

SIMPLE UNIVERSAL NONLINEAR LONGITUDINAL FLIGHT SIMULATION WITH AVOIDING OF STATIC AND DYNAMIC STABILITY DERIVATIVES

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Abstract: This paper shows simple method to simulate nonlinear longitudinal flight dynamic of an aircraft in the most direct way. The method is based on physical principles of an analytical-empiric flight dynamics. Non-linearity, as stall or thrust dependent on velocity, can be included. Static and dynamic stability derivatives are not required but can be computed as an output. The model is applicable for various aircraft conceptions. Higher level model, for example, based on flight tests data, can be modified by low-level analytical methods, e.g. for modification of horizontal tail area. No special simulation software is necessary. The model is compared to linear model and flight test experiment. This model, together with valuable analytical-empiric data, might be applied for fast flight dynamic computations. Model is easily accessible and understandable even with basic knowledge of flight dynamic and computer programming. The main application of the method is conceptual design when high precision is not expected, even if VLM, CFD or flight test data can improve the precision.

1 INTRODUCTION

Flight dynamic has been solved more than 100 years [1]. Classic analytical-empiric theory [2] is very useful tool for an aircraft design. Disadvantage is the time-consuming design process. In the time of high-performance computers, the simulation methods of flight dynamic [3] offer faster solution. However, combination of classic analytical methods, in its derived form, with simulation can significantly slow down the process. Non-dimensional, dimensional stability and control derivatives and derivatives in normal form are necessary to estimate [4], [5], [6], [7]. This means 3 equations for each of at least 15 stability and control derivatives for longitudinal simulation. This complexity means time-consuming work with high danger of making mistakes and problems with modification of the model. Classic linear computation has a lot of limitations and simplifications. It is usually valid only for classic conception and in close surrounding of trimmed flight. Non-linearities, as thrust and stall, are not easy to include. Drag and z component are usually neglected. Rotation of the forces from the local velocity vector to flight path a. s. is neglected. The only advantage is analytical solution.

2 FROM CLASSIC LINEAR THEORY TO LINEAR SIMULATION

Classic analytical-empiric linear aircraft computation requires estimation of stability and control derivatives. For example, horizontal tail influence on the aircraft flight dynamics can be expressed by following equations (do not read it, see only the complexity).

$$Z_{HT} = -p_D \cdot S \cdot \left(a_{HT} \cdot k_{HT} \frac{S_{VOP}}{S} \cdot (\varphi_{HT}^* - \varepsilon_0) + a_{HT} \cdot k_{HT} \cdot \frac{S_{HT}}{S} \cdot \left(1 - \frac{\partial \varepsilon}{\partial \alpha} \right) \cdot (\alpha + \varphi_{KT}) - \frac{2}{c_A} a_{HT} \cdot k_{HT} \frac{S_{HT}}{S} \left(q \frac{\tilde{l}_{HT}}{V} \right) + 2 a_{HT} k_{HT} \frac{S_{HT} \tilde{l}_{HT}}{S c_A} \frac{\partial \varepsilon}{\partial \alpha} \left(\frac{\dot{\alpha} c_A}{2V} \right) + a_{HT} \cdot k_{HT} \frac{S_{HT}}{S} \cdot \left(\frac{\partial \alpha}{\partial \delta} \right)_e \delta \right)$$

$$M_{HT} = p_D \cdot S \cdot c_A \cdot \left(-a_{HT} \cdot k_{HT} \frac{S_{HT} \tilde{l}_{HT}}{S c_A} \cdot (\varphi_{HT}^* - \varepsilon_0) - a_{HT} \cdot k_{HT} \frac{S_{HT} \tilde{l}_{HT}}{S c_A} \cdot \left(1 - \frac{\partial \varepsilon}{\partial \alpha} \right) \cdot (\alpha + \varphi_{KT}) - \frac{2}{c_A} a_{HT} \cdot k_{HT} \frac{S_{HT} \tilde{l}_{HT}}{S c_A} \left(q \frac{\tilde{l}_{HT}}{V} \right) - 2 \cdot a_{HT} \cdot k_{HT} \frac{S_{HT} \tilde{l}_{HT}}{S c_A} \frac{\partial \varepsilon}{\partial \alpha} \frac{\tilde{l}_{HT}}{c_A} \left(\frac{\dot{\alpha} c_A}{2V} \right) - a_{HT} \cdot k_{HT} \frac{S_{HT} \tilde{l}_{HT}}{S c_A} \cdot \left(\frac{\partial \alpha}{\partial \delta} \right)_e \delta \right)$$

After derivation of these equations by $\alpha, \dot{\alpha}, \bar{q}, \delta$, we obtain horizontal tail part of 8 stability and control derivatives. Each of the derivative is usually composed of wing, fuselage, tail and thrust part. Even if derivatives are very useful for comparison of different aircrafts and elimination of some parameters, the simplicity and clarity of the model can be more important. In case of using basic physics combined with simulation, we can write the same equations for horizontal tail in very simple way. There is no missing information in comparison to previous equations. Body correction constant K_B is added only.

$$Z_{BHT} = -k_{HT} p_D S_{HT} \cdot a_{HT} \left(\alpha + \varphi_{HT} - \varepsilon + K_B q \frac{\tilde{l}_{HT}}{V_\infty} + \left(\frac{\partial \alpha}{\partial \delta} \right)_e \delta \right) \quad M_{BHT} = Z_{ABHT} \tilde{l}_{HT}$$

For complete simple longitudinal model of classic piston engine aircraft with simplifications mentioned above we need to add only several following equations:

$$p_D = \frac{1}{2} \cdot \rho \cdot V^2 \quad \varepsilon = \frac{2 \cdot a_W \cdot (\alpha + \varphi_{WB} - \dot{\alpha} \frac{\tilde{l}_{HT}}{V})}{\pi A} \quad x_{WB} = x_W + K_W K_{AB} \frac{b_B c_{Wr}^2}{S}$$

$$Z_{WB} = -p_D S \cdot a_W (\alpha + \varphi_{WB}) \quad M_{WB} = p_D S c_A \cdot C_{m0,W} - Z_{WB} x_{WB}$$

$$X_{WBHT} = -C_{D0} p_D S - \frac{(Z_{WB} + Z_{BHT})^2}{\pi A e S p_D} \quad X_T = -\frac{P_a}{V} \quad X_G = -mg \sin(\gamma) \quad Z_G = mg \cos(\gamma)$$

$$X = X_{WBHT} + X_T + X_G \quad Z = Z_{WB} + Z_{BHT} + Z_G \quad M = M_{WB} + M_{BHT}$$

Trimming α for $t=0$: $P_{\alpha_{i+1}} = -X_i \cdot V_i + P_{\alpha_i} \quad \delta_{i+1} = \delta_i + \frac{M_i / \tilde{l}_{HT}}{k_{HT} p_{Di} S_{HT} a_{HT} \left(\frac{\partial \alpha}{\partial \delta} \right)_e}$

Simulation: $\dot{V} = \frac{X}{m} \quad V_{t+\delta t} = V_t + \dot{V} \cdot \delta t \quad \dot{\gamma} = -\frac{1}{V} \frac{Z}{m} \quad \dot{q} = \frac{M}{I_y} \quad \dot{\alpha} = q - \dot{\gamma}$

$$\theta_{t+\delta t} = \theta_t + q \cdot \delta t \quad \gamma_{t+\delta t} = \gamma_t + \dot{\gamma} \cdot \delta t \quad \alpha = \theta - \gamma \quad \delta_t = \delta_{t=0} + \Delta \delta_t$$

All the input parameters can be determined from aerodynamic analysis of the wing, body, horizontal tail and elevator and from mass and geometry analysis of an aircraft. Even if the model is very simple, experiment comparison at the end of this study shows, that it is close to measurement. In comparison to up to hundreds of the equations in the classic theory, which express the same model, it is high time save, even if simple solver and trimming algorithm is necessary to use.

3 NON-LINEAR MODEL

By using basic physics, we got only several equations. We can, therefore, afford level higher model in comparison to linear model. Rotation of the forces to the flight-path axis system is possible, z component as well as drag force can be included and non-linear coefficients can be used also. Every part of an aircraft can be defined by the position and by the forces which act on it. The aerodynamic forces are computed from airflow model and aerodynamic coefficients, estimated from VLM or CFD [8], [9]. Each of the model input characteristics can be linear, non-linear function or table of values. Avoiding of most of dynamic stability derivatives was possible due to modification of an angle of attack by angular velocity and by delayed downwash model of the wing. A transformation matrix is used to

move automatically the characteristics to the center of gravity and body fixed a. s. Trimmed flight is estimated by bisection method modified by interval expansion. The simulation is compared to Cessna 172 SP flight tests.

3.1 Axis systems and notation

Axis system is defined by an origin point and x-axis angle. Upper index of a symbols describes used axis system. Lower index describes a part of an aircraft.

$$Symbol_{part}^{coord. system}$$

Axis system origin symbol is omitted in forces symbols as the forces are independent of its origin. Axis system angle in moment symbols is omitted as the moment is independent of its angular displacement.

Table 1: The most used axis systems. (Red symbols, CG and θ are usually omitted for simpler notation).

Axis system	Origin	x direction	Force	Moment	Position in given a. s.	Ref-point position
Local air-path (moving air)	A_j	\vec{V}_j	D, L	$M_j^{A_j}$	$x_j^{A_j, \alpha_j}, z_j^{A_j, \alpha_j}$	$x_{A_j}^{ref}, z_{A_j}^{ref}$
Local body fixed	F_j	φ_j	$X_j^{\varphi_j}, Z_j^{\varphi_j}$	$M_j^{F_j}$	$x_j^{F_j, \varphi_j}, z_j^{F_j, \varphi_j}$	$x_{F_j}^{ref}, z_{F_j}^{ref}$
Body	CG	θ	$X_j^{\theta}, Z_j^{\theta}$	M^{CG}	$x^{CG, \theta}, z^{CG, \theta}$	$x_{CG}^{ref}, z_{CG}^{ref}$
Flight path	CG	γ	$X_j^{\gamma}, Z_j^{\gamma}$	M^{CG}	$x^{CG, \gamma}, z^{CG, \gamma}$	$x_{CG}^{ref}, z_{CG}^{ref}$
Earth	CG	$\gamma = 0, x \perp \vec{g}$	X^g, Z^g	M^{CG}	$x^{CG, g}, z^{CG, g}$	$x_{CG}^{ref}, z_{CG}^{ref}$
Reference	Ref. point	$\theta + 180^\circ$			x_j^{ref}, z_j^{ref}	

x-axis of local air-path a. s. has a direction of air velocity vector. x-axis of local body system has a direction of incidence angle at given position, for example propeller revolution axis. If the symbol in upper index is missing, it means θ orientation and CG origin.

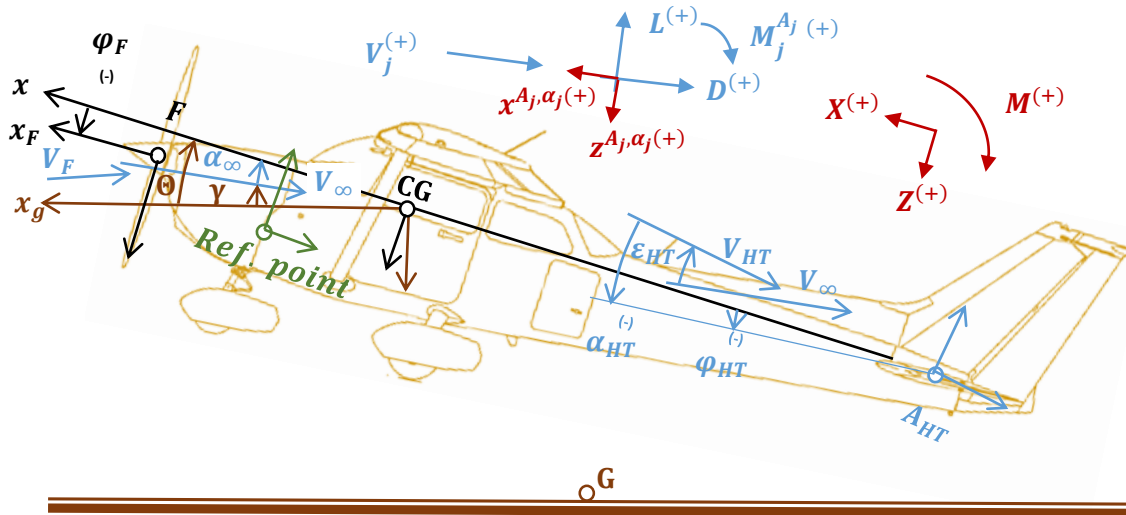


Figure 1: Axis systems and characteristic angles, mnemotechnic help in right up corner.

Angles and moments are controlled by right hand rule and dimensions and forces sign by axis system orientation. Plus (+) in upper index means the positive value of the vector in the picture, minus (-) means negative value of the vector in the picture. This mnemotechnic device helps to avoid mistake in derivation. Velocity in local air path axis system has opposite orientation, because we suppose, that the aircraft stands, and the air is moving. Lift and drag force have opposite orientation also, minus must be used in transformation.

3.2 Transformation

C. G. is the center of rotation of an aircraft. Therefore, we need to transform all the dimensions, forces and moments from the local systems to body a. s. Transformation of the moment and forces is two-step process. Rotation of forces Figure 2 and movement of moment to C. G. Figure 3. In case, we need to transform against the arrow direction, opposite sign must be used in front of the angle or dimension value in transformation matrixes.

Note: If we transform from local air path system, we must use “minus” in front of Drag and Lift in force and moment transformation.

The schemes in the Figure 2 and Figure 3 was created as a mnemotechnic device, because there is high danger of making a mistake.

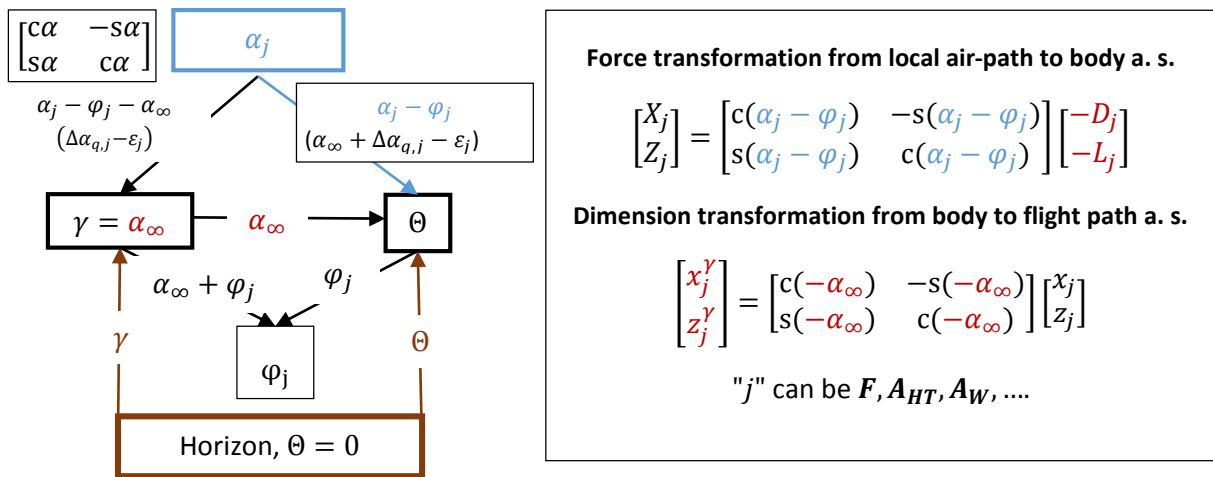


Figure 2: Rotation of the forces to different angle

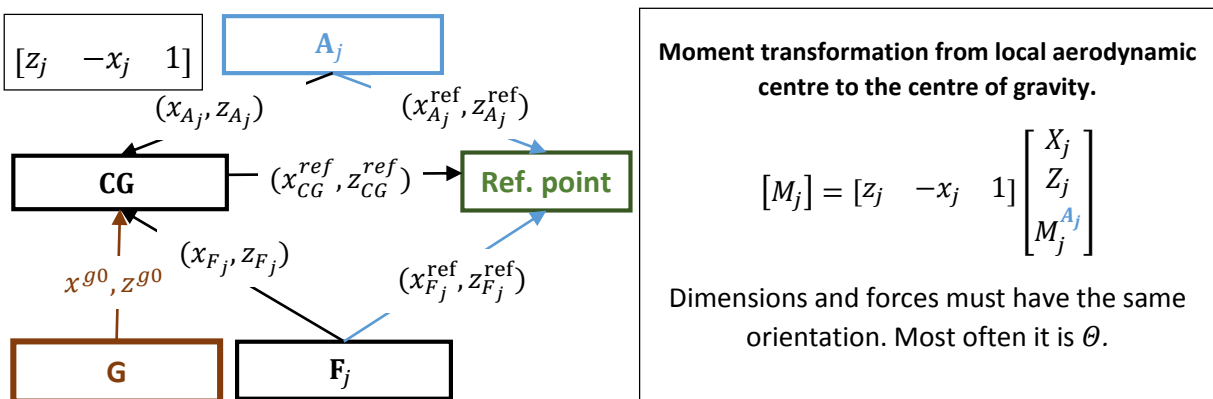


Figure 3: Movement moment to different position

Because the centre of gravity can change a position in time, every point on the aircraft is better to define by the distance from reference point Figure 1. Positions of the points on the aircraft is possible to transform to body fixed axis system with origin in CG and orientation of the x axis θ :

$$x_j = x_{CG}^{ref} - x_j^{ref} \quad \text{oriented x position in body axis system.}$$

$$z_j = z_{CG}^{ref} - z_j^{ref} \quad \text{oriented z position in body axis system.}$$

3.3 Airflow model

Airflow model for longitudinal flight dynamics can be described by undisturbed dynamic pressure p_D velocity V_∞ , AoA α_∞ , and its changes as dynamic pressure change k_j , and airflow angle change ε_j . These changes can be caused by wing - horizontal tail interaction, propeller, wind gust etc. Dynamic pressure change affects a local velocity V_j . Airflow angle change ε_j affects a local angle of attack α_j .

In this study we use time delayed downwash angle and angular velocity influence on local velocity and AoA. These characteristics are not usually used, but it helps to avoid using other than local dynamic stability derivatives. V_j and α_j are derived in following chapter.

3.4 Dynamic stability derivatives avoiding: Downwash delay and angular velocity influence.

Following part is showed on horizontal tail, but it can be general to all the aerodynamic parts on the aircraft, which can be replaced by the characteristics in one point.

Time angle of attack change will cause lift on the wing change. Time lift on the wing change will cause time downwash change at horizontal tail position. The change of downwash angle, due to lift, will affect the tail with $\Delta t_\varepsilon^{(+)} \cong -\frac{l_\varepsilon^{(-)}}{V_\infty}$ delay, because the horizontal tail is placed at given distance behind the wing. Figure 4

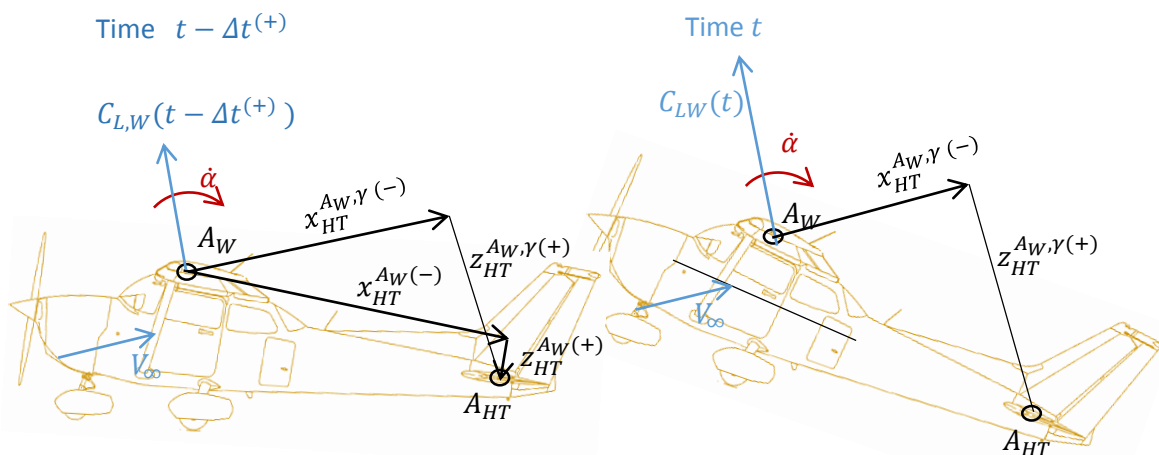


Figure 4: Time delayed downwash angle estimation

For explanation, very simple downwash model, which is useful for aspect ratio > 4 and low AoA and short $z_{HT}^{A_W,γ}$, was chosen. For chosen model, downwash angle is a function of wing lift coefficient.

Exact time is not necessary and it's not easy to integrate. Delay of the downwash is approximately:

$$\Delta t_\varepsilon^{(+)} \cong -\frac{x_{HT}^{A_W,γ(-)}}{V_\infty}, \text{ where } \begin{bmatrix} x_{HT}^{A_W,γ} \\ z_{HT}^{A_W,γ} \end{bmatrix} = \begin{bmatrix} c(-\alpha_\infty) & -s(-\alpha_\infty) \\ s(-\alpha_\infty) & c(-\alpha_\infty) \end{bmatrix} \begin{bmatrix} x_{HT}^{A_W} \\ z_{HT}^{A_W} \end{bmatrix}$$

Downwash is given by lift coefficient in $t - \Delta t_\varepsilon$ time.

$$\varepsilon_{HT} = \varepsilon_{HT} \left(t - \Delta t_\varepsilon^{(+)} \right) \cong \frac{2C_{LW} \left(t - \Delta t_\varepsilon^{(+)} \right)}{\pi A}$$

Simulation needs saving of delayed characteristics (wing lift coefficient) during the simulation.

3.5 Aerodynamic forces in local air path axis system

Part of an aircraft, which produces aerodynamic forces, as a wing, a fuselage, a horizontal tail, are usually described by aerodynamic coefficients. These coefficients can be a function of its angle of attack AoA, flap deflection δ , Reynolds number Re , angular velocity q , Mach number M , and so on.

$$C_{L,j}, C_{m,j}, C_{D,j} = f(\alpha_j, \delta_j, Re_j, M_j, q_j, \dots)$$

The model in this study allows to define the aerodynamic coefficients by linear function, general function or by a table of values. The reference point A_j is usually close to aerodynamic centre of the part. Angle of attack α_j is referenced to the characteristic line of the part. It can be zero lift line or mean aerodynamic chord.

The forces in local air-path axis system Figure 6 can be described by these formulas.

$$p_{D,j} = k_j \cdot p_D$$

$$D_j = p_{D,j} \cdot C_{D,j} \cdot S_j \quad L_j = p_{D,j} \cdot C_{L,j} \cdot S_j \quad M_j^{A_j} = p_{D,j} \cdot C_{m,j} \cdot S_j \cdot c_{A,j}$$

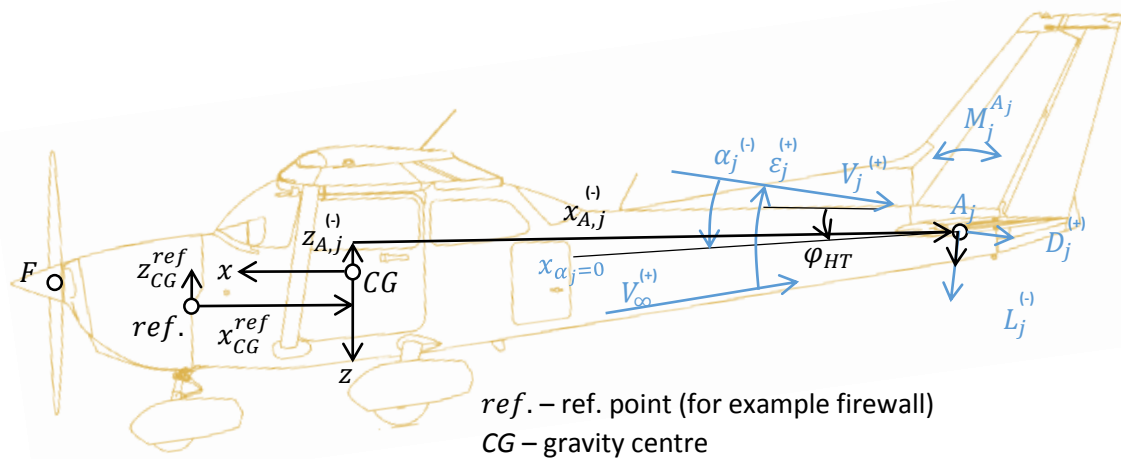


Figure 6: Local air-path a. s. definition on part $j = HT$

Lift and drag force have an opposite orientation, minus must be used in transformation therefore.

$$\begin{bmatrix} X_j \\ Z_j \end{bmatrix} = \begin{bmatrix} c(\alpha_j - \varphi_j) & -s(\alpha_j - \varphi_j) \\ s(\alpha_j - \varphi_j) & c(\alpha_j - \varphi_j) \end{bmatrix} \begin{bmatrix} -D \\ -L \end{bmatrix} \quad M_j = M_j^A + (-D_j) \cdot z_j - (-L_j) x_j$$

3.6 Gravity forces

Gravity force must be transformed to aircraft body fixed a. s. $X_j^\Theta = -m \cdot g \cdot \sin(\Theta)$, $Z_j^\Theta = m \cdot g \cdot \cos(\Theta)$.

3.7 Propulsion forces

Propulsion AoA definition is $\alpha_j = \alpha_\infty + \varphi_j - \varepsilon_j + \Delta\alpha_{j,q}$, where $\Delta\alpha_{j,q} \cong -q^{(+)} \cdot x_j^{(+)} / V_\infty^{(+)}$.

In case of constant speed propeller, power available $P_{a,j}$ is approximately constant, thrust can be expressed as $X_j^{\varphi_j} = P_{a,j} / V$. Perpendicular force can be also important $Z_j^{\varphi_j} = -p_{D,j} \cdot C_K(\alpha_j) S_j$. Transformation to body fixed a. s. is necessary Figure 7.

$$\begin{bmatrix} X_j \\ Z_j \end{bmatrix} = \begin{bmatrix} c(-\varphi_j) & -s(-\varphi_j) \\ s(-\varphi_j) & c(-\varphi_j) \end{bmatrix} \begin{bmatrix} X_j^{\varphi_j} \\ Z_j^{\varphi_j} \end{bmatrix} \quad M_j = M_j^F + X_j \cdot z_j - Z_j \cdot x_j$$

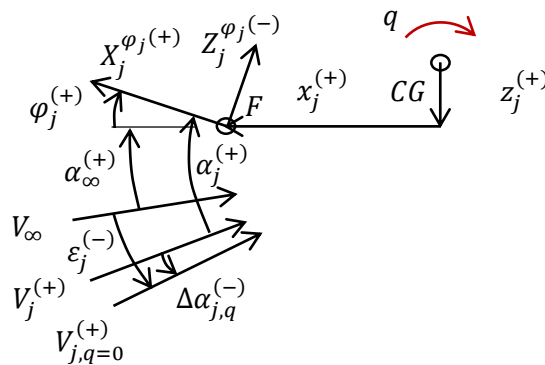


Figure 7: Propulsion force

3.8 Equations of motion

The forces, transformed to body fixed axis system, must be summed.

$$X = \sum_j X_j \quad Z = \sum_j Z_j \quad M = \sum_j M_j$$

For having simple equations of motion, we can transform summed forces to flight path a. s.:

$$\begin{bmatrix} X^Y \\ Z^Y \end{bmatrix} = \begin{bmatrix} c(-\alpha_\infty) & -s(-\alpha_\infty) \\ s(-\alpha_\infty) & c(-\alpha_\infty) \end{bmatrix} \begin{bmatrix} X^\theta \\ Z^\theta \end{bmatrix}$$

Note: Transformation to the flight path a. s. is convenient only for longitudinal equations of motions. For 6 DOF model, inertia moment would have to be transformed to flight path a. s. in every time step.

Equation of motion are derived from [10]. In every step we compute new velocity and flight path angle. Velocity perpendicular to flight path velocity is assumed to be zero.

$$\dot{V} = \frac{X^Y}{m} \quad \dot{\gamma} = -\frac{1}{V} \frac{Z^Y}{m} \quad \dot{q} = \frac{M^{CG}}{I_y} \quad \dot{\alpha} = q - \dot{\gamma} \quad (\dot{w} = \dot{\alpha} \cdot V) \quad (\dot{\theta} = q)$$

$$\dot{x}_g = V \cdot \cos(\gamma) \quad \dot{z}_g = V \cdot \sin(\gamma) \quad (w_{i+1} \cong 0)$$

$$x_{i+1} = x_i + \dot{x}_{i+1} \Delta t \quad x \in V, q, \gamma, \theta, x_g, z_g \quad \alpha_\infty = \theta - \gamma \quad u = [\delta_V, \delta_T, \dots]$$

Even if first order numerical solver can be sufficient, fourth order Runge-Kutta solver with error estimation was chosen [10]. The simulation is two steps. Approximate error is determined by first simulation with rough time step. According to the error in rough simulation, the time step is refined.

3.9 Trimming by bisection method with interval expansion

Bisection method for trimming was not sufficient. Aircraft model parameters are changing during trimming process, so the solution can move out of the bisection interval. Bisection with interval expansion was developed therefore. Example of use is below.

We look for an angle of attack and we control the power of the engine and elevator deflection, so that forces and moment are at equilibrium.

1. Choosing steady state flight initial conditions: $V_{ref}, \gamma_{ref}, \rho_{ref}, m_{ref}, x_{CG,ref}, \delta_T$
2. Control parameters δ_{HTref}, P_{ref} can be zero.
3. Choosing precision conditions of trimmed state: $|X| < 0.01N, |Z| < 0.1N, |M| < 0.1Nm$
4. Choosing minimum and maximum α_{ref} interval.
5. Computing required change in power and elevator deflection from the moment and X force.

$$P_{i+1} \cong -X^Y \cdot V + P_i$$

$$Z_{HT_{i+1}} \cong \frac{M}{x_{AHT}} + Z_{HT_i} \quad \delta_{HT_{i+1}} = -\frac{Z_{HT_{i+1}}}{p_{DHT} \cdot S_{HT} \cdot C_{L,HT\delta}}$$

The change in power is not exact, but it is enough for fulfilling the precision conditions.

6. Computing aircraft model for new initial power and elevator deflection.
7. The change in Z force result, from trimming process, is the condition for angle of attack variation.
8. If the difference of the change of Z force is smaller than its magnitude $|Z_{ref,i+1} - Z_{ref,i}| < |Z_{ref,i}|$, it is possible, that the solution is out of examined interval. Expansion of the angle of attack interval is applied.
9. Otherwise bisection of the interval is applied.

Core part of Matlab code of bisection method with interval expansion is provided below:

```
% If X, Z and M does not fullfill precision condition, find new initial conditions.
if abs(X) > 0.00001 || abs(M) > 0.00001 || abs(Z) > 0.00001
    % If the change of Z is smaller, than its magnitude, expand the interval.
    if abs(Z-tempZ) < abs(Z)
        if -Z > 0 % if the aircraft climb, decrease the angle.
            alp1 = alp1 - (alp3 - alp1); % expand minimum by interval range
        else % otherwise increase the angle
            alp3 = alp3 + (alp3 - alp1); % expand maximum by interval range
        end
    % otherwise bisect the interval
    else
        if -Z > 0 % if the aircraft climb, decrease the angle.
            alp3 = alp; % set used angle of attack as maximum
        else % otherwise increase the angle.
            alp1 = alp; % set used angle of attack as minimum
        end
        alp = (alp1 + alp3) / 2; % compute half of the interval
    end
    tempZ = Z; % remember the Z value in this time step.
```

This trimming method is very fast. Only 37 iterations were necessary for steady state flight with maximally $1e-6$ N parasitic forces in steady state. Two interval expansions were necessary during trimming process. Robustness of the method was not examined, but it works even for multidimensional trimming simultaneously. For example, finding the AoA and velocity, both by bisection with expansion method, for maximum thrust. Any other trimming algorithm can be used.

4 LINEAR, NONLINEAR SIMULATION COMPARISON WITH EXPERIMENT

Comparison of experiment with classic linear simulation and simple nonlinear simulation of the same model is presented in the Figure 8. It is a reaction of an aircraft to impulse to elevator deflection in $t = 10$ s taken out of experiment data.

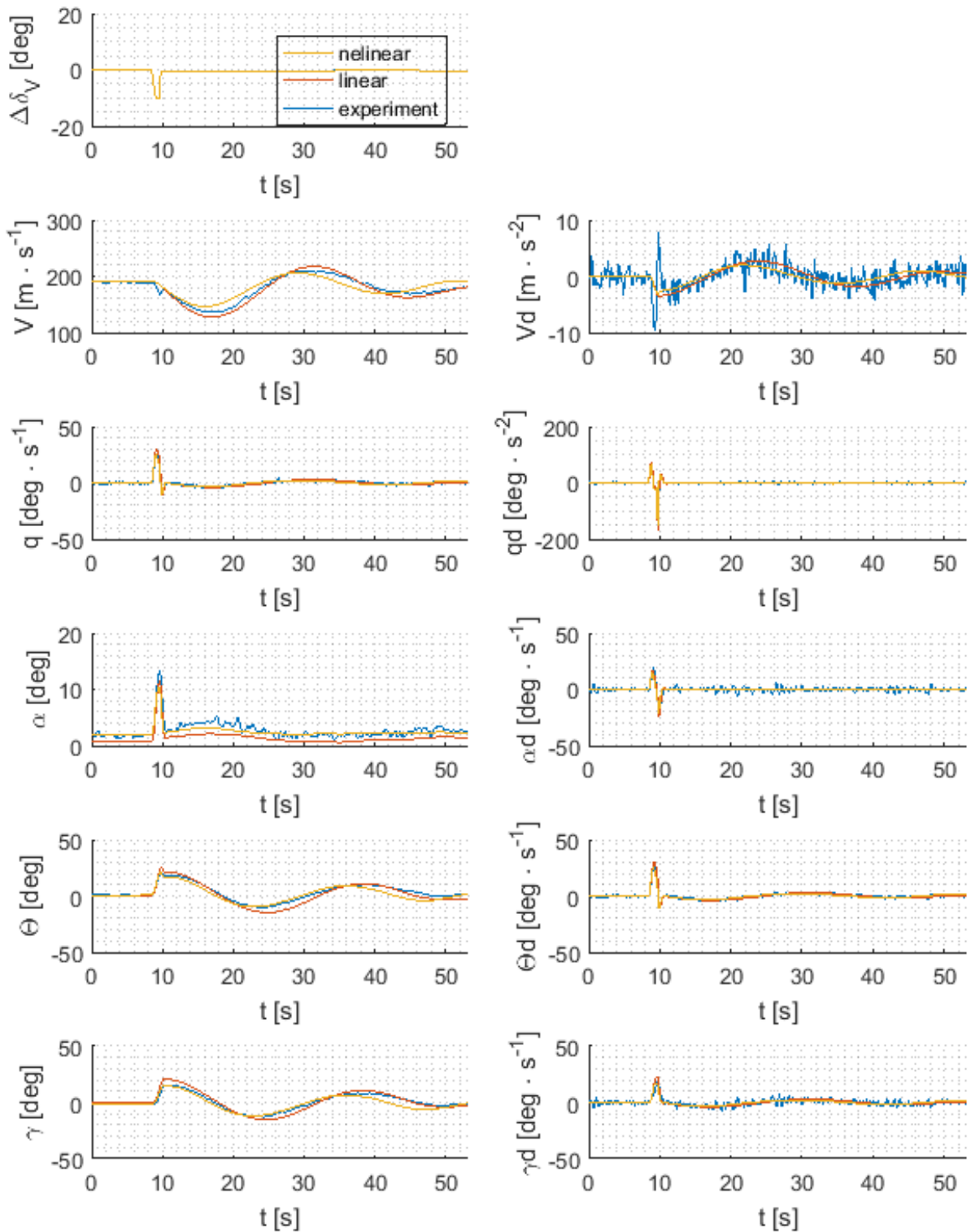


Figure 8: Comparison of experiment with linear and nonlinear simulation

Input characteristics of the models are the same and are derived from analytical methods [4]. Any of the models was not able to catch fast change of the velocity at the 10 s time. Model is probably too simple to describe the phenomena. It can be caused by fuel swing from end to end or local stall during the maneuver on examined aircraft. Both simulations are close to the experiment. Linear simulation estimated better the velocity change. Non-linear estimated better the γ , Θ angle. The precision of the models is on the same level.

CONCLUSION

The paper describes alternative non-linear simulation of an aircraft and compare it to the linear and an experiment. The precision of the method is comparable to the linear simulation on examined case. Non-linear simulation allows easier modifications of the model. Non-linear coefficients can be included. Simplification of the model is not necessary compared to classic theory. The physics of the model is more obvious, and number of required equations is low. Disadvantage is iterative trimming of an aircraft. Following problems were solved. Dynamic stability derivatives were succeeded by angle of attack modification with angular velocity and by delayed downwash model implementation. Bisection with interval expansion was used for trimming process. Time step refinement was used in the simulation.

Non-linear simulation of the whole aircraft is well known nowadays [11], but it is not usually used at the very beginning phase of conceptual design. Flight stability and control is not solved in the first part of conceptual design very often because it is too complex process. It was shown in introduction chapter, that linear simulation can be very simple process, can be finished in the very first part of conceptual design. Other chapters show, that non-linearities can be included quite easily.

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REFERENCES

- [1] BRYAN, G. H., (1911). *Stability in aviation: an introduction to dynamical stability as applied to the motions of aeroplanes*. London: Macmillan and co., limited.
- [2] Finck, R. D. (1978). *USAF (United States Air Force) Stability and Control DATCOM (Data Compendium)*. McDonnell Aircraft Co St Louis Mo.
- [3] Gato, William, et al. (2015). *Flight simulation modeling of aircraft dynamic stall aerodynamics*. U.S. Patent Application No. 14/823,507.
- [4] Daněk, Vladimír, (2011). *Mechanika letu*. Brno: Akademické nakladatelství CERM.
- [5] Phillips, Warren F. (2010). *Mechanics of flight*. 2nd ed. Hoboken, N.J.: J. Wiley
- [6] Cook, M. V. (2012). *Flight dynamics principles: a linear systems approach to aircraft stability and control*. Butterworth-Heinemann.
- [7] Etkin, B., & Reid, L. D. (1996). *Dynamics of flight: stability and control* (Vol. 3). New York: Wiley.
- [8] Kostić, Ivan, et al. (2013). Hybrid approach in the initial aerodynamic, stability and performance calculations of a light airplane. *Tehnički vjesnik*.
- [9] Kostić, Ivan A., Zoran A. Stefanović, and Olivera P. Kostić. (2014) Aerodynamic analysis of a light aircraft at different design stages. *FME Transactions*.
- [10] Stevens, B. L., et al. (2015). *Aircraft control and simulation: dynamics, controls design, and autonomous systems*. John Wiley & Sons.
- [11] Garza, F. R., & Morelli, E. A. (2003). A collection of nonlinear aircraft simulations in matlab.
- [12] DATA SHEETS. Aerodynamics, Vol. I, II a III. Royal Aeronautical Society (UK), 1965.

NOMENCLATURE

AoA, α	angle of attack	α	angle of attack
A	local air-path a. s. center or aspect ratio.	$\bar{\alpha} = \dot{\alpha}c_A/2V$	nondimensional AoA time change
e	Oswald efficiency number	$\bar{q} = qc_A/2V$	nondimensional q time change
b	wing span or body width	γ	flight path angle
CFD	Computational Fluid Dynamics	δ	deflection, control variable
CG	Centre of gravity	ε	downwash angle
C_L, C_D, C_m	lift, drag and moment coefficient	φ	incidence angle
c	reference dimension of coefficient, cosine	Θ	pitch angle
a. s.	axis system	ρ	air density
G	reference ground point	0	coefficient value at given angle equal 0
k_{HT}	dynamic pressure reduction at HT	A	aerodynamic
\tilde{l}_{HT}	$= -x_{HT}$ distance of HT behind CG	air	airflow
m	weight	B	body
M	moment, Mach number	CG	center of gravity
P_a	power available	e	elevator
p_D	dynamic pressure	F	fuselage
Re	Reynolds number	g	dimensions in moving normal earth a. s.
F	reference thrust point	i	trimming iteration
S	reference area of the coefficient	j	any aerodynamic part
s	sinus	F	thrust force
V	velocity	HT	horizontal tail
VLM	Vortex Lattice Method	r	root
x, z, α	oriented position and angle	$t, \delta t$	time and time increment
u, w, q	velocities components	W	wing
X, Z, M	forces and moment	∞	freestream characteristic
\cdot or "d"	time derivative		

K_W Correction factor of wing position. Typically, high-wing monoplane 0.95, low-wing 1.1.

K_{AB} Correction factor of aerodynamic center shift due to body [12].

K_B Correction factor of dynamic stability derivative due to body. Typical value is $1.1 \div 1.2$ for classic conception aircraft. It is derived from horizontal tail dynamic derivatives correction [4]

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