi_{bc}=\varphi_{cb}= (1/24) \,.\, (q.l_{bc}^3/EI_{bc}) = (1/24) \,.\, (26,355 \,.\, 4^3/ 3,46 \,.\, 10^{+07}) = 2,03.10^{-06}$

$\varphi_{cd}=\varphi_{dc}= (1/24) \,.\, (q.l_{cd}^3/EI_{cd}) = (1/24) \,.\, (26,355 \,.\, 8^3/ 4,30 \,.\, 10^{+07}) = 1,31.10^{-05}$

After getting them we go use them in equation where bending moments over the supports are determined as follows:

$M_a \,.\, \beta_{ba} + M_b \,.\, (\alpha_{ba} + \alpha_{bc}) + M_c \,.\, \beta_{bc} + \varphi_{ba} + \varphi_{bc} = 0$

$M_b \,.\, \beta_{cb} + M_c \,.\, (\alpha_{cb} + \alpha_{cd}) + \; M_d \,.\, \beta_{cd} + \varphi_{cb} + \varphi_{cd} = 0$

Since there no bending moment over the side hinge supports, where: $M_a = 0\ kN/m$, $M_d = 0\ kN/m$ then:

$M_b \,.\, (\alpha_{ba} + \alpha_{bc}) + \; M_c \,.\, \beta_{bc} + \varphi_{ba} + \varphi_{bc} = 0$

$M_b \,.\, \beta_{cb} + M_c \,.\, (\alpha_{cb} + \alpha_{cd}) + \varphi_{cb} + \varphi_{cd} = 0$

After substituting the symbols by numbers we have then the equations look as follows:

$M_b \,.\, (6{,}20.10^{-08} + 3{,}86.10^{-08}) + \; M_c \,.\, 1{,}93.10^{-08} + 1{,}31.10^{-05} + 2{,}03.10^{-06} = 0$

$M_b \,.\, 1{,}93.10^{-08} + M_c \,.\, (3{,}86.10^{-08} + 6{,}20.10^{-08}) + 2{,}03.10^{-06} + 1{,}31.10^{-05} = 0$

Once we have two equations with two variables then we get the bending moments over the supports *b* and *c* that equal to:

$M_b = -126{,}01\ kN/m$

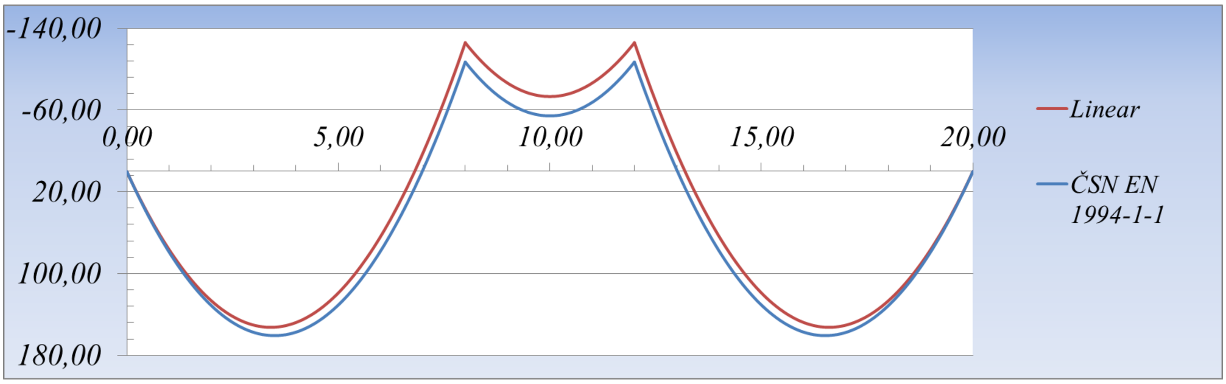
$M_c = -126{,}01\ kN/m$

As the redistribution of the internal forces is limited by *15%* according to ČSN EN 1994-1-1 , then we obtain the bending moments over the internal supports as follows:

	M_b , M_c [kN.m]	Max M in *ab* and *dc* spans [kN.m]
Linear Step	-126,01	152,52
EN 1994-1-1 Redistribution	-107,11	160,68

Table 4.11: *Bending moment values over the internal support and the maximum positive bending moments in the side spans, with its change due to distribution determined by the national code ČSN EN 1994-1-1*

Where it affects the distribution of internal forces along the beam as shown in bending moments diagram:



Graf 4.11: *Redistribution of bending moments along the 3-span continuous composite beam regarding the national code ČSN EN 1994-1-1*

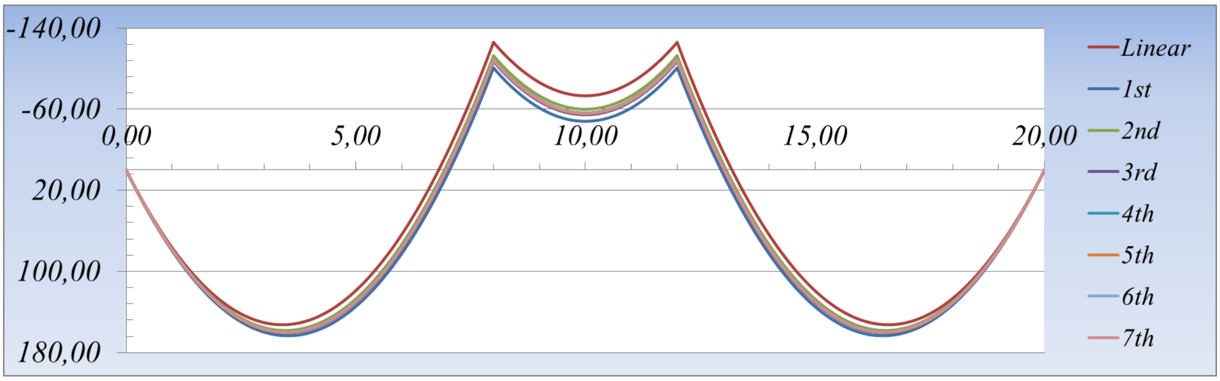
By using the conditions of the nonlinear solution of our program made using Microsoft Excel, we will get the oscillation of bending moments until it gets to the optimal redistibution of bending moments in the 4th iteration as shown in the table:

No. of Step	No.	M_b [kN.m]	M_c [kN.m]
Linear step	0	-126,008	-126,008
1st	1	-100,856	-100,856
2nd	2	-112,794	-112,794
3rd	3	-107,297	-107,297
4th	4	-109,254	-109,254
5th	5	-108,384	-108,384
6th	6	-108,958	-108,958
7th	7	-108,958	-108,958

Table 4.12: *results for both internal supports b and c of every step*

after applying the non-linearity using the 2-stiffness value condition

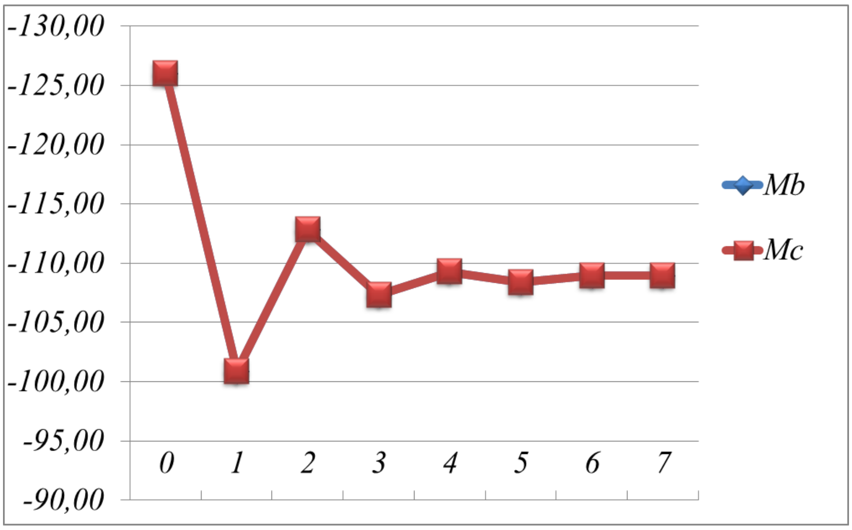
Where the redistribution along the whole beam for all iterations look as follows:



Graph 4.12: *redistribution of bending moments in 7 iterations*

along the 3-span continuous beam using the 2 stiffness values condition

As we note here we can see the redistribution of bending moments over the supports reaches **13,53%** .



Graph 4.13: *the iterational non-linear process of bending moments obtained using the 2-stiffness values condition (here M_b and M_c are the same)*

We will try with the option of having three stiffness values where:

- For the first and third spans (*ab* and *cd* span):

$EI_{ab1} = 4{,}30 \,.\, 10^{+07}$ (Concrete slab + Steel bar in the span, no cracks in concrete part)

$EI_{ab2} = 2{,}04 \,.\, 10^{+07}$ (Steel bar + Reinforcement over the support, cracks occur)

$EI_{ab3} = 4{,}38 \,.\, 10^{+07}$ (Concrete slab + Steel bar + Reinforcement in the negative moment area before it gets to cracking)

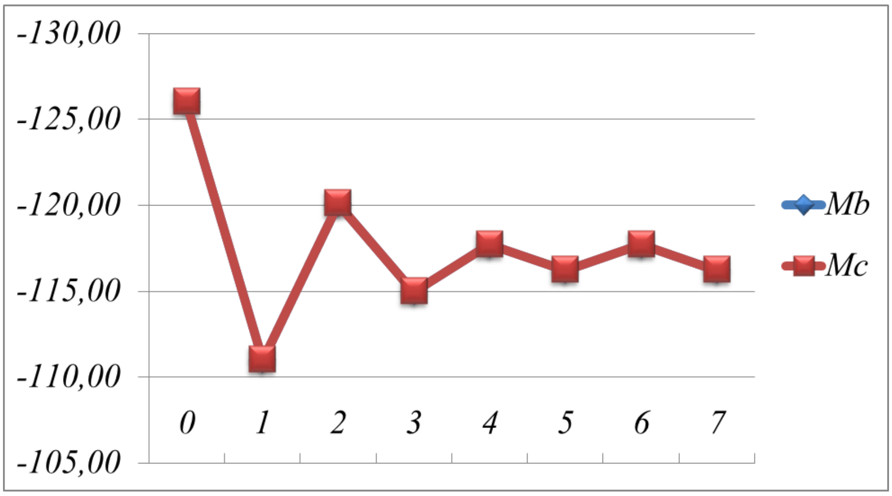
- For the second span (*bc* span):

$EI_{bc1} = 3{,}46 \,.\, 10^{+07}$ (Concrete slab + Steel bar in the span, no cracks in concrete part)

$EI_{bc2} = 2{,}04 \,.\, 10^{+07}$ (Steel bar + Reinforcement over the support, cracks occur)

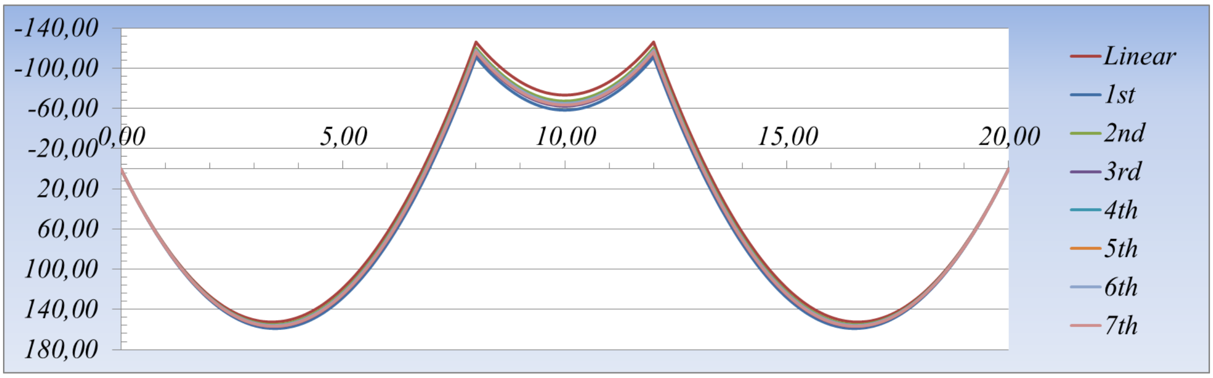
$EI_{bc3} = 3{,}66 \,.\, 10^{+07}$ (Concrete slab + Steel bar + Reinforcement in the negative moment area before it gets to cracking)

Then we execute the non-linear calculations where we get the following redistribution:

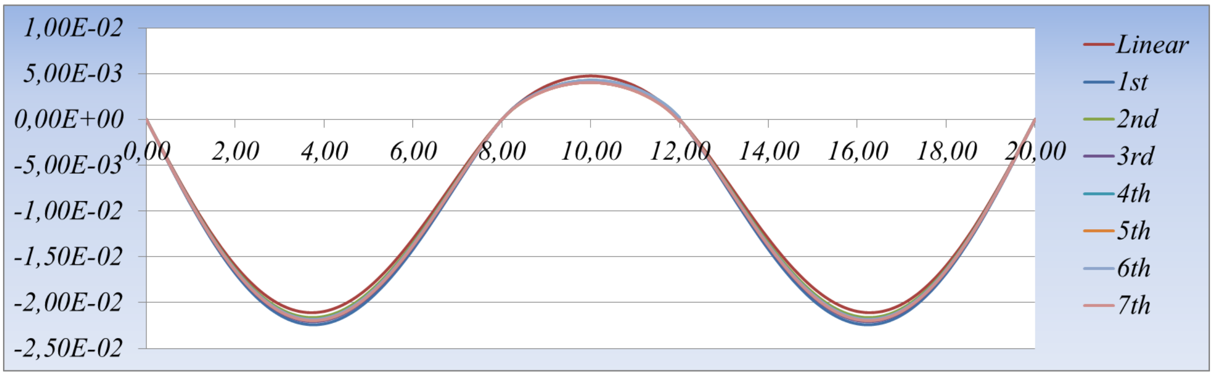


Graph 4.14: *the iterational non-linear process of bending moments obtained using the 3-stiffness values condition (here M_b and M_c are the same)*

Where we see that the redistribution reaches its optimal stage after the 7th iteration by **7,77%** when the results oscillate influencing the redistribution of bending moments along the whole beam for all iterations will look as follows:



Graph 4.15: *redistribution of bending moments in 7 iterations along the 3-span continuous beam using the 3 stiffness values condition*

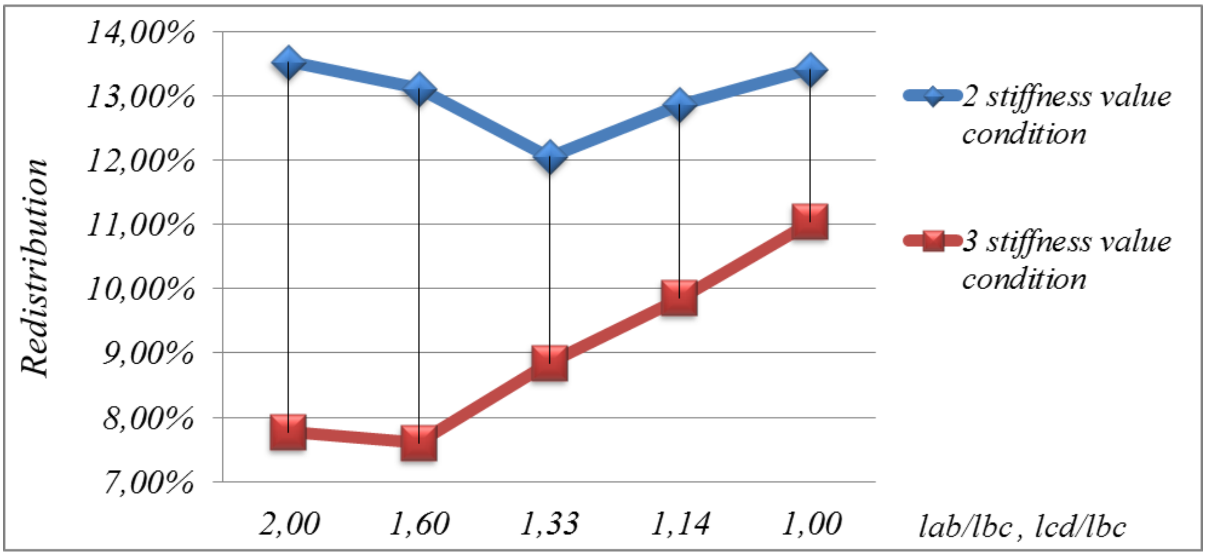


Graph 4.16: *effect on deflection using 3 stiffness values condition on 3 spans*

We would check the redistribution along the whole 3-span continuous beam of different span lengths by keeping span lengths l_{ab} and l_{cd} equal to *8 m*, in time we will change the middle span length l_{bc} . The review will be on the same cross-section with *IPE 270* and the same load of *26,355 kN/m*. For which we obtain the following:

	Redistribution over support *b* and *c*	
l_{ab}/l_{bc} , l_{cd}/l_{bc}	2 stiffness value condition	3 stiffness value condition
2,00	13,53%	7,77%
1,60	13,11%	7,60%
1,33	12,05%	8,84%
1,14	12,86%	9,85%
1,00	13,41%	11,03%

Table 4.13: *comparison between 2 stiffness values and 3 stiffness values conditions in their final redistributions for various span lengths reffered by the ratio* l_{ab}/l_{bc} *and* l_{cd}/l_{bc}



Graph 4.17: *Redistribution comparison of 2 stiffness and 3 stiffness values conditions on a 3-span continuous composite beams having the same applied load of 26,355 kN/m and fixed span lengths* $l_{ab} = l_{cd} = 8$ *m, where the change of span length is described using* l_{ab}/l_{bc} *and* l_{cd}/l_{bc} *ratios*

We will repeaat the previous calculation of the same previous example with the difference of subjecting the same load on span ab and bc, in time cd span will be loaded by its self-weight.

Where this example has the following properties:

$l_{ab} = 8\ m$, $l_{bc} = 4\ m$, $l_{cd} = 8\ m$

- For the first and third spans (*ab* and *cd* span):

$EI_{ab1} = 4{,}30 \cdot 10^{+07}$ (Concrete slab + Steel bar in the span, no cracks in concrete part)

$EI_{ab2} = 2,04 . 10^{+07}$ (Steel bar + Reinforcement over the support, cracks occur)

$EI_{ab3} = 4,38 . 10^{+07}$ (Concrete slab + Steel bar + Reinforcement in the negative moment area before it gets to cracking)

- For the second span (*bc* span):

$EI_{bc1} = 3,46 . 10^{+07}$ (Concrete slab + Steel bar in the span, no cracks in concrete part)

$EI_{bc2} = 2,04 . 10^{+07}$ (Steel bar + Reinforcement over the support, cracks occur)

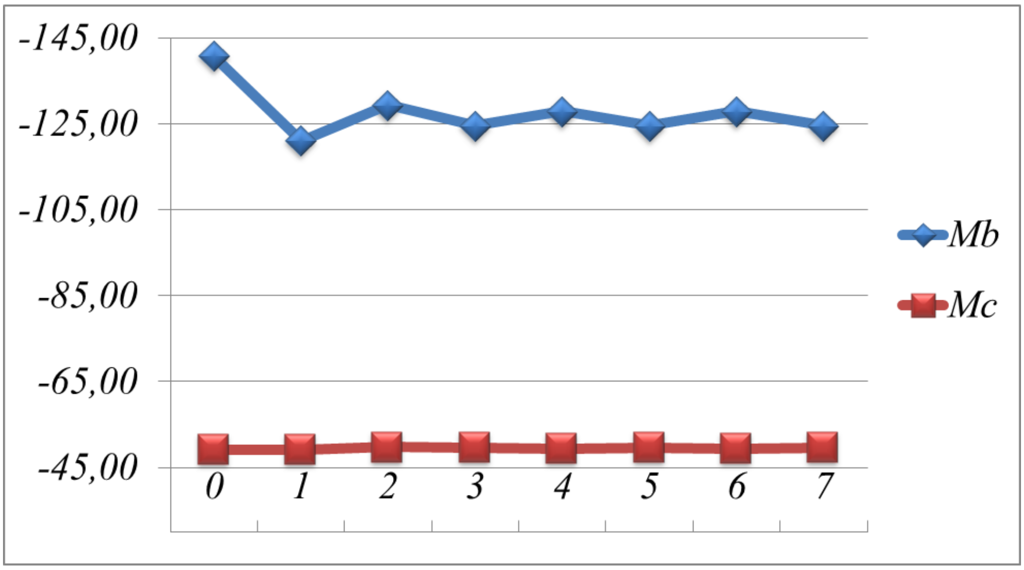
$EI_{bc3} = 3,66 . 10^{+07}$ (Concrete slab + Steel bar + Reinforcement in the negative moment area before it gets to cracking)

Loads:

Total load on span cd = Permanent load = 8,411 . 1,35 = 11,355 kN/m

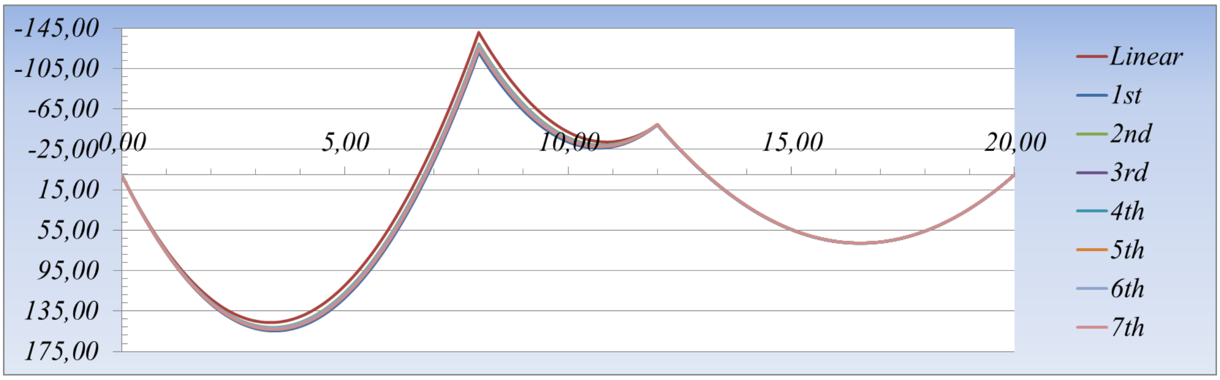
Total design load on spans ab and bc = Permanent load + living load = 11,355 + 15,0 = 26,355 kN/m

Then redistribution using the 3-stiffness condition goes as follows:



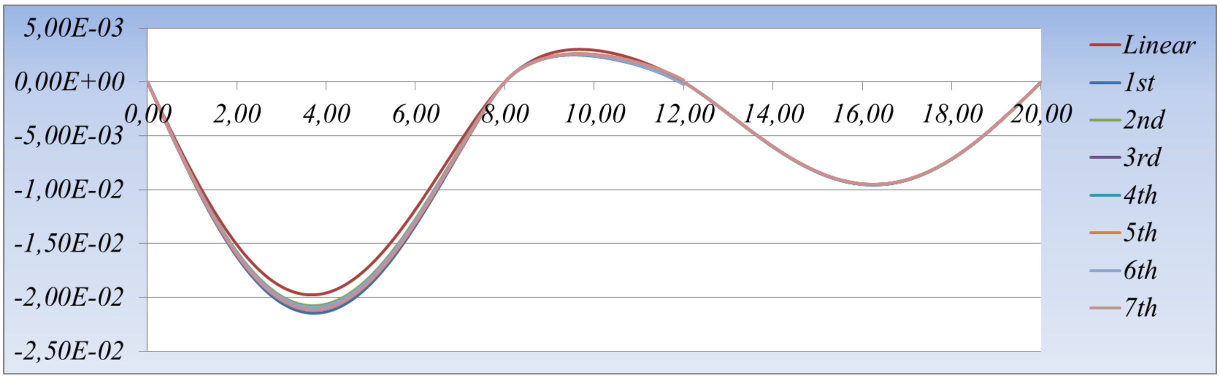
Graph 4.18: *Redistribution comparison of 2 stiffness and 3 stiffness values conditions on a 3-span continuous composite beams having the same applied load of 26,355 kN/m on ab and bc spans, while 11,355 kN/m on cd span and fixed span lengths $l_{ab} = l_{cd} = 8$ m, where the change of span length is described using l_{ab}/l_{bc} and l_{cd}/l_{bc} ratio*

Which is along the whole beam as follows:



Graph 4.19: *redistribution of bending moments in 7 iterations on 3-span continuous beam using the 3 stiffness values condition with different loads*

And its deflection goes as:



Graph 4.20: *effect on deflection using 3 stiffness values condition on 3 spans with different loads applied on the spans, where 26,355 kN/m is on spans ab and bc in time 11,355 kN/m is applied on cd*

By getting the results of bothe conditions of redistribution we would put them in the following:

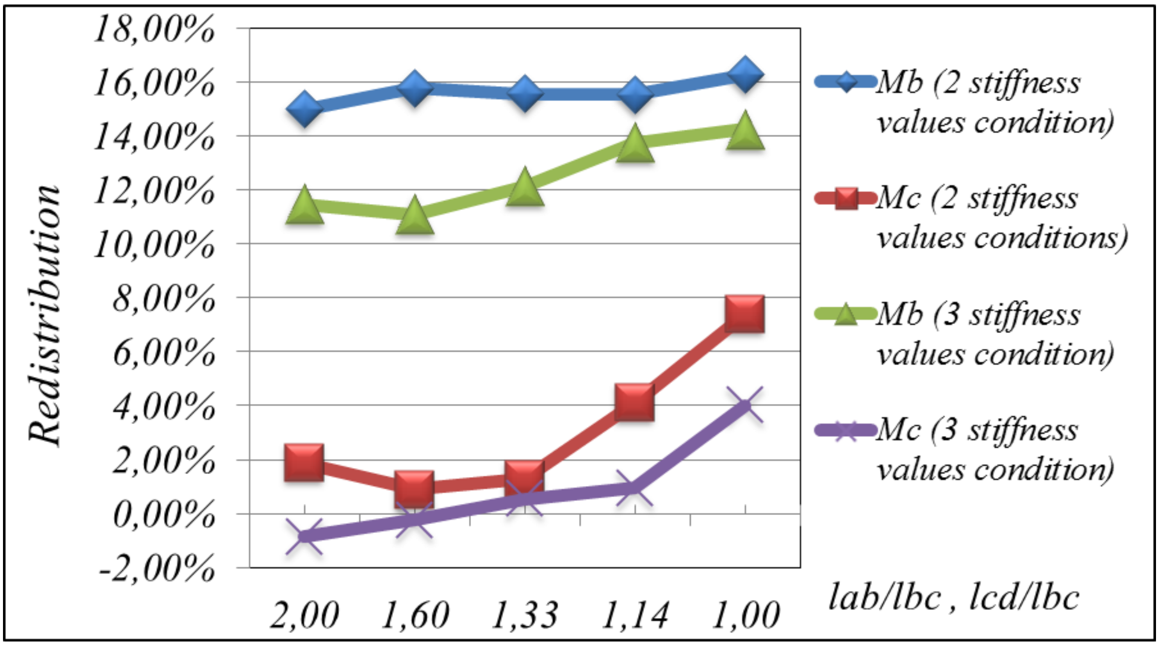
Redistribution			
2 stiffness values		3 stiffness values	
M_b	M_c	M_b	M_c
14,99%	1,86%	11,48%	-0,86%

Table 4.14: *comparison between 2 stiffness values and 3 stiffness values conditions on 3-span continuous beam*

Using different lengths of the middle span gives us the following result:

	Redistribution			
	2 stiffness values		3 stiffness values	
l_{ab}/l_{bc} , l_{cd}/l_{bc}	M_b	M_c	M_b	M_c
2,00	14,99%	1,86%	11,48%	-0,86%
1,60	15,78%	0,90%	11,08%	-0,24%
1,33	15,56%	1,27%	12,14%	0,54%
1,14	15,54%	4,11%	13,76%	0,94%
1,00	16,26%	7,38%	14,25%	4,02%

Table 4.15: *comparison between 2 stiffness values and 3 stiffness values conditions on 3-span continuous beam of loads 26,355kN/m on ab and cd, 11,355 kN/m on bc; with various span lengths using* l_{ab}/l_{bc} *and* l_{cd}/l_{bc}*. And fixed side span lengths* $l_{ab} = l_{cd} = 8m$ *are presumed*



Graph 4.21: *Redistribution comparison of 2 stiffness and 3 stiffness values conditions on a 3-span continuous composite beams having different applied loads of 26,355 kN/m on ab and cd in time 11,355 kN/m is applied on bc; and fixed span lengths* $l_{ab} = l_{cd} = 8$ *m are presumed, where the change of span length is described using* l_{ab}/l_{bc} *and* l_{cd}/l_{bc} *ratios*

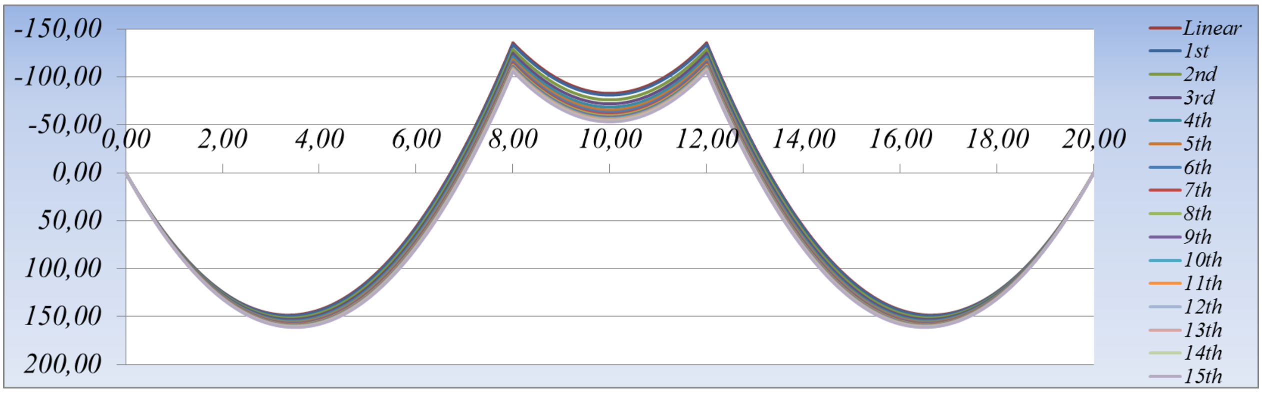
5.4. 3-span continuous beam – plasticity redistribution:

Since we are focusing on the same cross-section we dealt with in the 2-span continuous beam, we can note the fact that we can use the exact same stress strain diagram as a base for our non-linear calculations of plasticity effect. For this we will execute the calculation on our 3-span beam that has the following geometrical properties:

$l_{ab} = 8$ m, $l_{bc} = 4$m, $l_{cd} = 8$ m, Steel S235, IPE270, C20/25, a = 0,05m,

$b_{eff,ab}$ = 1,6m, $b_{eff,bc}$ = 0,7m, $b_{eff,cd}$ = 1,6m .

And th results go as follows:



Graph 4.22: *redistribution using plasticity condition over the 3-span beam by 15 iterations*

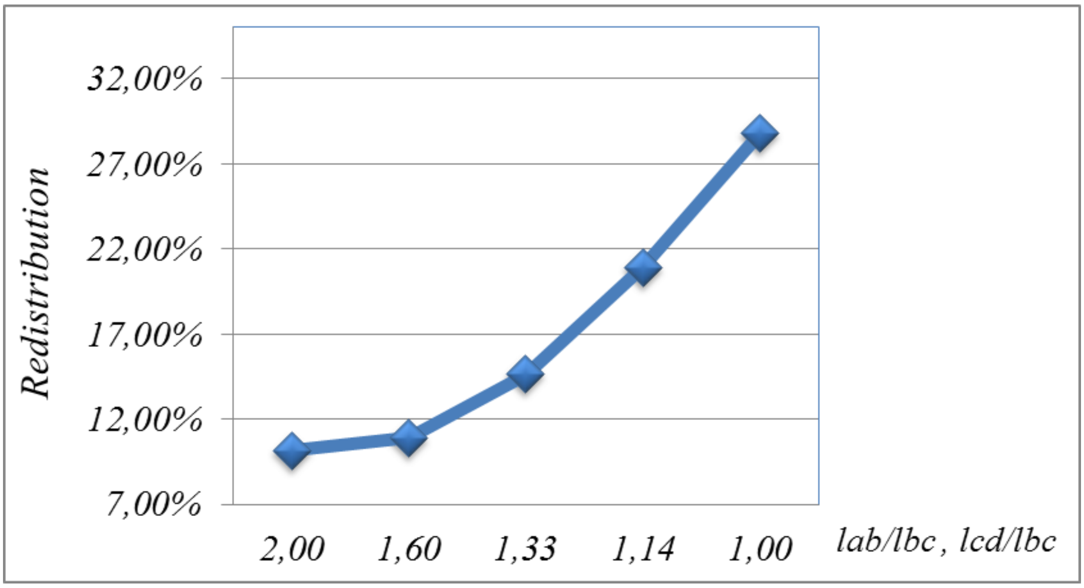
As we notice the reduction of bending moments over the supports *b* and *c* comes up to **10,16%**.

Making a review of the whole 3-span continuous beam for distribution of internal forces will be by giving the side spans *ab*, *cd* a fixed length which is equal to 8m and the middle span *bc* will be changed and according to each length we will get a different distribution that is shown in the following table:

	Redistribution over support *b* and *c*
l_{ab}/l_{bc} , l_{cd}/l_{bc}	2 stiffness value condition
2,00	10,16%
1,60	10,88%
1,33	14,62%
1,14	20,90%
1,00	28,79%

Table 4.16: *plasticity condition effect on 3-span continuous beam with various span lengths*

using the change of the middle span bc due to ratios l_{ab}/l_{bc} and l_{cd}/l_{bc} with assuming fixed span lengths l_{ab} = l_{cd} = 8m, and the same load on all spans that is equal to 26,355 kN/m



Graph 4.23: *deflection rise for 3-span continuous beam with fixed* $l_{ab} = l_{cd} = 8m$ *and changing* l_{bc} *according to the ratio* l_{ab}/l_{bc} *and* l_{cd}/l_{bc}. $M_b = M_c$

We will repaeat the previous calculation of the same previous example with the difference of subjecting the same load on span ab and bc, in time cd span will be loaded by its self-weight.

Where this example has the following properties:

$l_{ab} = 8\ m$, $l_{bc} = 4\ m$, $l_{cd} = 8\ m$

- For the first and third spans (*ab* and *cd* span):

$EI_{ab1} = 4{,}30 \cdot 10^{+07}$ (Concrete slab + Steel bar in the span, no cracks in concrete part)

$EI_{ab2} = 2{,}04 \cdot 10^{+07}$ (Steel bar + Reinforcement over the support, cracks occur)

$EI_{ab3} = 4{,}38 \cdot 10^{+07}$ (Concrete slab + Steel bar + Reinforcement in the negative moment area before it gets to cracking)

- For the second span (*bc* span):

$EI_{bc1} = 3{,}46 \cdot 10^{+07}$ (Concrete slab + Steel bar in the span, no cracks in concrete part)

$EI_{bc2} = 2{,}04 \cdot 10^{+07}$ (Steel bar + Reinforcement over the support, cracks occur)

$EI_{bc3} = 3{,}66 \cdot 10^{+07}$ (Concrete slab + Steel bar + Reinforcement in the negative moment area before it gets to cracking)

Loads:

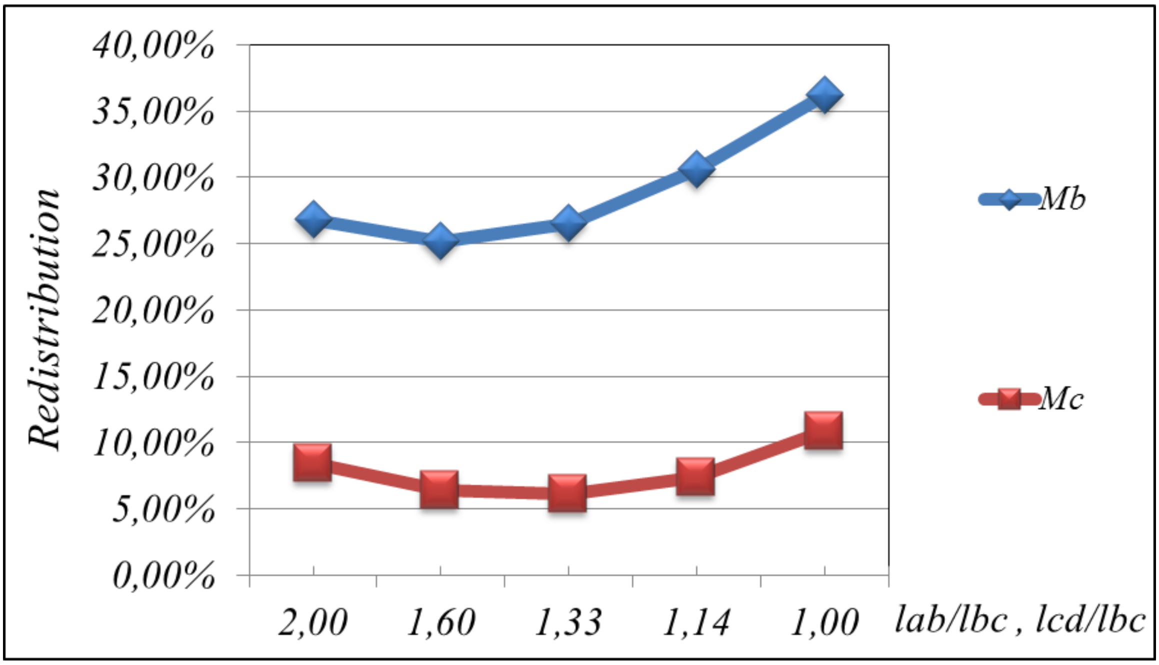
Total load on span cd = Permanent load = 8,411 . 1,35 = 11,355 kN/m

Total design load on spans ab and bc = Permanent load + living load = 11,355 + 15,0 = 26,355 kN/m

Then redistribution of this example added by other examples using various lengths of the middle span gives us the following:

	Redistribution due to plasticity	
l_{ab}/l_{bc} , l_{cd}/l_{bc}	M_b	M_c
2,00	26,84%	8,41%
1,60	25,17%	6,38%
1,33	26,55%	6,10%
1,14	30,59%	7,34%
1,00	36,21%	10,83%

Table 4.17: *plasticity condition effect on 3-span continuous beam with various span lengths using the change of the middle span bc due to ratios l_{ab}/l_{bc} and l_{cd}/l_{bc} with assuming fixed span lengths l_{ab} = l_{cd} = 8m, and the same load on spans ab and bc that is equal to 26,355 kN/m in time cd span has the load 11,355 kN/m*



Graph 4.24: *deflection rise for 3-span continuous beam with fixed l_{ab} = l_{cd} = 8m and changing l_{bc} according to the ratio l_{ab}/l_{bc} and l_{cd}/l_{bc}. With different loads on ab and bc on a hand and cd on the other hand*

5.5. Simple beam – The effect of creep (SLS):

Problems affecting the stress distribution on the beam cross-section and its deflection as shown in all examples before must have the major role in the continuous beams since a big drop in stiffness values could appear by the start of cracking, as it gives steel the chance to be an added effect on the deflection of beams when it is plasticized. These major effects on the continuous beam make other material effects as it is with creep of concrete not a big deal of consideration.

The reason for this is the nature of creep effect because of being the final result related to a long-time dependent process of stiffness drop that cannot be bigger than the one done by cracks of concrete part of the beam under the negative bending moments, therefore, to eliminate the possibility of obtained negative bending moments by eliminating the reasons causing it. For this we chose to deal with a simple beam to clear it from all effects of cracks that there is no way to appear since there is no tension on concrete side, a reason why there is no effect of plasticization of steel.

A simple beam is designed which means it is simply supported, a span length of 8 meters, which gives us the following:

Effective width: $b_{eff} = L / 4 = 8 / 4 = 2\ m$

Loads:

Permanent loads:

- Concrete slab of concrete type C20/25: $25 \,.\, 0{,}05 \,.\, 2 = 2{,}5\ kN/m$
- Concrete filling of the trapezoidal plate: $25 \,.\, 0{,}6 \,.\, 0{,}05 \,.\, 2 = 1{,}5\ kN/m$
- Steel section *IPE 240* of steel S235: $0{,}307\ kN/m$
- Floor tiles with partition walls: $6\ kN/m$

Characteristic permanent load: $q_k = 2{,}5 + 1{,}5 + 0{,}307 + 6 = 10{,}31\ kN/m$

And because of respecting the codes, we need to apply the load according to the quasipermanent combination that goes as follows:

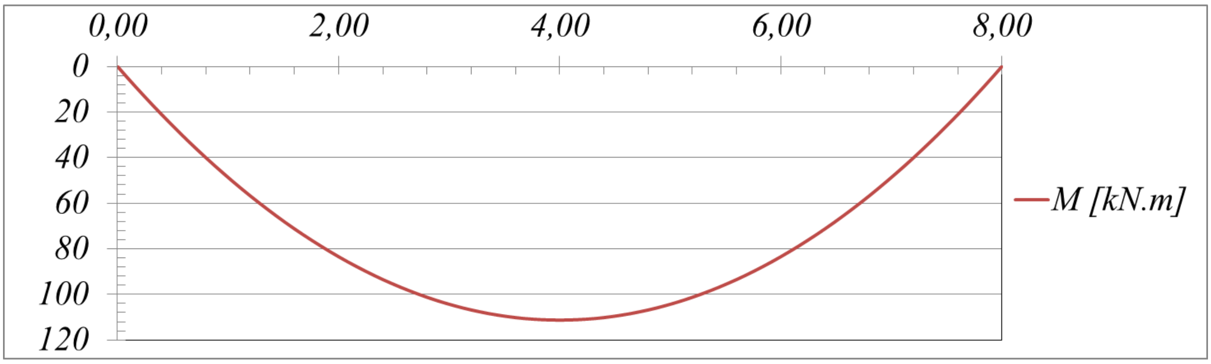
$\Sigma\ Q_{k,j} + \Sigma\ \psi_{2,1} \,.\, Q_{k,i} = q_k \,.\, 1{,}35 = 10{,}31 + 6 \,.\, 0{,}6 = 13{,}91\ kN/m$

Using the made programme in Excel we get the stiffness of cross-section, where the steel part is *IPE 240* of type *S235*, and concrete slab of thickness $a = 0{,}05\ m$ and effective width $b_{eff} = 2\ m$. The value obtained is $EI_i = 3{,}33.10^{+07}\ N.m^2$

After applying the the mentioned load on the simple beam, we get the following results using the statical Excel programme as:

Maximum bending moment:

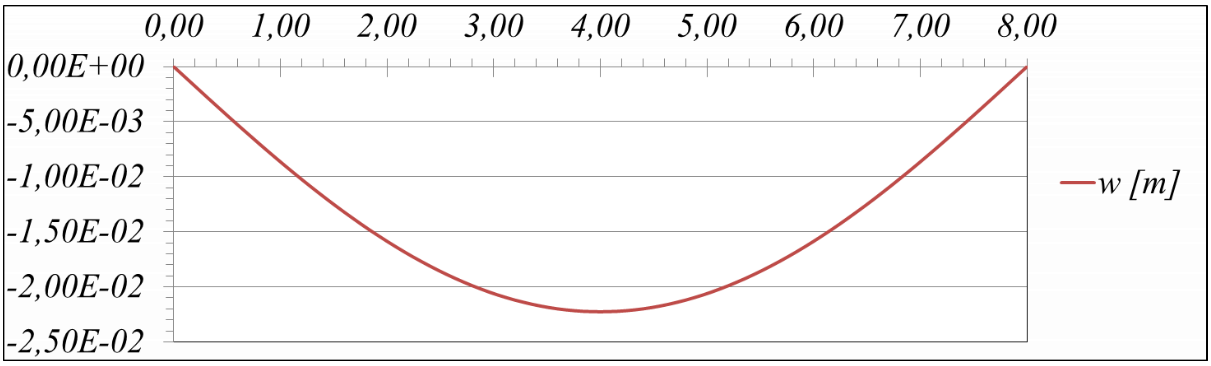
$M_{max} = 111{,}28\ kN.m$



Graph 4.25: *distribution of bending moments along the simply supported beam with a composite cross-section of IPE 240 for steel S235 and 2 m width, 0,05 m thickness of a concrete slab made of C20/25*

Maximum deflection:

w_{max} = *-0,02226 m*



Graph 4.26: *distribution of deflections along the simply supported beam with a composite cross-section of IPE 240 for steel S235 and 2 m width, 0,05 m thickness of a concrete slab made of C20/25*

The cross-sectional area A_c must be equal to $A_c = b_{eff} \,.\, a = 2 \,.\, 0{,}05 = 0{,}1\ m^2$ where it is a fixed value taken from the effective width of the beam.

We are going to try different varieties of creep effect on our beam by the following changes:

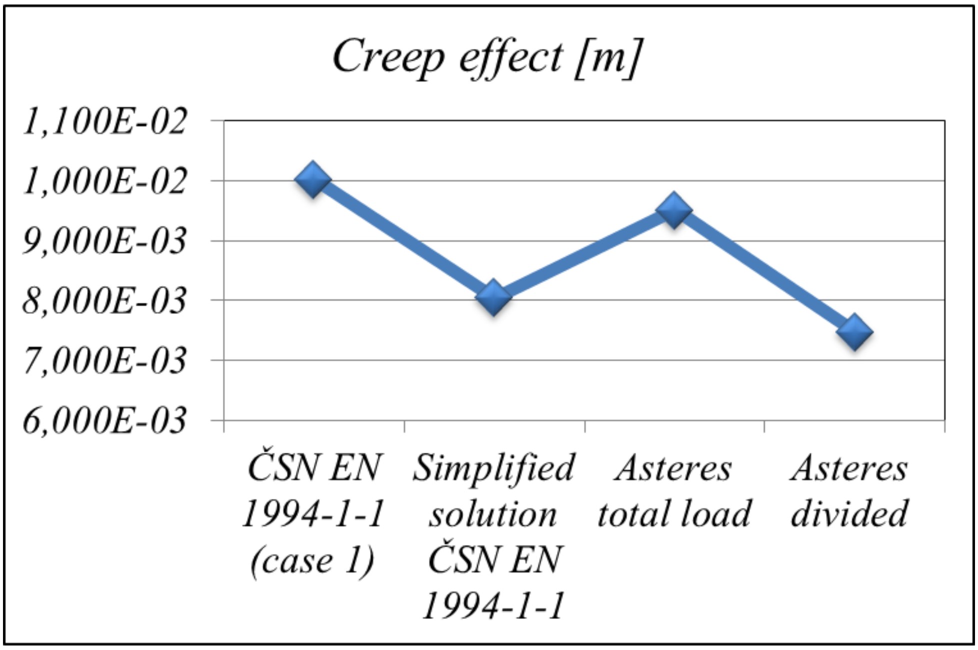
For our case we will consider $u = 2$ since the only face that is in contact with the air is the upper face of the slab in time all other sides are covered by the trapezoidal steel plate.

- Using the advanced method through software ASTERES, that the reference axis is installed at the point between the trapezoidal plate and the steel bar.

Then after putting different varieties of calculation we get the following results:

	Quasipermanent comb. [m]	Creep effect [m]	Shrinkage effect [m]	Total [m]
ČSN EN 1994-1-1	0,022274	1,003E-02	-	-
Simplified solution ČSN EN 1994-1-1	0,026101	8,044E-03	-	-
Asteres total load	0,022946	9,51E-03	6,17E-03	3,86E-02
Asteres divided	0,022946	7,48E-03	6,17E-03	3,66E-02

Table 4.18: *comparison of creep effect calculated by the national code, its simlplified method, ASTERES software with applied the load at once and ASTERES with permanent load applied and living one afterwards. The simplified method of the national code, unlike all the others, is obtained after enrolling the whole load without counting with combination factors. The load that must be 16,31 kN/m instead of 13,91 kN/m for the simplified method.*



Graph 4.27: *comparison of various methods used to reach creep effect in a composite beam*

The same time we could consider to take only a part of the concrete slab to be under creep by applying it on the tenth of the concrete face that is under contact with the air where $u = 0,2$. While the time of consideration we take of the creep effect approaching the final value we take the time of almost 30 years that for simplicity we consider $t = 10000$ *days*, and for knowing the development of creep in an earlier stage we take $t = 30$ *days* as a try for the first 3 months.

In time the usual relative humidity of the surrounding atmosphere taken is $RH = 60$ *%* we consider the value $RH = 90$ *%* for a more humid atmosphere.

By all the changes in values covering the possible varieties of creep coefficient we put them in such an order:

		u [m]	A_c [m^2]	h_0 [mm]	RH [%]	t_0 [days]	t [days]
ČSN EN 1994-1-1	1	2	0,1	100	60	28	10000
	2	2	0,1	100	60	28	90
	3	0,2	0,1	1000	60	28	10000
	4	2	0,1	100	90	28	10000

Table 4.19: *changing varieties of effecting factors on creep for the same beam*

Where the total characteristic load = *16,31 kN/m* is applied on the simple beam in all cases. And the stiffnesses EI_{iy} are obtained by the application of stiffness values Excel programme. In time the maximum deflection w_{max} is taken from the statical solution programme made using Excel.

	$\phi(t,t_0)$	EI_{iy} [$N.m^2$]	w_{cr} [m]	w_{Tot} [m]
1	2,85	2,30E+07	-3,229E-02	-3,701E-02
2	1,58	2,63E+07	-2,825E-02	-3,257E-02
3	2,06	2,77E+07	-2,675E-02	-3,424E-02
4	1,83	2,55E+07	-2,909E-02	-3,345E-02
	Loads Applied [kN/m]:		13,91	16,31

Table 4.20: *varieties effect on factor of creep $\varphi(t,t_0)$ and its deflection*

This search was done to make an approach for creep affected by less concrete drying from the comvering of the trapezoidal plate and bedding during the construction, which has a little effect.

6. Overall conclusions

To fullfill safety and effectivity along continuous composite beams‘ validity life with their most economic design was counted with the defects appearing over the internal supports. The time concrete cracks occur and the steel bar is plasticized at those supports.

This reach for which iterational conditions are set up for getting the most precise length over the supports that was involved by concrete cracks, and iterational conditions for the almost reached total plasticization of cross-section over the support. These influences give us the redistributions allowed to be counted with for various span lengths of continuous composite beams.

From all the results we got before we see that the national code ČSN EN 1994-1-1 is under-rated in many cases, and included in the interval for various cases of numerical approach calculations.

The 2-span continuous beams are having the redistribution due to concrete cracks in a range from ***17,5%*** to ***29,00%***, in time the national code stays with ***15%***. The interval of results is close using the two conditions of 2 stiffness and 3-stiffness values. This result makes the national code under-rated and less accurate in comparing to our approach. For plasticization of steel in the same kind of beams ranges from ***30%*** to ***51%*** which is also a higher rated including the national code assumption which is ***40%***.

The 3-span continuous beams have lower redistribution values than the ones of 2-span continous beams. And they differ whether is was a 2 or 3 stifness value condition, where in the 2-stiffness condition the distribution range from ***12,00%*** to ***13,50%***, in time 3-stiffness condition has the interval from ***7,00%*** to ***11,00%***, which in both cases the national code is over-rated claims of reaching a redistribution up to ***15%***. For plasticity of steel beam it gives the redistribution from ***10%*** up to ***30%***, which these values in turn up reaching ***40%*** that the national code gives for plasticity effect on distribution.

All the previous results were for a constant continuous load that is the same for all spans. Once the load in the side span of the 3-span beam is only the permanent one which makes a loss of the half of load in comparing to the other spans, then we get into other results. Where we get redistributions ***37%*** in the support surrounded by the same load in time the other reaches ***11%*** only. Which is way various in comparing with the constant value of the national code. More specifications could be noted in table 5.1 .

The serviceability limit state is fulfilled in all cases by being under the limit of deflection that is equal to *L/250*, where *L* is the span length.

		Redistribution		
		2 stiffness values condition	3 stiffness values condition	Plasticity condition
2-span beam (26,355 kN/m on both spans)	M_b	18,43% to 28,58%	18,33% to 28,06%	30,16% to 50,31%
3-span beam (26,355 kN/m on all spans)	M_b M_c	13,53% to 12,05%	7,60% to 11,03%	10,16% to 28,79%
3-span beam (26,355 kN/m on first two spans and 11,355 kN/m in the third span)	M_b	14,99% to 16,26%	11,08% to 14,25%	25,17% to 36,21%
	M_c	0,90% to 7,38%	(-0,85)% to 4,02%	6,10% to 10,83%
ČSN EN 1994-1-1		15,00%		40,00%

Table 5.1: *Total comparison of internal forces redistributions of various conditions used with the national code*

In time creep was calculated from different approaches for the same conditions of a simply supported beam. Where the results do not differ much, but from Graph 4.25 we can consider the result coming up from ASTERES software approach with a divided load by time is more precise than using the same approach and subjecting all the load a once regardless of time dependancy. Then according to result of the more complicated approach proceeded by ASTERES and the simplicity of the structure we can assume that the simplified approach of the national code ČSN EN 1994-1-1 is suitable, and the other more complicated one is just a safer approach which is the safest among all other calculations. But this must be only for simple structures as it is in simply supported beam, otherwise, in more complicated structures there is a need to use the more advanced methods of solving creep.

Literature

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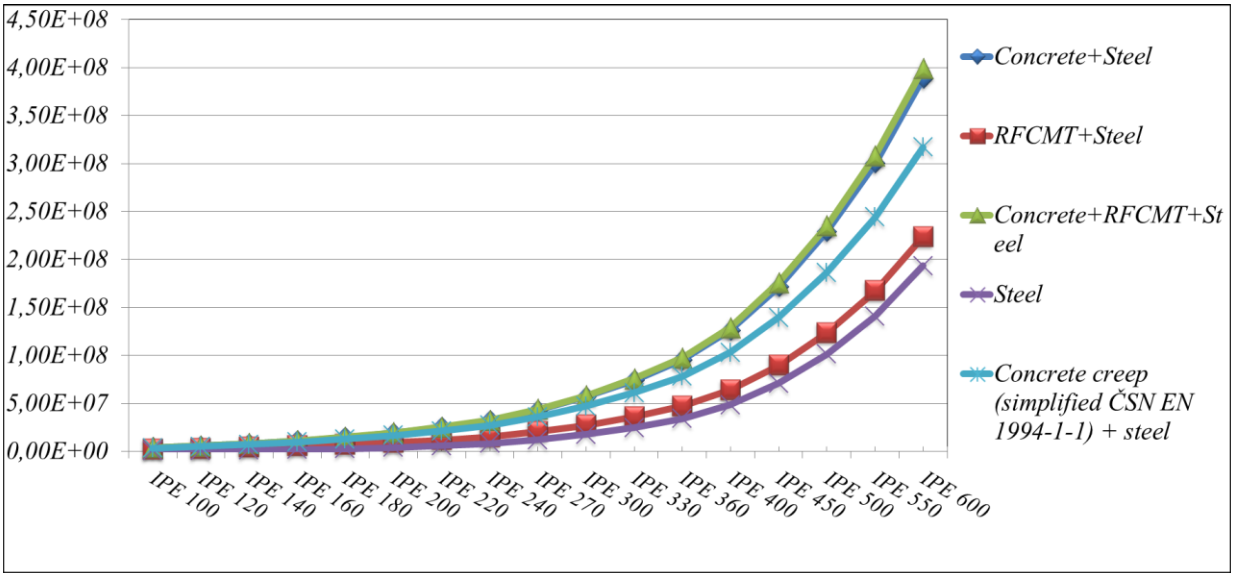
Appendix

A.1 Classification of IPE section used:

	h_{IPE}	t_w	c	c/t		$72*\varepsilon$	
	m	m	m				
IPE 100	0,1	0,0041	0,0877	21,3902	<	72	CLASS 1
IPE 120	0,12	0,0044	0,1068	24,2727	<	72	CLASS 1
IPE 140	0,14	0,0047	0,1259	26,7872	<	72	CLASS 1
IPE 160	0,16	0,0050	0,1450	29,0000	<	72	CLASS 1
IPE 180	0,18	0,0053	0,1641	30,9623	<	72	CLASS 1
IPE 200	0,2	0,0056	0,1832	32,7143	<	72	CLASS 1
IPE 220	0,22	0,0059	0,2023	34,2881	<	72	CLASS 1
IPE 240	0,24	0,0062	0,2214	35,7097	<	72	CLASS 1
IPE 270	0,27	0,0066	0,2502	37,9091	<	72	CLASS 1
IPE 300	0,3	0,0071	0,2787	39,2535	<	72	CLASS 1
IPE 330	0,33	0,0075	0,3075	41,0000	<	72	CLASS 1
IPE 360	0,36	0,0080	0,3360	42,0000	<	72	CLASS 1
IPE 400	0,4	0,0086	0,3742	43,5116	<	72	CLASS 1
IPE 450	0,45	0,0094	0,4218	44,8723	<	72	CLASS 1
IPE 500	0,5	0,0102	0,4694	46,0196	<	72	CLASS 1
IPE 550	0,55	0,0111	0,5167	46,5495	<	72	CLASS 1
IPE 600	0,6	0,0120	0,5640	47,0000	<	72	CLASS 1

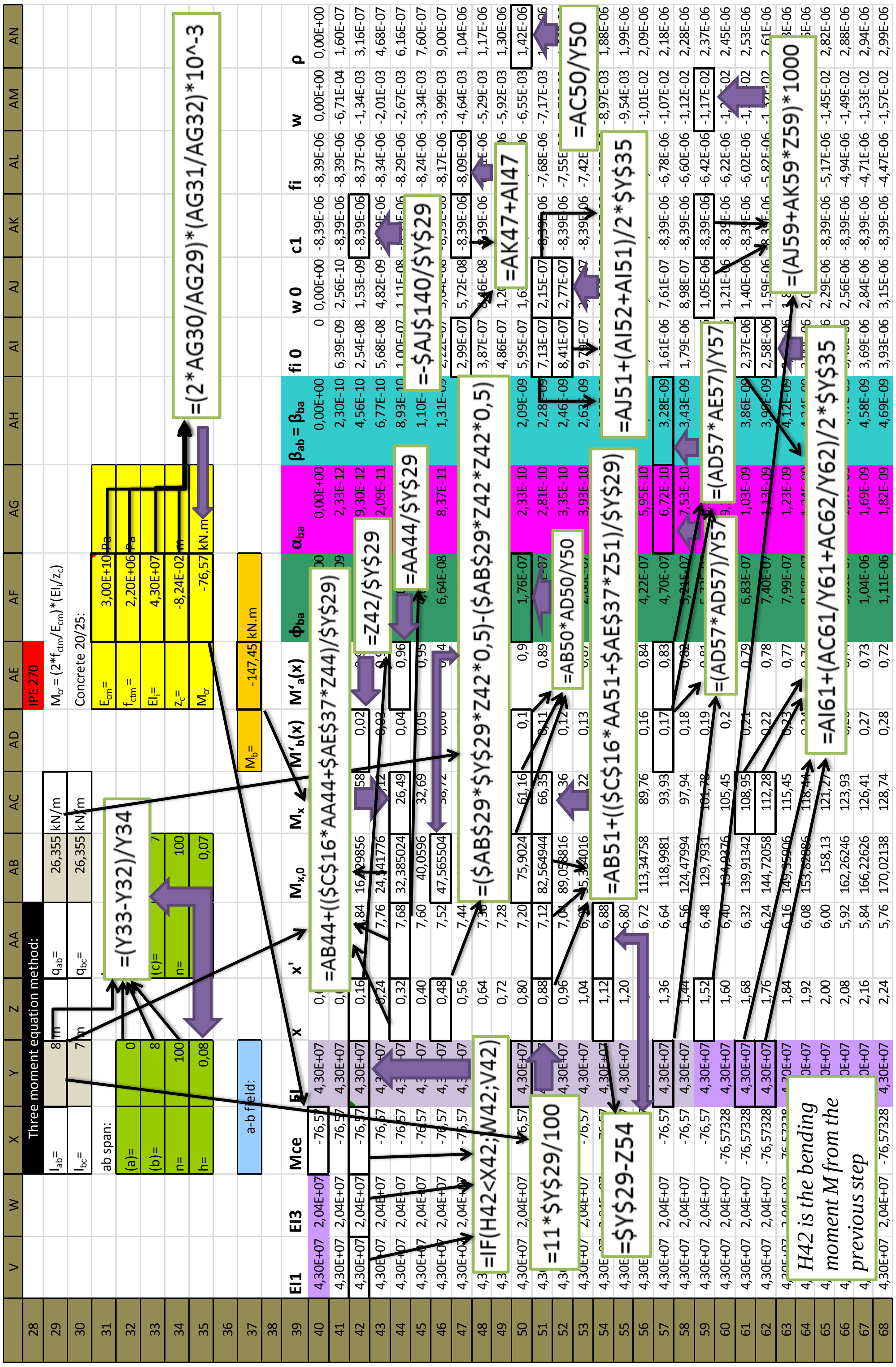
The classification of IPE designations according to cross-section dimensions

A.2 IPE composite sections and their stiffnesses section used:



Comparison of IPE section with their stiffnesses for an effective width of 1,6m (same as the side 8m spas in our continuous beams)

A.3 Excel programming view:



A programming view on the second step of non-linear 2 stiffness value condition and its effect on deflection

A.4 ASTERES input file:

```
# Simple beam - with connected elements
# elastic connection + concrete slab + (RFCMT)
# Nodes
addgen2, Node, 1, 0,    0,  0,  0
addgen2, Node, 2, 0,    4,  0,  0
addgen2, Node, 3, 0,    8,  0,  0
addgen2, Node, 4, 0,    0,  0,  0
addgen2, Node, 5, 0,    4,  0,  0
addgen2, Node, 6, 0,    8,  0,  0

# Macroentities
# Num, Div
addgen2, MEntit, 1, 10
addgen2, MEntit, 2, 10
addgen2, MEntit, 3, 10
addgen2, MEntit, 4, 10

# Macroentity properties
# Num, Item, Nod
addgen2, MEntitChar, 1, 1, 1
addgen2, MEntitChar, 1, 2, 2
addgen2, MEntitChar, 2, 1, 2
addgen2, MEntitChar, 2, 2, 3

addgen2, MEntitChar, 3, 1, 4
addgen2, MEntitChar, 3, 2, 5
addgen2, MEntitChar, 4, 1, 5
addgen2, MEntitChar, 4, 2, 6

# Macroelement - tab. MElem
# 1-2 ... Steel beam
```

```
# 3-4 ... concrete slab
# 5-6 ... RFCMT

#            Num, Real, Mater, TElem, MEntit, Con
addgen2, MElem, 1,   1,    1,     4,     1,     0
addgen2, MElem, 2,   1,    1,     4,     2,     0
addgen2, MElem, 3,   2,    2,     5,     3,     0
addgen2, MElem, 4,   2,    2,     5,     4,     0
#addgen2, MElem, 5,   3,    3,     6,     3,     0
#addgen2, MElem, 6,   3,    3,     6,     4,     0

# MEleGrpxMEle table
# Grp, MEleaddgen2, Node, 3, 0,    8,  0,  0
addgen2, MEleGrpxMEle, 1, 1
addgen2, MEleGrpxMEle, 1, 2
addgen2, MEleGrpxMEle, 1, 3
addgen2, MEleGrpxMEle, 1, 4

# table of real. characteristic element type groups - Real table
# nosnik - typ 4 ... set up by cross-section characteristics
# deska  - typ 5 ... set up by layers
# vyytuz - typ 6 ... element influence by tension and compression
# Typ, Pack
addgen2, Real,4, 1
addgen2, Real,5, 2

# Table of real characteristics - tabulka RealChar
# pro typ 3: Pack, A, A_kappa, I, S, e_d, e_h
# pro typ 4 (beam_lin): A, A_kappa, I, ey_t, e_d, e_h, -1
# pro typ 5:(concrete_layers n, y_d, b, h, Ak (No. of layers, sour. bottom face, width, height, shear area)
# pro typ 6 (RFCMT): n_s, D_s, e_y (No. of bars,diameter,excentricity)
# Pack, Item, Char
```

```
addgen2, RealChar, 1, 1, 0.00391

addgen2, RealChar, 1, 2, 0.0029325

addgen2, RealChar, 1, 3, 0.00003892

addgen2, RealChar, 1, 4, -0.120

addgen2, RealChar, 1, 5, -0.240

addgen2, RealChar, 1, 6, 0.0

addgen2, RealChar, 1, 7, -1


addgen2, RealChar, 2, 1, 10

addgen2, RealChar, 2, 2, 0.05

addgen2, RealChar, 2, 3, 2

addgen2, RealChar, 2, 4, 0.05

addgen2, RealChar, 2, 5, 0.0375


# table of material characteristics - tabule Mater

# nosnik - typ 1 - ocel  .... linear material

# deska  - typ 6 - beton .... non-linear material with creep

# vyztuz - typ 7 - ocel  .... bilinear stress-strain diagram

# Pack, TypMataddgen2, Node, 3, 0,    8,  0,  0

addgen2, Mater, 1,1

addgen2, Mater, 2,6

#addgen2, Mater, 3,7


# table of material charakteristics - MatChar table

# pro typ materialu 1: E, Mi, G, Dens (linear, isotropic)

# pro typ materialu 4: 1:E, 2:Mi, 3:G, 4:TimeDep, 5:Density, 6:time of concreting, 7:treatment time, 8:fcm, 9:Alfa, 10:RH,
11:H_0, 12:Cem

# pro typ materialu 5: Ecm, G, fcm, fctm, epsc1, epscu, Gf, Lcr

# pro typ materialu 6: 1:Ecm, 2:G, 3:fcm, 4:fctm, 5:epsc1, 6:epscu, 7:Gf, 8:Lcr,

#                   9:TimeDep, 10:Density, 11:Erec, 12:Cur, 13:Alfa, 14:RH, 15:H_0

# pro typ materialu 7: E, fy (bilinear RFCMT)


# Pack, Item, Val
```

```
# ocel
addgen2, MatChar, 1, 1, 210e9
addgen2, MatChar, 1, 2, 0.2
addgen2, MatChar, 1, 3, 87.5e9
addgen2, MatChar, 1, 4, 7850

# beton
addgen2, MatChar, 2, 1, 30.0e9
addgen2, MatChar, 2, 2, 12.5e9
addgen2, MatChar, 2, 3, 28e6
addgen2, MatChar, 2, 4, 2.2e6

addgen2, MatChar, 2, 5, 0.002
addgen2, MatChar, 2, 6, 0.0035
addgen2, MatChar, 2, 7, 65
addgen2, MatChar, 2, 8, 0.05

addgen2, MatChar, 2,  9, 0
addgen2, MatChar, 2, 10, 2500
addgen2, MatChar, 2, 11, -28
addgen2, MatChar, 2, 12, -21

addgen2, MatChar, 2, 13, 0
addgen2, MatChar, 2, 14, 0.6
addgen2, MatChar, 2, 15, 0.1

# boundary conditions tabule
# BounCon table
#  Num, Nod, Typ, Coord, Dir, Val
# horizontal support (dir 1) - node 1
addgen2, BounCon, 1, 1, 0, 0, 1, 0
# vertical support (dir 2) - node 1
```

```
addgen2, BounCon, 2, 1, 0, 0, 2, 0
# vertical support (dir 2)- node 3
addgen2, BounCon, 3, 3, 0, 0, 2, 0

# boundary conditions group table
# BounConGrp table
# Grp, BCon, Ref
addgen2, BounConGrp, 1, 1, -1
addgen2, BounConGrp, 1, 2, -1
addgen2, BounConGrp, 1, 3, -1

# connected macroelements table
# tabulka DepenMElem
# Num, Master, Slave, StifBeg, StifEnd

addgen2, DepenMElem, 1, 1, 3, 6.e9 ,6.e9
addgen2, DepenMElem, 2, 2, 4, 6.e9, 6.e9

# connected macroelements groups table
# tabulka GrDepMEl
# Grp, DepMEl
addgen2, GrDepMEl, 1, 1
addgen2, GrDepMEl, 1, 2

# node load table
# LoadNode table
# Nod, Stage, Coord, Dir, Val
#addgen2, LoadNode, 2, 1, 0, 2,  -1000
#addgen2, LoadNode, 2, 2, 0, 2,  -3000

# element load table
# LoadMEle table
```

```
# MElem, Stage, Coord, Dir, Val1, Val2, Val3, Val4

# Long-Term - cast 1
addgen2, LoadMEle, 3, 1, -1, 2, -4310, -4310, 0, 0
addgen2, LoadMEle, 4, 1, -1, 2, -4310, -4310, 0, 0

# Long-Term - cast 2
addgen2, LoadMEle, 3, 2, -1, 2, -10310, -10310, 0 ,0
addgen2, LoadMEle, 4, 2, -1, 2, -10310, -10310, 0, 0

# Short-Term
addgen2, LoadMEle, 3, 3, -1, 2, -13910, -13910, 0, 0
addgen2, LoadMEle, 4, 3, -1, 2, -13910, -13910, 0, 0

# load combination table
# LoadComb table
# Num, Stage, Coef
addgen2, LoadComb, 1, 1, 1
addgen2, LoadComb, 2, 2, 1
addgen2, LoadComb, 3, 3, 1
addgen2, LoadComb, 4, 4, 1
addgen2, LoadComb, 5, 5, 1

# tabulka parametru fyzikalne nelinearniho vypoctu
# tabulka ParFyz
# item: 1 ... maximal step length
#       2 ... minimal step length
#       3 ... maximal steps
#       4 ... ideal No. of steps
#       5 ... bending moment konvegence criterion
#       6 ... normal forces konvegence criterion
# Pack, Item, Val
```

```
addgen2, ParFyz, 1, 1, 1.0
addgen2, ParFyz, 1, 2, 0.0005
addgen2, ParFyz, 1, 3, 28
addgen2, ParFyz, 1, 4, 10
addgen2, ParFyz, 1, 5, 100
addgen2, ParFyz, 1, 6, 100

# macronodes timetable
# time of bearing the 1st long-term load  .... 0   days
# time of bearing the 2nd long-term load  .... 100 days
# final time .... 10000 dnu
# MTime table
#           Num,  Type, Total,  Ref, MEleGrp, BConGrp, Load, DepGrp, DepElemGrp, Solu, Matrix, Solver, ParSolu, ParFyz,
Analyse 000
addgen2, MTime, 1,    0,     0,  -1,      1,      1,   1,    -1,        1,   0,     0,     1,     -1,     1,     1
addgen2, MTime, 2,    0,   180,   1,      1,      1,   1,    -1,        1,   0,     0,     1,     -1,     1,     1
addgen2, MTime, 3,    0,   180,   2,      1,      1,   2,    -1,        1,   0,     0,     1,     -1,     1,     1
addgen2, MTime, 4,    0,   360,   3,      1,      1,   2,    -1,        1,   0,     0,     1,     -1,     1,     1
addgen2, MTime, 5,    0,   360,   4,      1,      1,   3,    -1,        1,   0,     0,     1,     -1,     1,     1
addgen2, MTime, 6,    0, 10000,   5,      1,      1,   3,    -1,        1,   0,     0,     1,     -1,     1,     1

# TimeInti table
# dividing time intervals table
# TNumFrom, TNumTo, TypInter, NInter, Char1
addgen2, TimeInt, 1, 2, 2, 10, 1.5
addgen2, TimeInt, 3, 4, 2, 5, 1.5
addgen2, TimeInt, 5, 6, 2, 5, 1.5

prepar

matnelin
```