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ADVANCED METHODS FOR THE SOLUTION OF JOURNAL BEARING DYNAMICS

POKROČILÉ METODY PRO ŘEŠENÍ DYNAMIKY KLUZNÝCH LOŽISEK

DOCTORAL THESIS

DISERTAČNÍ PRÁCE

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ABSTRACT

The scope of this dissertation is the description of a complex and generally applicable strategy for computational modelling of slide bearing in mixed lubrication regime. Also taking into consideration real pattern of rough surfaces of both slide bearing parts – bearing shells and bearing pins. Detailed ways to get characteristic data describing rough surfaces and the description of numerical algorithm used for subsequent surface data processing are listed in this doctoral thesis. Furthermore, all the implemented computational models for contact pressure modelling are discussed. The impact on hydrodynamic lubrication layer of slide bearings is given by the size and trend of flow factors, whose numerical determination is also described in detail in this thesis. Alternative assessment of flow factors using analytical equations is also listed for the completeness. In the closing section of this thesis a description of the numerical solver of slide bearing behaviour is included, together with stated and discussed results of realised simulations. The verification of individual computational models and the comparison of their results are presented in relevant chapters.

KEYWORDS

Surface roughness, flow factor, hydrodynamic lubrication, slide bearing, mixed lubrication.

ABSTRAKT

Náplní této disertační práce je popis komplexní a obecně použitelné strategie výpočetního modelování chování kluzných ložisek, pracujících v režimu smíšeného mazání a uvažujících reálné struktury drsných povrchů obou členů kluzných ložisek – ložiskových pánví a ložiskových čepů. V práci jsou uvedeny a detailně popsány způsoby získání charakteristických popisných dat těchto drsných povrchů a její nedílnou součástí je i popis numerických algoritmů, sloužících k následnému zpracování těchto dat. Dále jsou zde rozebrány implementované výpočetní přístupy k modelování kontaktního tlaku. Ovlivnění hydrodynamické mazací vrstvy kluzných ložisek je dáno velikostí a průběhem faktorů toku, jejichž numerické stanovování je zde detailně popsáno. Alternativní stanovení faktorů toku pomocí analytických funkcí je zde pro úplnost rovněž obsaženo. V závěru práce je uveden popis numerického řešiče modelování chování kluzných ložisek spolu s uvedenými výsledky provedených simulací a jejich diskuzí. Ověření jednotlivých výpočetních modelů, popřípadě porovnání výsledků s výsledky získanými alternativním způsobem, je uvedeno v příslušných kapitolách.

KLÍČOVÁ SLOVA

Drsnost povrchu, faktor toku, hydrodynamické mazání, kluzné ložisko, smíšené mazání.



BIBLIOGRAPHIC CITATION

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DECLARATION OF AUTHORSHIP

I, Ondřej Maršálek, hereby declare that this doctoral thesis – “Advanced Methods for the Solution of Journal Bearing Dynamics” – is my own work based on my own original research job, and all the sources have been quoted and acknowledged by means of a complete list of references in this document.

Brno 5.6. 2015

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Ondřej Maršálek



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INTRODUCTION

Slide bearings are still irreplaceable in many engineering applications, especially because of their positive features – e.g. the ability to absorb shocks and vibrations, their simplicity, low price and weight, and many others.

If we focus on slide bearings of contemporary internal combustion engines, then among the most important bearings, regarding their basic function, belong the main slide bearings of crankshaft. In the case of cranktrain bearings of conventional internal combustion engines (except piston pin bearings), in the past only very rarely occurred a contact between the bearing pin and the shell (in addition to fault condition practically only during cold start, and only for an insignificantly short moment). The continuous downsizing trend results in sharply increased combustion pressures, causing extreme loading of (not only) the slide bearings of (not only) the cranktrain mechanism. At the same time the effective load carrying surface of hydrodynamic bearings is decreased due to the overall reduction of the combustion engines size. Thereby, it logically decreases the overall load bearing capacity and increases the rate of contact between the bearing pins and shells over a unit of time. Example of crankshaft evolution, affected by the downsizing trend, is illustrated in Fig. 1.

Due to slide bearing development (in terms of materials and design) and due to requirements from the car producers' side, the level of development of each machinery part in the virtual prototype state is important. Therefore, the perfection and credibility of the design of individual components nowadays play an important role in the development of any product already in the process of virtual prototyping.

Another fact that demonstrates the need to develop internal combustion engine slide bearings is the annually increasing pressure from the side of legislation regarding exhaust emissions. To explain the previous statement it is necessary to devolve into this issue deeper. Among the exhaust emissions also belongs the CO_2 gas. This gas is included among the products of complete combustion and therefore the decrease of its volume in the overall volume of emissions requires the fuel consumption decrease as well – increase of the combustion engine efficiency. The overall combustion engine efficiency also consists of mechanical efficiency which is given by mechanical losses. These mechanical losses of combustion engine cranktrain are predominantly given by losses in slide bearings.

That brings us back to the need for the development of computational models to help with the improvement of slide bearings not only in terms of design, but also in terms of used materials, which is also one of the main motivations for the creation of this work.



Fig. 1: Illustration of crankshaft design evolution



1 PROBLEM DEFINITION

Internal combustion engines of modern passenger cars and commercial vehicles in terms of sophistication and development are at a very high level. This fact forces the manufacturers to go more in depth and take focus more on the detailed design of individual parts, so their power units are at higher level when compared to the competitors. The information technology with time has also noticeably progressed. Therefore, computational models also need to adapt to the contemporary hardware, to utilize its potential to the maximum and to enable the emergence of more comprehensive, as well as more sophisticated, software tools. This statement is supported by the fact that development, especially the employment of experimental tools, is very expensive and time consuming in comparison with the application of computational models. These models, after completion, provide results quickly and inexpensively.

Various companies from around the world have made great effort and have put considerable financial resources into research methods for real components' surface scanning, as well as into the development of instruments for the surface scanning itself, with the effort to include real surface pattern into computational models. From participation in scientific conferences, from information gained during internships at foreign scientific institutions and personal visits in foreign and domestic companies dealing with research and development in the automotive industry, it is evident that only a small percentage of the tools developed in this way are commercially available for companies just from the field of automotive industry. An expressed interest from the side of potential industrial partners in a modern and comprehensive software tool for slide bearing dynamics solution thus became one of the main impulses for writing this doctoral thesis. It was also an essential proof that this doctoral thesis is not only pure academic work without any connection to the industry, but on the contrary – it is a doctoral thesis with considerable potential for industrial application.

One of the main questions of any computational modelling is still a compromise between time efficiency and the accuracy of results. Among the aforementioned accuracy of results also belongs the degree of capturing of the physical essence of the problem. Evaluation of this accuracy can be assessed on the basis of concordance between the results of numerical simulations and experiments. In the case of this thesis there are two possible ways for results verification (or possibly for code fine-tuning):

- Commercially available software tools which, although they do not affect the issues in such detail as it is required nowadays, can at least provide information regarding the nature and trend of the expected partial results.
- Experimental validation of computational simulations.

Both options are included in this thesis.

In the conclusion of this chapter the definition of the problem may be summarised as follows: this doctoral thesis describes comprehensive computational modelling of slide bearing, maximally utilizing the potential of contemporary hardware and taking into consideration all perspective partial knowledge published in foreign and domestic scientific publications so far and also own, so far acquired, knowledge about this field, including the experimental verification of partial results obtained by computer simulations.



For logical reasons, the traditional separation of chapters describing the current state of knowledge in the investigated field was dropped. One of the goals of this thesis is to describe the whole issue in as much detail as possible. Therefore, the current state of knowledge is smoothly integrated into the individual chapters. All chapters are arranged in logical succession and gradually describe everything from the developed software tools.

This doctoral thesis is divided into two main parts, just as there are two main programs. These are: a flow factor database generator (F-CALC) and the detailed slide bearing dynamics solver (BearDYN) in which the above mentioned databases are used as one of the inputs for the calculation. This slide bearing solver thus takes into account the influence of the surface pattern on hydrodynamic lubricating layer and contact pressure. In other words – it maps the transition from pure hydrodynamic lubrication regime to mixed lubrication regime in detail.

From chapter 2 (Surface Pattern of Real Machinery Parts) up to chapter 9 (Comparison of Results from Numerical Simulation and from Analytical Formulae) the database generator is described, which is then used for detailed solution of slide bearing dynamics, described in chapter 10 (Slide Bearing Dynamics Solution).

Based on the advanced information about the surface pattern the database of flow factors, contact pressures, contact area percentage and shear stress factors depending on the distance between analysed surfaces are created. The following chapters discuss all these issues in detail. Modern software tool of this kind definitely has to take into consideration surface pattern of the acting part.



2 SURFACE PATTERN OF REAL MACHINERY PARTS

There are two possibilities to obtain data characterising rough surface:

- Measured (scanned) data from profilometer (scanner).
- Numerically generated data.

For the purpose of effective use of developed software tool these both options are needed. Positive and negative aspects of these two options are listed below along with their descriptions and examples, including generated and scanned surfaces examples.

2.1 SURFACE ROUGHNESS MEASUREMENT

Among the main disadvantages of the surface roughness measurement process belongs:

- the need to purchase an expensive equipment,
- highly time-consuming measuring process and subsequent data postprocessing,
- dependence on the preparation of the real surface sample.

Among the advantages of this option is possible to include the fact that if we provide sufficient measuring accuracy and meaningfulness of the subsequent data filtering, we undeniably have data, characterizing the surface of the real component.

Surface measuring of samples were carried out on the profilometer Contour GT-X from BRUKER. This is a very precise optical profilometer, working on the principle of transmission and subsequent (after reflection from the scanned surface) receiving light rays. More details about this measuring principle and the measuring device itself can be found on the official website of the company BRUKER ([1]).

Among the disadvantages of this principle can be listed the fact that during the measuring process of coarse surface an imperfect lighting of surface roughness valley (or incomplete reflection of the light beam back to the lens) may occur. In such cases, the value of the height of surface roughness is not recorded in these places. In case of the data postprocessing directly from the software, provided by the manufacturer, this is never a problem. Problem occurs in the case of export of this matrix with values and its subsequent processing. This issue is described in detail in chapter 3.2, which deals with the surface data postprocessing.

Another, significantly cheaper, alternative of rough surfaces scanning is the utilization of contact profilometer. This option is, however, significantly limited by the curvature radii of the measuring stylus, which is used for data scanning (only two-dimensional surface profile is scanned). With such measuring device it is, logically, not possible to scan as smooth surfaces and with as high precision as in the case of the optical profilometer. Another disadvantage is the limited possibility of capturing the directional dependence of scanned surfaces. Taking into account that all surfaces which were manufactured by using any machining are highly directionally dependent, it can be stated that this is a considerable disadvantage.

To get the picture of the surface appearance of mechanical components measurement (scanning) of surfaces of several samples was carried out. An overview of the measured values with illustrative pictures follows. If in the picture of the component are marked points, their number corresponds to the line number in the table of measurements. The number



of measurements for individual samples differs. The reason is that, for example, in the case of slide bearings places of measuring (then also the number of these points) were selected with regard to the number of “interesting” places on its surface.

- Mercedes-Benz Vito – the bottom part of connecting rod bearing shell

It is a worn machinery part. Scanned area is $94 \times 94 \mu\text{m}$. The detailed view of the surface roughness pattern shows signs of wear – scratches in the scanned surface.

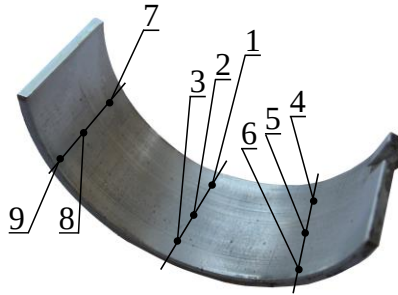


Fig. 2: The bottom part of Mercedes-Benz slide bearing shell

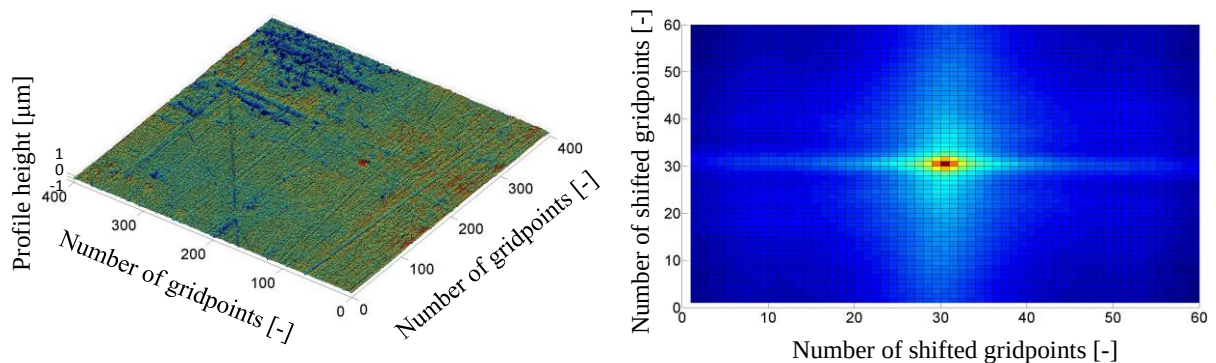


Fig. 3: Detail of surface roughness pattern and the appropriate ACF

Table 1: Measured statistical description of the bottom Mercedes-Benz bearing shell surface

Measurement number	S_a [μm]	S_{ku} [-]	S_p [μm]	S_q [μm]	S_{sk} [-]	S_v [μm]	S_z [μm]	S_{dq} [$^\circ$]	S_{dr} [-]	S_{ds} [$1/\text{mm}^2$]	S_{sc} [$1/\text{mm}$]
1	0.1636	8.9363	0.9591	0.2403	-1.6501	-2.0177	2.9769	15.7791	-	-	-
2	0.2763	10.3772	1.9512	0.3655	-0.8248	-4.8717	6.8229	17.8529	4.6796	42151.1680	547.5310
3	0.5725	13.9107	1.8003	0.8981	-2.9282	-7.4648	9.2651	17.4573	4.3834	44234.8570	593.0490
4	0.2330	8.3537	3.0976	0.3238	-0.4460	-3.6753	6.7729	25.3400	-	-	-
5	0.2040	15.9911	3.2503	0.2882	0.3966	-4.1641	7.4144	18.8596	5.2570	35599.2056	558.4062
6	0.2513	7.1793	2.4237	0.3560	-1.0512	-4.3645	6.7882	19.3745	5.3968	45971.8965	512.8701
7	0.2092	11.4547	2.8717	0.2965	0.0983	-4.1713	7.0430	18.0262	4.7583	43217.4914	430.1988
8	0.9456	2.6355	3.5428	1.1881	0.5501	-3.8745	7.4172	18.2955	-	-	-
9	0.2945	9.7396	2.2824	0.4209	-1.2961	-3.1488	5.4312	17.8000	-	-	-
Average	0.3500	9.8420	2.4644	0.4864	-0.7946	-4.1947	6.6591	18.7539	4.8950	42234.9237	528.4110



- Mercedes-Benz Vito – the top part of connecting rod bearing shell

Again, it is a worn machinery part. Scanned area in this case is $118 \times 118 \mu\text{m}$.

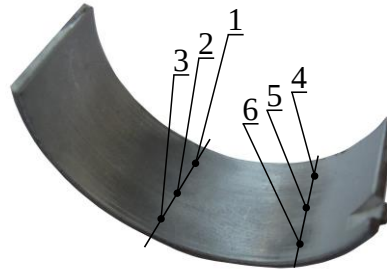


Fig. 4: The top part of Mercedes-Benz slide bearing shell

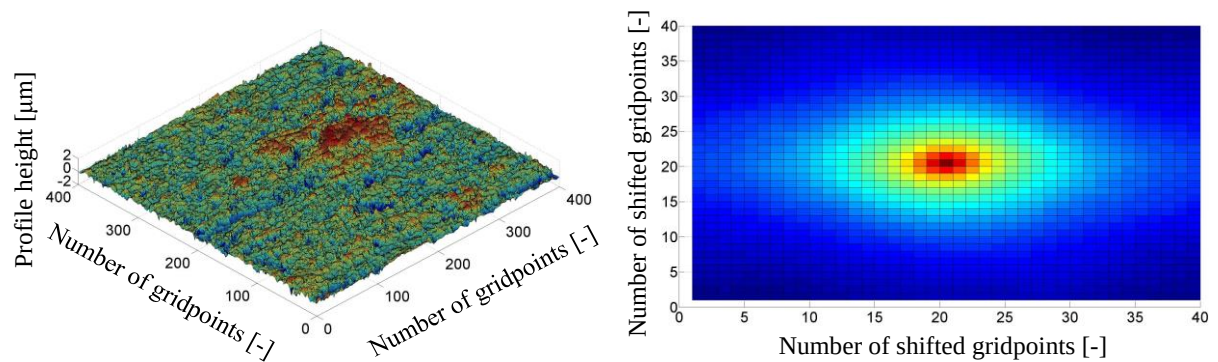


Fig. 5: Detailed view of surface roughness pattern and the appropriate ACF

Table 2: Statistical descriptive data of the top part of Mercedes-Benz slide bearing shell

Measurement number	S_a [μm]	S_{ku} [-]	S_p [μm]	S_q [μm]	S_{sk} [-]	S_v [μm]	S_z [μm]	S_{dq} [$^\circ$]	S_{dr} [-]	S_{ds} [$1/\mu\text{m}^2$]	S_{sc} [$1/\mu\text{m}$]
1	0.3320	13.3265	1.8715	0.4759	-1.6319	-6.7369	-	-	-	-	-
2	0.3595	6.0605	2.5156	0.5005	-0.7887	-4.6604	-	-	-	-	-
3	0.2409	10.0365	1.5173	0.3355	-0.9330	-5.1298	-	-	-	-	-
4	0.2538	7.7405	1.4720	0.3497	-1.1967	-4.0256	-	-	-	-	-
5	0.1676	18.6774	1.2550	0.2480	-1.9458	-3.6556	4.9106	31.5382	15.9772	0.1145	2.5652
6	0.2578	6.4239	1.7131	0.3474	-0.9373	-3.8887	5.6018	41.6582	31.6916	0.1568	3.8745
Average	0.2686	10.3776	1.7241	0.3762	-1.2389	-4.6828	5.2562	36.5982	23.8344	0.1357	3.2199



- Ford Tourneo – the bottom part of connecting rod bearing shell

As in the previous cases – it is a worn machinery part. Scanned area is 118x118 μm .

The shape of Areal Autocorrelation Function (AACF) is not derived from the sample of surface roughness pattern introduced. The shape of AACF would be significantly affected by this noticeable scratch of the surface. The example of the surface with the scratch is here just to demonstrate the accuracy of the scanning process by using the optical profilometer.

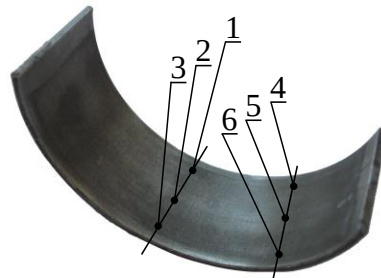


Fig. 6: The bottom part of Ford slide bearing shell

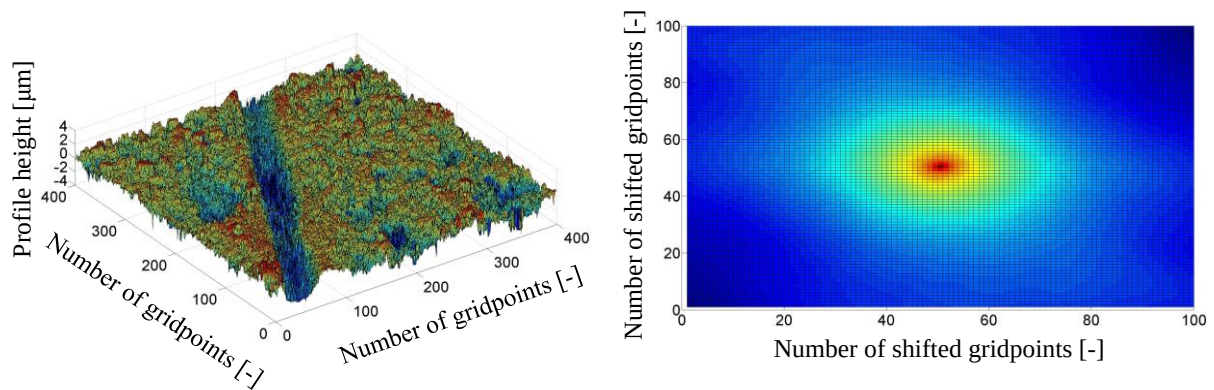


Fig. 7: Measured surface roughness pattern of the bottom part of Ford slide bearing shell with its AACF

Table 3: Statistical descriptive data of the bottom part of Ford slide bearing shell

Measurement number	S_a [μm]	S_{ku} [-]	S_p [μm]	S_q [μm]	S_{sk} [-]	S_v [μm]	S_z [μm]	S_{dq} [$^\circ$]	S_{dr} [-]	S_{ds} [$1/\mu\text{m}^2$]	S_{sc} [$1/\mu\text{m}$]
1	0.7542	3.8361	2.8022	0.9495	-0.6901	-9.6143	12.4164	29.9227	14.4038	0.0160	0.5798
2	0.7534	3.3052	3.4987	0.9333	-0.6223	-6.0810	9.5797	51.4591	60.1865	0.1370	4.3250
3	0.7608	3.7969	2.8288	0.9586	-0.6671	-5.5241	8.3529	50.1894	55.6697	0.1328	3.7080
4	0.6981	3.5903	2.7074	0.8684	-0.6163	-5.5075	8.2149	45.8271	41.7431	0.1172	3.8524
5	0.3600	6.2946	1.6028	0.4954	-1.2800	-3.5818	5.1846	38.9719	26.1709	0.1288	3.2232
6	0.4316	6.1560	1.9132	0.5814	-1.4329	-5.0613	6.9744	41.4205	30.7875	0.1353	3.5137
Average	0.6263	4.4965	2.5588	0.7978	-0.8848	-5.8950	8.4538	42.9651	38.1603	0.1112	3.2004



- Ford Tourneo – the top part of connecting rod bearing shell

It is again a worn machinery part. Scanned area in this case is 118x118 μm .

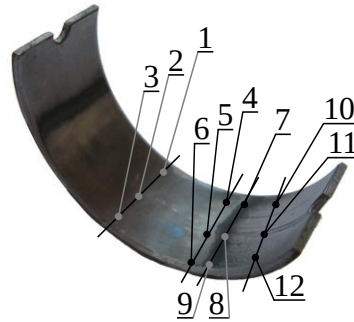


Fig. 8: The top part of Ford slide bearing shell

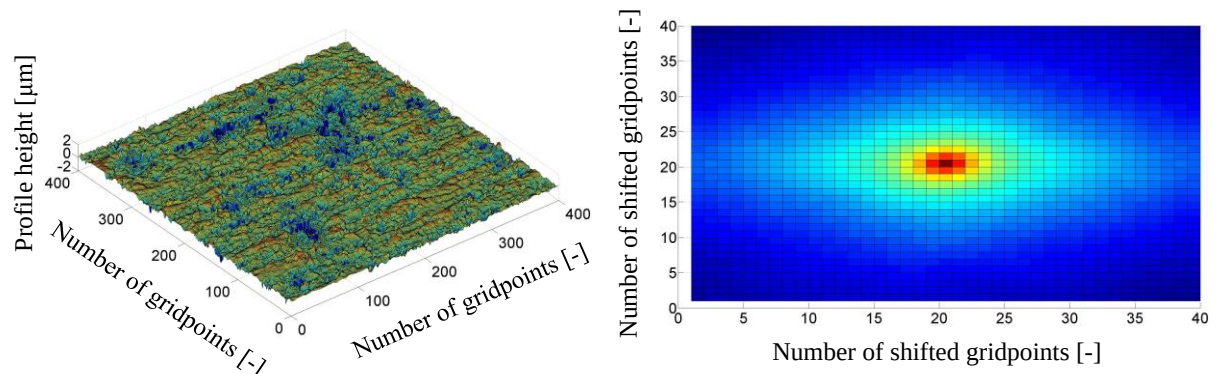


Fig. 9: Detailed view of surface roughness pattern of the top part of Ford slide bearing shell and the shape of appropriate AACF

Table 4: Statistical descriptive data of the top part of Ford slide bearing shell

Measurement number	Sa [μm]	Sku [-]	Sp [μm]	Sq [μm]	Ssk [-]	Sv [μm]	Sz [μm]	Sdq [$^\circ$]	Sdr [-]	Sds [$1/\mu\text{m}^2$]	Ssc [$1/\mu\text{m}$]
1	0.2495	15.2002	1.2392	0.3723	-2.4956	-5.4955	6.7347	34.8158	19.8343	0.1372	2.4488
2	0.1738	10.0639	1.1571	0.2458	-1.7495	-2.7522	3.9092	29.5637	13.4269	0.1290	1.9949
3	0.3624	12.9868	1.5120	0.5262	-2.1923	-7.2309	8.7429	35.9173	21.0653	0.1140	2.4267
4	0.3832	22.5573	1.8577	0.5439	-2.4008	-8.4653	10.3230	40.5596	29.2829	0.0989	3.7227
5	0.3373	65.2303	2.0209	0.5559	-5.2328	-10.1069	12.1278	34.0126	18.7173	0.0696	3.1924
6	0.3176	78.0384	1.5721	0.5703	-6.4590	-10.3940	11.9661	36.9933	23.0917	0.0787	3.7640
7	0.7319	8.6088	4.1113	1.0175	-1.0219	-9.3261	13.4374	60.7907	111.8070	0.1054	4.5044
8	0.6906	4.7502	3.3124	0.9065	-0.6572	-7.4281	10.7405	61.6063	123.1740	0.1280	5.1856
9	0.6018	4.5815	3.4157	0.7870	-0.6353	-5.4096	8.8253	59.1928	101.1480	0.1347	4.1654
10	0.4710	4.8154	1.9865	0.6056	-1.0321	-5.1498	7.1363	43.1223	35.0842	0.1329	4.5139
11	0.5012	5.0075	2.6898	0.6486	-0.9043	-6.0905	8.7803	39.7887	27.5973	0.1176	3.3171
12	0.6019	6.6265	3.0265	0.8055	-1.3760	-6.5344	9.5609	49.1346	51.4550	0.1429	5.1142
Average	0.4519	19.8722	2.3251	0.6321	-2.1797	-7.0319	9.3570	43.7915	47.9737	0.1157	3.6958



- Compression piston ring

The following picture shows the trace signs of honing process – the resulting surface pattern is not isotropic and therefore the AACF of this surface doesn't have a symmetrical shape.

Scanned area is 47x47 μm .



Fig. 10: Compression piston ring

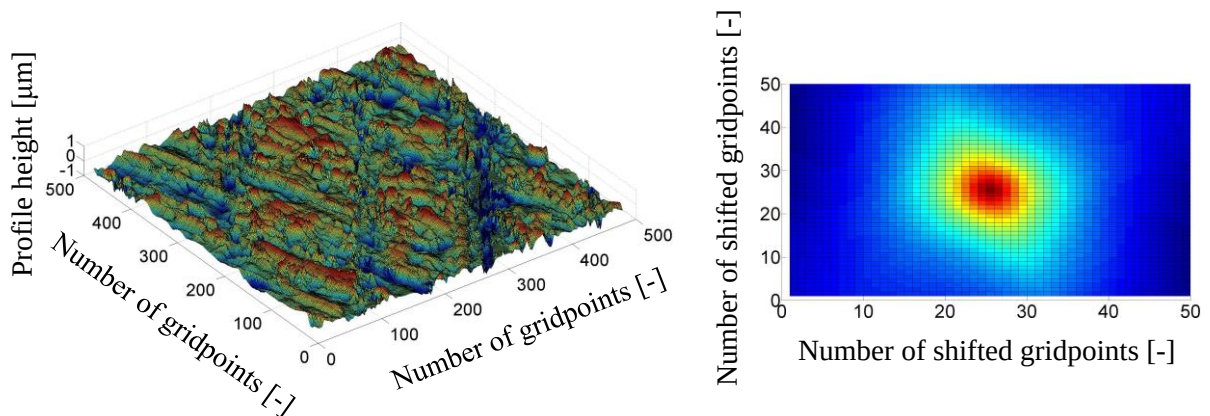


Fig. 11: The compression piston ring surface pattern together with the AACF of this surface

Table 5: Statistical descriptive data of compression piston ring

Measurement number	Sa [μm]	Sku [-]	Sp [μm]	Sq [μm]	Ssk [-]	Sv [μm]	Sz [μm]	Sdq [$^\circ$]	Sdr [-]	Sds [$1/\text{mm}^2$]	Ssc [$1/\text{mm}$]
1	0.1697	3.8937	1.0389	0.2214	-0.1723	-1.3373	2.3762	35.9043	21.9626	479131.4006	5904.3303
2	0.1928	3.4187	1.2488	0.2477	-0.1120	-1.1883	2.4371	37.8352	25.0194	507493.7940	6427.1917
3	0.2210	3.1492	1.2298	0.2798	0.0043	-1.2000	-	-	-	-	-
4	0.2092	3.5694	1.2505	0.2687	-0.1170	-1.5300	-	-	-	-	-
5	0.2341	3.4774	1.2719	0.3005	-0.1365	-1.7602	-	-	-	-	-
6	0.2485	3.0858	1.3002	0.3117	0.0742	-1.4579	2.7581	48.7432	51.8483	717177.0334	10346.1361
Average	0.2126	3.4324	1.2234	0.2716	-0.0765	-1.4123	2.5238	40.8276	32.9434	567934.0760	7559.2194



- Si-DLC coated test sample

Scanned area has the size of 94x94 μm .

Here can be seen (from the shape of AACF) that also after the surface coating of the tooled surface (in this case the final tooling was grinding) the surface pattern dependency is visible – it is again significantly directionally dependent surface pattern.

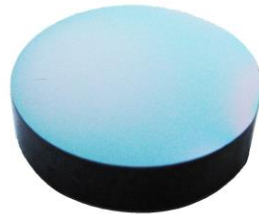


Fig. 12: Si-DLC coated test sample

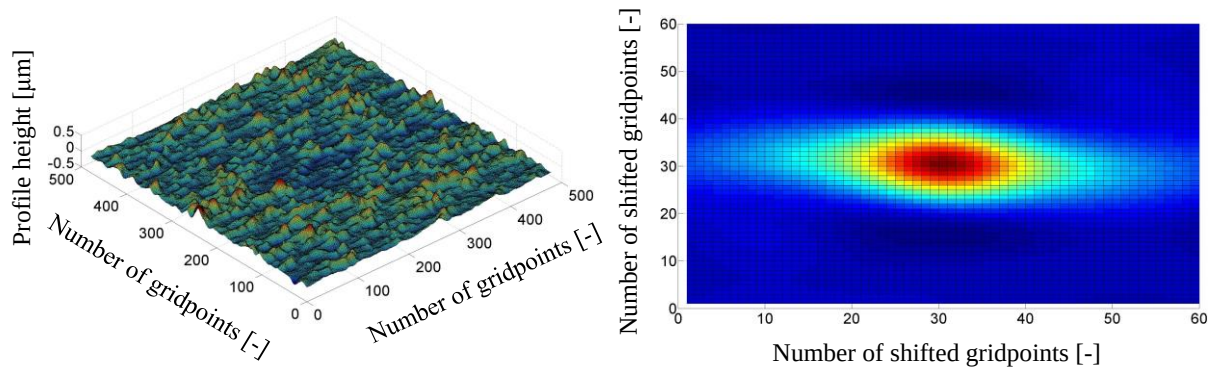


Fig. 13: Surface pattern of Si-DLC coated test sample together with its AACF

Table 6: Statistical descriptive data of Si-DLC coated test sample

Measurement number	S_a [μm]	S_{ku} [-]	S_p [μm]	S_q [μm]	S_{sk} [-]	S_v [μm]	S_z [μm]	S_{dq} [$^\circ$]	S_{dr} [-]	S_{ds} [$1/\mu\text{m}^2$]	S_{sc} [$1/\mu\text{m}$]
1	0.0524	5.0500	0.4243	0.0688	1.2780	-0.2735	0.6978	4.8727	0.2660	-	-
2	0.0531	5.7021	0.4816	0.0703	1.3076	-0.4868	0.9683	5.2017	0.3340	-	-
3	0.0513	5.6501	0.3998	0.0685	1.3442	-0.3018	0.7015	4.6804	0.2543	0.0662	0.3229
4	0.0446	5.5373	0.3382	0.0592	1.2101	-0.4264	0.7647	4.4570	0.1998	0.0734	0.3226
5	0.0500	4.6053	0.3159	0.0651	1.1427	-0.3628	0.6787	4.5662	0.2311	0.0633	0.3102
Average	0.0503	5.3090	0.3919	0.0664	1.2565	-0.3703	0.7622	4.7556	0.2570	0.0676	0.3185



- DLC coated test sample

Scanned area has in this case the size of 125x125 μm .

The following figure demonstrates how great effect can a scratch on the surface pattern have on the evaluation of the directional dependency of this surface pattern. Without the inclusion of the impact of this scratch this surface pattern might be considered an isotropic (the shown AACF would have symmetrical shape).



Fig. 14: DLC coated sample

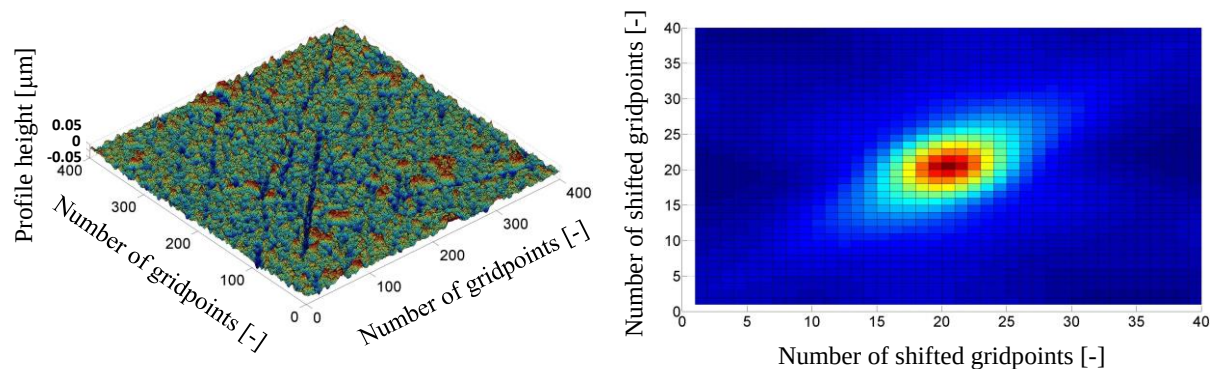


Fig. 15: Surface pattern of DLC coated test sample together with the AACF of this surface pattern

Table 7: Statistical descriptive data of DLC coated test sample

Measurement number	S_a [μm]	S_{ku} [-]	S_p [μm]	S_q [μm]	S_{sk} [-]	S_v [μm]	S_z [μm]	S_{dq} [$^\circ$]	S_{dr} [-]	S_{ds} [1/ μm^2]	S_{sc} [1/ μm]
1	0.0112	759.6540	1.8768	0.0261	17.7881	-1.0542	2.9310	4.3560	-	-	-
2	0.0110	424.6820	0.9885	0.0198	12.9221	-0.2193	1.2078	3.1181	0.4904	0.0040	0.1846
3	0.0185	204.7930	2.2226	0.0593	11.8004	-0.7144	2.9370	8.8601	1.2821	0.0046	0.2822
4	0.0107	1297.9900	1.8868	0.0278	27.2072	-0.5092	2.3960	3.8000	-	-	-
5	0.0117	870.5330	1.4354	0.0194	17.0689	-0.2887	1.7241	2.7542	0.4565	0.0254	0.1710
6	0.0134	396.5820	2.0085	0.0402	16.4582	-0.4183	2.4268	4.9938	0.6650	0.0022	0.1929
7	0.0112	759.6540	1.8768	0.0261	17.7881	-1.0542	2.9310	4.3560	-	-	-
Average	0.0125	673.4126	1.7565	0.0313	17.2904	-0.6083	2.3648	4.6055	0.7235	0.0090	0.2077



- ZETOR piston liner

This machinery part was used to conduct an experiment (see chapter 4) mainly because it was an available part from the company ZETOR a.s. residing in Brno. The picture with surface pattern, again, shows the characteristic structure of honing process which logically affects the shape of AACF of this surface pattern. Because of the necessity to put each measured part under the lens of the profilometer it was necessary to create a cut-out from the original piston liner.

The size of the scanned area is $933 \times 933 \mu\text{m}$.

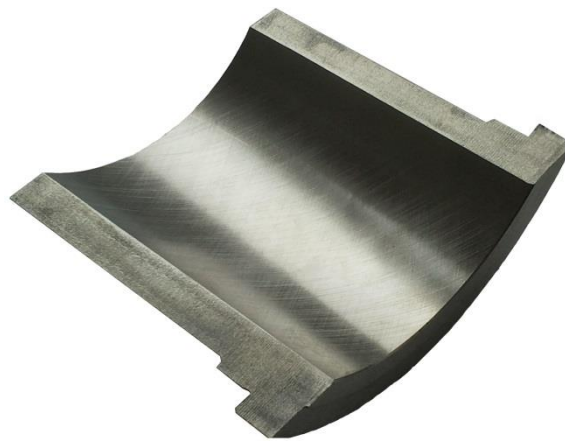


Fig. 16: Cut-out of the original ZETOR piston liner

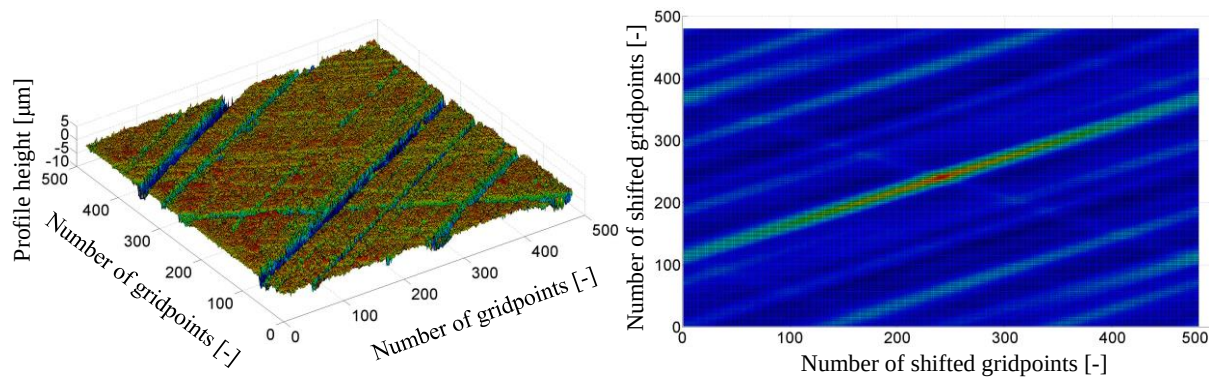


Fig. 17: Surface pattern of the scanned ZETOR piston liner with the shape of its AACF



Table 8: Statistical descriptive data of ZETOR piston liner

Measurement number	Sa [μm]	Sku [-]	Sp [μm]	Sq [μm]	Ssk [-]	Sv [μm]	Sz [μm]	Sds [1/μm ²]	Ssc [1/μm]	Sdr [-]	Sdq [°]
1	0.9807	7.2366	3.7958	1.4051	-1.9434	-10.9034	14.6991	0.0037	0.3014	7.3536	21.7797
2	0.8335	9.4346	3.3374	1.2117	-2.1113	-15.6313	18.9687	0.0035	0.3022	7.1028	21.4740
3	0.7729	10.1459	3.9211	1.0884	-1.9473	-15.4049	19.3260	0.0033	0.3002	7.0973	21.4266
4	0.7118	8.1740	19.5433	0.9718	-1.5086	-10.7181	30.2615	0.0033	0.2961	6.4349	20.3276
5	0.8554	6.5707	7.0061	1.1988	-1.7272	-7.2163	14.2223	0.0037	0.2885	6.3746	20.2087
6	1.2428	5.3599	4.2445	1.6555	-1.5952	-13.5339	17.7784	0.0036	0.3015	7.3049	21.5772
7	0.7566	15.4893	4.8297	1.0832	-2.3513	-20.2033	25.0330	0.0032	0.2992	7.1103	21.3856
8	0.7247	9.3257	3.8416	1.0248	-1.9945	-10.9586	14.8002	0.0036	0.2790	5.9669	19.5898
9	0.9798	9.4950	3.4884	1.4087	-2.1330	-16.2461	19.7345	0.0034	0.2934	6.3195	20.0826
10	1.0607	5.5013	4.4451	1.4066	-1.5621	-8.5200	12.9651	0.0035	0.2873	6.2739	20.0412
Average	0.8919	8.6733	5.8453	1.2455	-1.8874	-12.9336	18.7789	0.0035	0.2949	6.7339	20.7893

- Test piston for experiments

A special piston was made for the purpose of the experiment. To reach a required quality of the surface two types of machining were used – turning and grinding. This manufacturing process left again significant marks on the surface which affect the resulting shape of AACF. Here it is possible to see a significant and dominant direction of surface roughness direction dependence which corresponds to the direction of rotation of the piston during the manufacturing process.

The size of the measured surface sample is 933x933 μm.



Fig. 18: Experimental piston for testing device

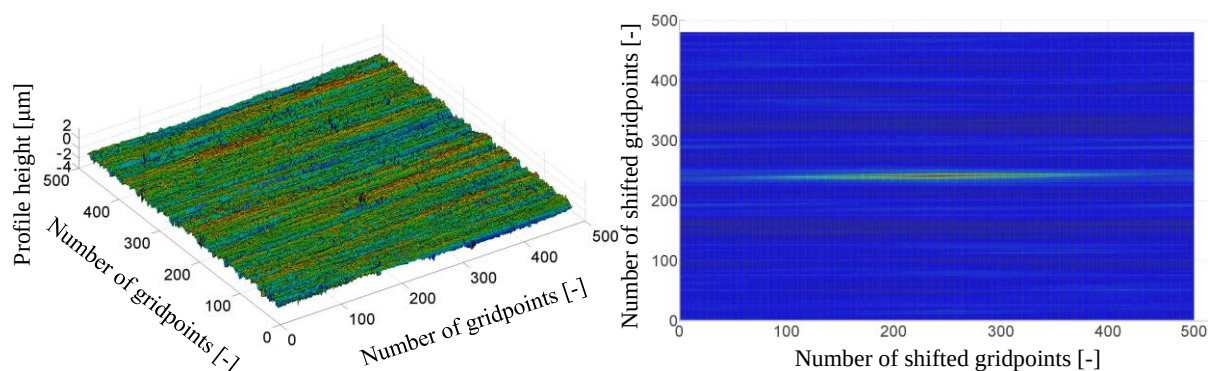


Fig. 19: Scanned surface pattern of experimental piston with its AACF

Table 9: Statistical descriptive data of grinded experimental piston

Measurement number	Sa [μm]	Sku [-]	Sp [μm]	Sq [μm]	Ssk [-]	Sv [μm]	Sz [μm]	Sds [$1/\mu\text{m}^2$]	Ssc [$1/\mu\text{m}$]	Sdr [-]	Sdq [$^\circ$]
1	0.5904	2.8486	3.3404	0.7338	-0.1347	-4.4983	7.8387	0.0026	0.1240	2.1415	11.8194
2	0.5902	2.8518	3.4646	0.7335	-0.1347	-4.4554	7.9200	0.0026	0.1230	2.1406	11.8172
3	0.6794	2.9909	4.6292	0.8573	0.0557	-3.8569	8.4861	0.0025	0.1366	2.4450	12.6477
4	0.6825	2.9879	5.2066	0.8609	0.0703	-3.8449	9.0515	0.0025	0.1360	2.4554	12.6660
5	0.5874	3.7580	3.6758	0.7484	-0.1980	-6.2298	9.9056	0.0025	0.1393	2.6978	13.3100
6	0.5893	3.7685	3.7954	0.7505	-0.2032	-5.9661	9.7615	0.0025	0.1368	2.6912	13.2893
7	0.5089	2.9646	2.7426	0.6352	-0.1485	-3.7111	6.4536	0.0027	0.1220	1.9207	11.0846
8	0.5102	2.9424	2.2744	0.6367	-0.1452	-3.8816	6.1560	0.0028	0.1212	1.9251	11.0926
9	0.6776	3.1139	2.9040	0.8546	-0.1695	-4.3178	7.2218	0.0025	0.1194	2.5567	13.0060
10	0.6738	3.1349	2.8598	0.8502	-0.1928	-4.1642	7.0240	0.0026	0.1207	2.5495	12.9760
Average	0.6090	3.1361	3.4893	0.7661	-0.1201	-4.4926	7.9819	0.0026	0.1279	2.3524	12.3709



2.2 NUMERICALLY GENERATED ROUGH SURFACES

Among the greatest advantages of the numerical generation of rough surfaces in comparison with the surface scanning (surface measurement) belong:

- time effectivity,
- except for standard computer no additional equipment is needed,
- unlimited possibilities for creation of different surface patterns without limiting any production technologies.

However, the main disadvantage is then the need to develop a specialized software tool for numerical generation of rough surfaces.

There are two most widely used methods for numerical generation of surface pattern. The first one is based on the fractal geometry and the second one uses statistical descriptive data for this purpose. In order to generate directionally dependent rough surface it is necessary to generate three-dimensional data. Therefore, this thesis does not present any equation for the two-dimensional rough surfaces generation.

2.2.1 FRACTAL GEOMETRY BASED ROUGH SURFACES

The fractal geometry based approach was applied mainly because of the fact that many authors use fractal based surfaces for further calculations. In connection to the fractal surfaces, a contact mechanic is, for example, solved in the information source [52]. Fractal surfaces are also widely used for an adhesive wear determination – see references [54], [53]. Moreover, because of this reason, for a subsequently planned implementation of adhesive wear computational model, it is also useful to have at hand rough surfaces generated this way.

For this purpose, the following two methods are most widely used:

- the midpoint displacement method and
- the Weierstrass-Mandelbrot function.

The first mentioned method, in detail described in the reference [43], finds its application in the area of images scans reconstructions of various kinds of material fractures. For the purpose to numerically generate rough surfaces of machinery parts the second mentioned method seems to be the best – the Weierstrass-Mandelbrot function. This function was published in 1998 by authors Yan and Komvopoulos [52] in order to generate the 3D rough surface. Fractal formations, among which are logically counted also fractal rough surfaces, are based on the wide chaos theory. Pioneer in this branch of science was Benoit Mandelbrot [4].

The above mentioned function has the following shape:

$$z_r(x, y) = L \left(\frac{G}{L} \right)^{(D_f-2)} \left(\frac{\ln \gamma_f}{M} \right)^{1/2} \sum_{m=1}^M \sum_{n=0}^{n_{\max}} \gamma_f^{(D_f-3)n} \left\{ \cos \phi_{m,n} - \right. \\ \left. - \cos \left[\frac{2\pi \gamma_f^n (x^2 + y^2)^{1/2}}{L} \cos \left(\tan^{-1} \left(\frac{y}{x} \right) - \frac{\pi m}{M} \right) + \phi_{m,n} \right] \right\}, \quad (1)$$



where L is the sample length [m], G is the fractal parameter of asperity heights changes [m], D_f is the fractal dimension [-], γ_f is the parameter of scale changes [-], M is the number of superposed ridges used for surface roughness profile construction [-], n is the frequency index [-] and $\phi_{m,n}$ is the random phase [-].

$$n_{\max} = \text{int} \left[\frac{\log(L / L_s)}{\log \gamma_f} \right], \quad (2)$$

where $\text{int}[\dots]$ denotes the maximum integer value of the number in the bracket and L_s is the “cut off” length [m].

Examples of the three-dimensional fractal surface and the cut through this surface (2D rough profile), generated by using this function, are presented in the figures below. The size of one edge of this surface sample is $1 \cdot 10^{-5}$ m.

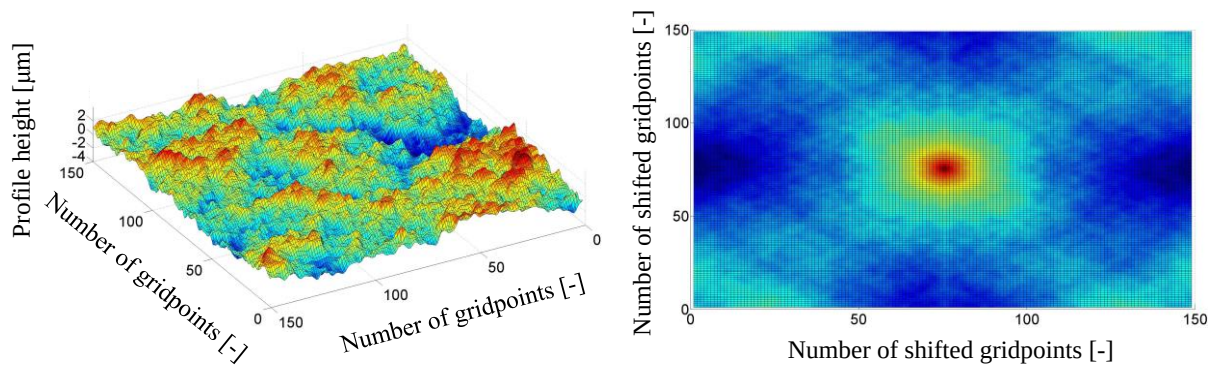


Fig. 20: Three-dimensional fractal surface together with view on its ACF

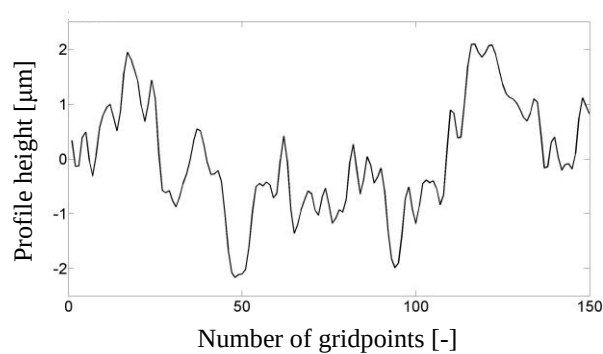


Fig. 21: Cut through 3D fractal rough profile

In the case of need to generate fractal rough profile based on measured data it is essential to know the main parameter for the Weierstrass-Mandelbrot function – the fractal dimension D_f . Therefore, the following chapter, dealing with the fractal dimension determination, is incorporated.



2.2.2 FRACTAL DIMENSION D_f DETERMINATION

Although the fractal geometry is known since the 1970's, there is no exact or universal definition of this concept. Mandelbrot defined fractal using its main feature, the self-similarity. This shows that fractal is created by the infinite number of geometric patterns that repeat themselves in the basic body changing only their view of scale. Fractal formation can be described on two examples known worldwide – the Koch snowflake or the calculation of the length of Britain's coastline. [4]

The fractal dimension can take non-integer values in range $0 < D_f < 3$. For example simple line would have fractal dimension of $D_f = 1$, plain surface $D_f = 2$, but surface with fractal dimension of $D_f = 2.9$ would fill the space more like a volume.

Fractal dimension can be used to describe the complexity of the analysed surface, as it is shown in the next figure. Even though these two surfaces have the same roughness ($Sa = 0.8 \mu\text{m}$), it can be clearly seen that they look different. This variance is caused by the fractal dimension.

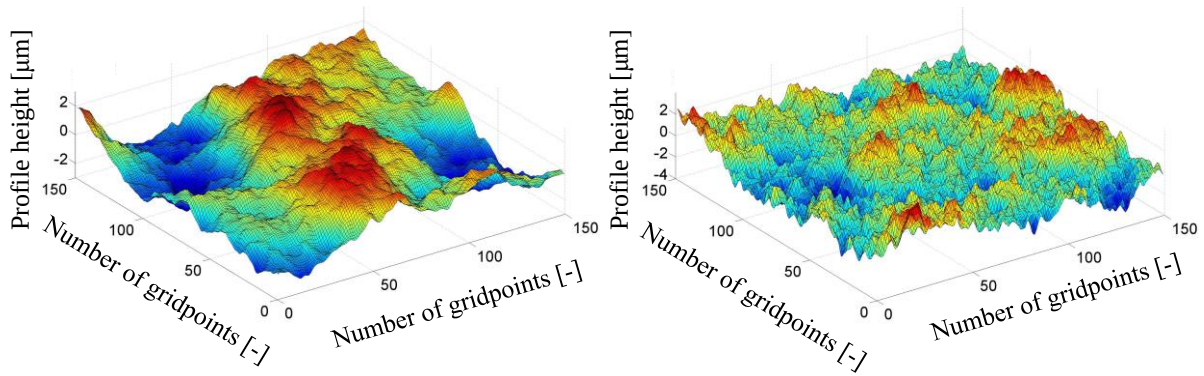


Fig. 22: Three-dimensional fractal surface with fractal dimension 2.1 (on the left side) and with the dimension 2.7 (on the right side)

To determine fractal dimension (FD) a specialized software tool in MATLAB programming environment was developed, it can provide the real value of FD of the given surface profile. As mentioned before, fractals are created by repetitions of geometric patterns with the change of scale. This characteristic is used in the FD analysis tool, which means that FD calculation is based on the surface area depending on the scale.

For this purpose, the surface is divided into a known number of elements. For each element the area is calculated, which are then summed up for the whole surface. The element size is gradually decreasing from the equal size to the size of surface multiplied by 0.9.

Because it is hard to determine the FD accurately, three different ways to calculate the area of one element were implemented. The reasons for this step are explained in the following passage.

First approach suggested by Kwaśny (2009) is described by the following equation: [18]

$$A_{ei}(\delta_e) = \frac{1}{2} \delta_e \left(\sqrt{\delta_e^2 + (h_{ai} - h_{bi})^2 + (h_{ci} - h_{bi})^2} + \sqrt{\delta_e^2 + (h_{ai} - h_{di})^2 + (h_{ci} - h_{di})^2} \right). \quad (3)$$



Where $A_{ei}(\delta_e)$ is the area of the i -th element [m^2], δ_e is the length of each element side [m] and h_{ai} , h_{bi} , h_{ci} , h_{di} are values of element height in each of its corners [m].

As the second approach the formula developed by Xie (1998) was implemented: [51]

$$A_{ei}(\delta_e) = \frac{1}{2} \left(\sqrt{\delta_e^2 + (h_{ai} - h_{di})^2} \sqrt{\delta_e^2 + (h_{di} - h_{ci})^2} + \sqrt{\delta_e^2 + (h_{ai} - h_{bi})^2} \sqrt{\delta_e^2 + (h_{bi} - h_{ci})^2} \right). \quad (4)$$

And finally the third approach, based on Heron's formula for the calculation of the area of a triangle, is expressed by the following equation:

$$A_{ei}(\delta_e) = \sqrt{s_{abc} |s_{abc} - l_a| |s_{abc} - l_b| |s_{abc} - l_c|} + \sqrt{s_{acd} |s_{acd} - l_a| |s_{acd} - l_c| |s_{acd} - l_d|}. \quad (5)$$

Where l_a , l_b , l_c and l_d are lengths of element sides [m], s_{abc} and s_{acd} are circumferences of triangular elements created by division of each squared element along its diagonal [m] and l_{ac} is the length of this element's diagonal [m].

If the whole area of surface is known for the present element size, then these two parameters are plotted to the fully logarithmic axes graph and linear regression is performed on the linear part of the data set, as shown in the following figure.

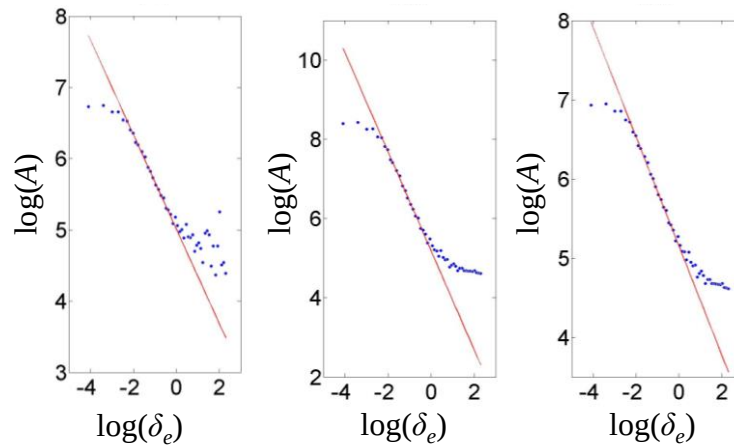


Fig. 23: Fully logarithmic interpretation of the dependence of the surface area on the element size. On the left: according the Kwaśny formula, in the middle: according the Xie approach and on the right: computation based on Heron's formula.

The line formed during the linear regression can be described as:

$$y_f = \alpha_f + \beta_f x_f. \quad (6)$$

Then the FD of the analysed surface is calculated according to the equation: [18]

$$D_f = 2 - \beta_f. \quad (7)$$

All three formulas are used because during the testing phase (with surfaces of known FD) of the FD analysis tool it was discovered that none of the formulas can be used for the whole range of FDs (D_f from 2.1 to 2.9), since none of the formulas are accurate enough throughout the whole range.



Furthermore, it was discovered that the first implemented approach (Kwaśny) works well for low values of FDs ($D_f < 2.2$). It was also discovered that the most accurate results for values $D_f > 2.2$ are given by arithmetic mean value of results from the second and the third approach (Xie and Heron's formula).

After the implementation of these changes into the FD analysis tool, the maximum error of the determined D_f value was within 1.5%, instead of up to 12% when only one formula was used. The effect of this improvement can be clearly seen in the graph below.

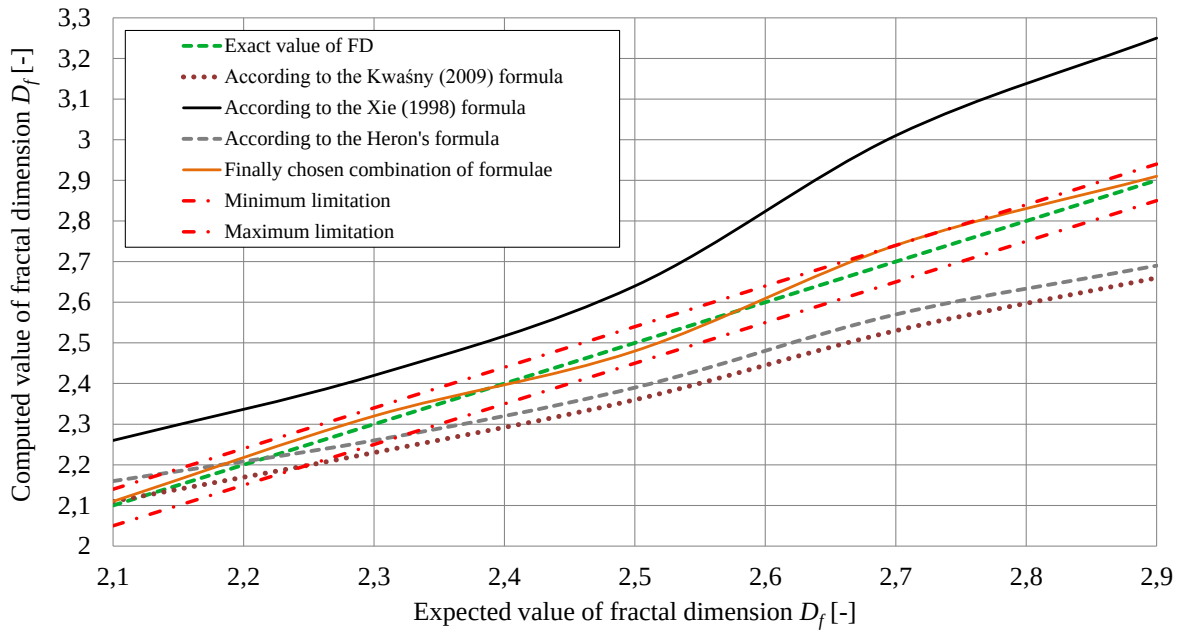


Fig. 24: Comparison of implemented approaches

2.2.3 NUMERICAL GENERATION OF ROUGH SURFACES BASED ON THE STATISTICAL DATA

Parameters used for rough surfaces description are listed in the norm ISO 25178-2:2012 [10]. After a detailed view into this reference, it can be seen that a big part of this norm is based on the information source [46].

For numerical generation of rough surfaces based on statistical data an equation listed in reference [48] is used.

$$z_r(x, y) = \frac{Sq}{\sqrt{n_s m_s}} \sum_{k=1}^{m_s} \sum_{l=0}^{n_s} \phi_{x+k, y+l} \quad (8)$$

Here $z_r(x, y)$ is the resulting matrix with surface roughness heights [m]. Variables n_s and m_s are autocorrelation lengths in both x and y directions [-]. Sq is a required value for the generated surface profile [m]. Variables x and y give the size (number of gridpoints) of matrix z_r and ϕ is, as in the case of fractal surfaces, random phase [-]. The random phase is in this case created by values of normal distribution, mean value of 0 and a standard deviation of 1. To generate these random numbers the function *normrnd* from the statistic toolbox of MATLAB is used.



The ratio between values m_s and n_s gives the directional dependence of the generated surface. By variation of these values it is possible to affect the surface density of surface asperities. Examples of surfaces with different areal density of asperities are shown in the following two figures. The length of each side of both surfaces samples is $7.5 \cdot 10^{-6}$ m. The third figure shows an example of directionally dependent surface. The side length of this sample is the same as in the previous case.

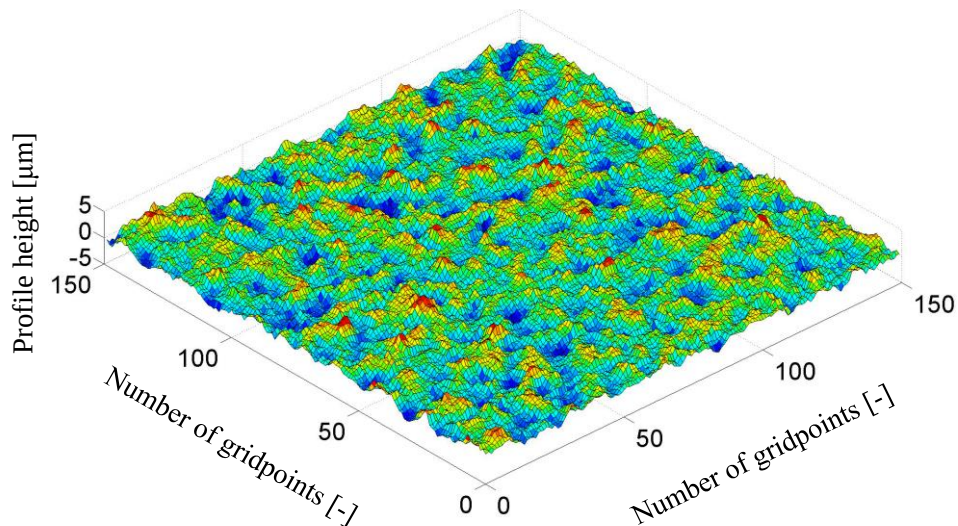


Fig. 25: Three-dimensional rough surface – isotropic surface with autocorrelation length 5

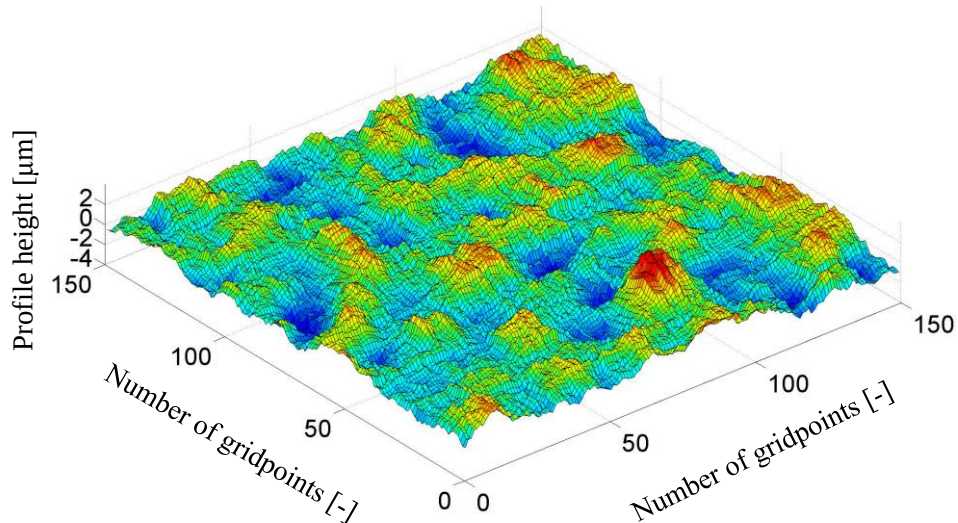


Fig. 26: Three-dimensional rough surface – isotropic surface with autocorrelation length 15

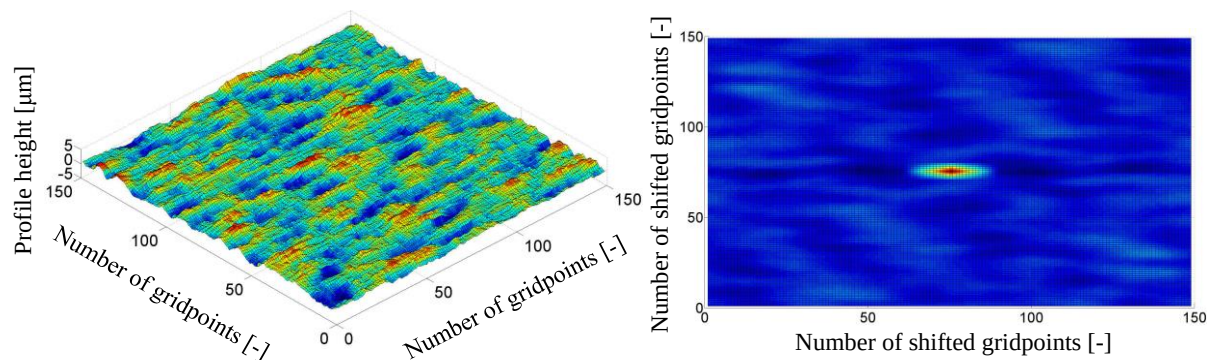


Fig. 27: Three-dimensional rough surface – anisotropic surf. with ratio of autocorrelation lengths 1/3

2.2.4 HONED SURFACES

Striation of the honing process is created into the generated surface roughness profile additionally, independently from the type of surface roughness creation. It is basically the same process as in the case of a real manufacturing process. The honing process follows after the final tooling and therefore also the numerical generation of characteristic honing furrows is implemented after the numerical generation of the 3D rough profile.

It is a purely numerical process – the given value in the matrix with surface roughness height needs to be replaced by a different value. Honing process creates in the surface roughness profile regular grooves under a given angle and with given spacing. The angular coefficient of each groove is then calculated from the angle between the grooves. These groove lines determine the width, position, and angle of the final honing grooves. The labelling of the keypoints of regular finite difference mesh becomes more challenging when the angular coefficient of the groove lines exceeds $\tan(45^\circ)$. In this case bigger gaps occur between the calculated values on the y axis, resulting in the creation of empty y axis values in the matrix. In that case an inverted process can be used, when the value on the x axis is determined from the value on the y axis of a given point. This situation is outlined in the following figures.

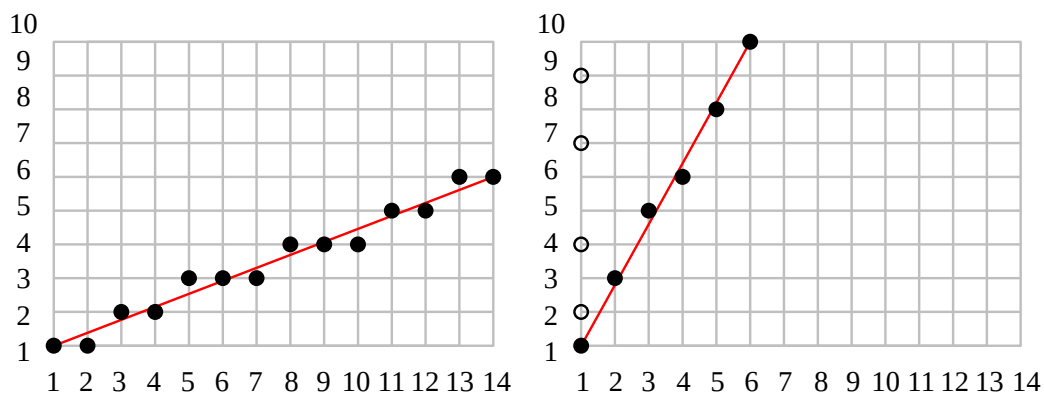


Fig. 28: Labelling of mesh grid points to make honing process grooves – for each value on the x axis a y axis value is determined

The figure on the left shows the case when the angular coefficient of each groove line is not higher than $\tan(45^\circ)$. In the case when angular coefficient of each groove line exceeds $\tan(45^\circ)$, the situation illustrated by the picture on the right, can occur. One unit step on the x axis causes bigger than one unit step on the y axis and therefore empty points about the positions of grooves, caused by honing process, appear in the resulting matrix.

The following figure shows the reversed process for the case when the line's angular coefficient is bigger than $\tan(45^\circ)$.

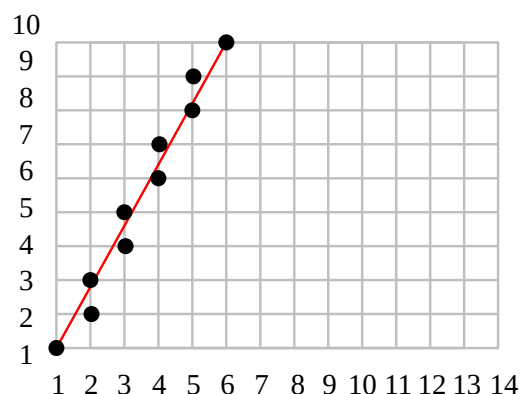


Fig. 29: Labelling of mesh grid points to make honing process grooves – for each value on y axis a x axis value is determined

User sets the value of the angle between the honing lines at their intersection, the distance between honing grooves, their width and depth. The character of this input data (for the modelling of the honing grooves in the rough surface) outlines that relatively advanced information about the honing tool and process is required from the user.

The following figures show numerically generated, directionally dependent rough surfaces with marks after honing process.

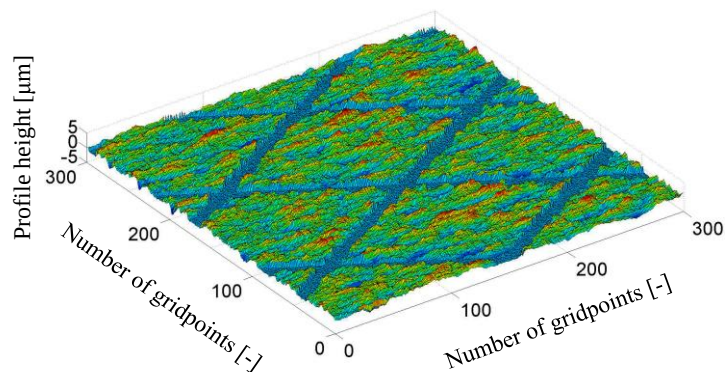


Fig. 30: Numerically generated, directionally dependent rough surface with angle between honing grooves 120°

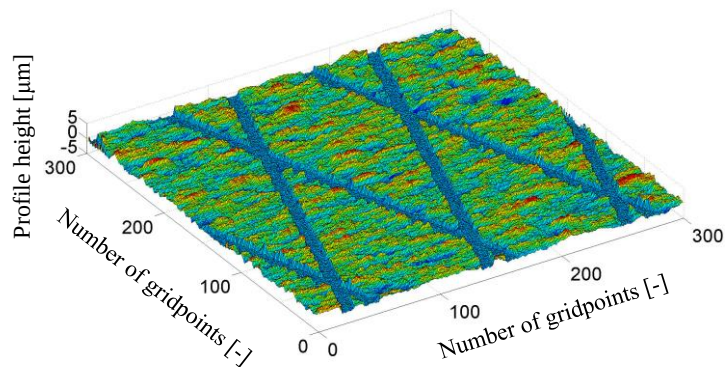


Fig. 31: Numerically generated, directionally dependent rough surface with angle between honing grooves 35°

2.3 ROUGH SURFACE PARAMETERS TAKEN FROM THE LITERATURE

Several examples are in this chapter included for better illustration of surface parameters of other surface structures. Information comprised in this chapter is gathered exclusively from literature research. The chapter is further divided according to the type of tooling process through which the final shape of surface pattern was obtained.

a) Turning

Four different surfaces were measured. From T1 (Turned 1) up to T4 (Turned 4).

T1 – the manufactured material was steel AISI 5140 with hardness 57 ± 1 HRC. Turning was done on a CNC turning machine without cooling by process liquid, the cutting speed was $150 \text{ m} \cdot \text{min}^{-1}$, feed rate $0.1 \text{ mm} \cdot \text{rev}^{-1}$ and chip width of turning tool 0.15 mm. Cutting tool was made of CBN, (CB7015) from company Sandvik Coromat. Turned surface was finished by ball burnishing or superfinishing. Parameters of final surfaces can be seen in tables for corresponding type of operation (see below). [6]



T2, T3 – turned material was steel AISI 52100 with the hardness of the surface 61–62 HRC. Both turning processes were without process liquid, feed rate was $0.0254 \text{ mm} \cdot \text{rev}^{-1}$ and depth of cut was 0.254 mm . For turning process T2 the cutting speed (velocity) was set to $1.78 \text{ m} \cdot \text{s}^{-1}$ and turning tool GE BZN 8100 ($0.015/15^\circ$ chamfer, 6.35 mm radius) was used. For T3 the operation was changed: the turning tool was substituted by coated tool GE BZN 8100 (VB 0.5 mm) ($0.015/15^\circ$ chamfer, 6.35 mm radius) and the cutting speed (velocity) was increased to $2.82 \text{ m} \cdot \text{s}^{-1}$. Turned surfaces were finished with grinding, see the parameters in the corresponding table. [50]

T4 – turned material was AZ91, which is magnesium alloy with tensile strength $R_m = 235 \text{ MPa}$, yield point $R_{p0.2} = 175 \text{ MPa}$ and hardness of the surface 83 HB. [16]

Table 10: Surface roughness parameters after turning [6][16][50]

	S_a [μm]	S_q [μm]	S_z [μm]	S_p [μm]	S_v [μm]	S_{vk} [μm]	S_{sk} [-]	S_{ku} [-]	S_{mr} [%]	S_{dr} [%]	S_{ds} [mm^{-2}]	S_{sc} [μm^{-1}]
T1	0.460	-	2.380	-	-	0.091	0.139	-	43.600	0.070	581	0.005
T2	0.229	0.280	1.350	0.980	0.653	-	0.530	2.880	48.900	0.442	3996	0.298
T3	0.175	0.219	1.300	0.773	0.715	-	0.228	2.890	85.200	0.588	4504	0.363
T4	0.908	1.040	5.460	3.030	3.200	-	0.308	1.910	-	0.148	799	0.006

b) Milling

Material of scanned surface profiles was AISI 1034.

Table 11: Parameters of surface structures after milling [40][17]

	S_a [μm]	S_q [μm]	S_z [μm]	S_{sk} [-]	S_{ku} [-]	S_{10Z} [μm]	S_{ds} [mm^{-2}]	S_{pd} [mm^{-2}]
M1	0.920	1.100	4.790	0.470	2.330	-	606	-
M2	4.090	-	-	-	2.510	511	-	129
M3	4.150	-	-	-	2.590	326	-	79
M4	3.620	-	-	-	1.990	748	-	148

c) Electroerosive machining

In the following table are listed values of settings for electroerosive machining and values of surface roughness parameters for tool steel (0.38 % C, 16 % Cr) and material removal 0.5 mm for all analysed samples.

Table 12: Surface roughness parameters after electroerosive surfacing [41]

Impuls [μs]	Electric current [A]	Cathode area [mm^2]	Time of process	S_q [μm]	S_{ds} [mm^{-2}]
750	22.850	750	11 min 30 s	26.865	176.600
750	11.180	3000	2 h 6 min 3 s	13.090	382.800
120	11.180	750	40 min 6 s	13.730	277.050



d) Ultrasonic assisted turning

The acquired information is for ultrasonic and conventional turning of the titanium Ti-15V-3Al-3Cr-3Sn. This material has high tensile strength $R_m = 1200$ MPa and low coefficient of thermal conductivity $(8.1 \text{ W} \cdot (\text{m} \cdot \text{K})^{-1})$.

Cutting conditions for ultrasonic turning:

- cutting speed $10 - 70 \text{ m} \cdot \text{min}^{-1}$,
- feed $0.1 \text{ mm} \cdot \text{rev}^{-1}$,
- depth of cut $50 - 500 \text{ } \mu\text{m}$,
- without coolant.

Cutting conditions for conventional turning:

- cutting speed $10 - 70 \text{ m} \cdot \text{min}^{-1}$,
- feed $0.1 \text{ mm} \cdot \text{rev}^{-1}$,
- depth of cut $50 - 500 \text{ } \mu\text{m}$,
- without coolant.

Table 13: Acquired surface roughness parameters after ultrasonic and conventional turning of titanium [25]

Mach. method	S_a [μm]	S_q [μm]	R_a [μm]	R_q [μm]	R_{sk} [-]	R_{ku} [-]	Spd [mm^{-2}]	Spc [mm^{-2}]
Ult.	0.89 ± 0.25	1.17 ± 0.35	1.21 ± 0.28	1.56 ± 0.34	0.73 ± 0.54	3.92 ± 1.58	764 ± 335	1.82 ± 2.64
Conv.	1.73 ± 0.33	1.98 ± 0.33	2.14 ± 0.27	2.58 ± 0.36	0.11 ± 0.24	1.94 ± 0.48	140 ± 53	1.05 ± 0.26

e) Grinding

Evaluated surfaces are labelled as G1 (Grinded 1) – G7 (Grinded 7).

G1, G2 – grinded material was steel AISI 52100. Machining tool was a grinding wheel with diameter of 254 mm, made of Al_2O_3 . [50]

Cutting conditions [50]:

	G1	G2
- cutting velocity	$23.94 \text{ m} \cdot \text{s}^{-1}$	$23.94 \text{ m} \cdot \text{s}^{-1}$,
- table speed	$15.24 \text{ m} \cdot \text{min}^{-1}$	$16.62 \text{ m} \cdot \text{min}^{-1}$,
- cross feed	$1.14 \text{ mm} \cdot \text{pass}^{-1}$	$1.14 \text{ mm} \cdot \text{pass}^{-1}$,
- down feed (rough)	$12.7 \text{ } \mu\text{m} \cdot \text{pass}^{-1}$	$12.7 \text{ } \mu\text{m} \cdot \text{pass}^{-1}$,
- number of passes (rough)	2	2,
- down feed (finish)	$5.08 \text{ } \mu\text{m} \cdot \text{pass}^{-1}$	$26.0 \text{ } \mu\text{m} \cdot \text{pass}^{-1}$,
- number of passes (finish)	1	1,
- coolant	yes	no.

G3 - surface G3 was created by grinding of chrom-molybden alloy, mark 42CrMo4. [15]

G4, G5 – grinded material was steel 100Cr6 refined to hardness 63 ± 2 HRC. [28]

G6 – gear made of steel ASM 6308 refined to hardness 35 – 45 HRC. [27]



G7 – grinded material was steel AISI 5140, hardness of surface 57 ± 1 HRC and initial $Sa = 0.4 \mu\text{m}$. Material was grinded by a grinding wheel made of Al_2O_3 with cutting velocity $11.9 \text{ m}\cdot\text{s}^{-1}$, depth of cut 0.025 mm and table speed $3.5 \text{ m}\cdot\text{min}^{-1}$. Cutting process was with coolant. [7]

Table 14: Surface parameters after grinding process [50][15][27][28][7]

	Sa [μm]	Sq [μm]	Sz [μm]	Sp [μm]	Sv [μm]	Svk [μm]	Ssk [-]	Sku [-]	Smr [%]	Sdr [%]	Sds [mm^{-2}]	Ssc [μm^{-1}]
G1	0.158	0.196	1.250	0.612	0.679	-	-0.127	2.790	97.300	0.402	8762	0.172
G2	0.202	0.256	1.800	1.110	1.010	-	0.427	3.190	30.900	3.680	47815	1.150
G3	0.638	0.642	6.060	3.710	3.430	0.714	-0.280	3.590	-	0.089	313	0.003
G4	79.600	104	749	-	-	-	-1.270	4.630	-	324	189	0.193
G5	57.200	72	538	-	-	-	-0.818	3.910	-	217	166	0.163
G6	1.300	1.600	11.700	6.090	5.610	1.630	-0.050	2.710	-	0.335	-	-
G7	0.280	-	1.600	-	-	-	-0.200	4.100	-	-	1305	0.018

f) Honing

Evaluated surfaces are labelled as H1 – H5.

H1 – honed material was steel ASM 6308 refined to hardness 35 – 40 HRC.[27]

H2, H3 - plateau honed piston liners. Surface H2 was first coarse-honed with honing pressure 4.1 MPa. Final operation was under honing pressure 1 MPa and it took 23 s. The final honing of surface H3 took 180 s, under honing pressure 1 MPa and the coarse-honing pressure was 18,1 MPa.[34]

H4, H5 – these two examples are presented in this text for comparison of surfaces with and without surface texture. Both surfaces were plateau honed. To surface H5 were added oil pockets, created by the burnishing technique. The average depth of oil pockets was $5 \mu\text{m}$. Oil pockets are on the 13% of the entire surface.[37]

Table 15: Honed surfaces parameters [27][34][37]

	Sa [μm]	Sq [μm]	Sz [μm]	Sp [μm]	Sv [μm]	Svk [μm]	Ssk [-]	Sku [-]	Spc [μm^{-1}]	Spd [μm^{-2}]	Sdr [%]	Sds [mm^{-2}]	Ssc [μm^{-1}]
H1	0.110	0.140	6.040	3.880	2.160	0.140	1.350	27.400	0.005	2505	0.015	-	-
H2	0.356	0.553	14.800	3.570	18.200	-	-4.300	49.400	-	-	0.549	7171	0.045
H3	0.889	1.270	25.200	7.820	38.100	-	-2.820	15.200	-	-	1.660	5422	0.045
H4	0.245	0.381	3.570	0.810	2.760	-	-2.740	13.800	-	-	-	-	-
H5	1.030	1.550	15.800	7.520	8.330	-	-1.620	6.640	2.060	1299	8.350	-	-

g) Polishing

Surface parameters after polishing with diamond powder are listed in the following table. Polished material was steel S235JR.[55]



Table 16: Surface parameters of polished surface [55]

	Sa [μm]	Sq [μm]	Sz [μm]	Sv [μm]	Spc [μm^{-1}]	Spd [μm^{-2}]	Sdr [%]
P1	0.798	1.020	15.300	8.750	0.007	449	0.208

h) Rolling

Surface parameters of rolled surface made of steel S235JR [55] are listed in the following table.

Table 17: Surface parameters of rolled surface structure [55]

	Sa [μm]	Sq [μm]	Sz [μm]	Sv [μm]	Spc [μm^{-1}]	Spd [μm^{-2}]	Sdr [%]
R1	1.890	2.350	18.900	7.560	0.010	263	0.519

i) Ball penning

BP1, BP2 – analysed samples are made from AZ91 material. It is magnesium alloy with tensile strength $R_m = 235$ MPa, yield point $R_{p0.2} = 175$ MPa and hardness of surface 83 HB. The ball penning process was done with hardened bearing balls with diameter 2.5 mm. [16]

BP3 – gear-wheel under the ball penning process made of steel ASM 6308 and hardened to 35 – 40 HRC.[27]

Table 18: Surface parameters after ball penning [16][27]

	Sa [μm]	Sq [μm]	Sz [μm]	Sp [μm]	Sv [μm]	Svk [μm]	Ssk [-]	Sku [-]	Spc [μm^{-1}]	Spd [μm^{-2}]	Sdr [%]	Sds [mm^{-2}]	Ssc [μm^{-1}]
BP1	2.780	3.470	18.200	10.400	12.400	-	-0.250	2.930	-	-	0.231	77.300	0.006
BP2	2.160	2.720	15.700	8.910	10.500	-	0.056	2.890	-	-	0.215	64.000	0.006
BP3	0.330	0.420	6.010	3.490	2.520	0.430	-0.230	4.200	0.006	1617	0.050	-	-

j) Burnishing

Burnishing by diamond ball is in the following table labelled as DB and burnishing process with ceramic ball has an acronym CB.

Conditions of burnishing [6][15]:

	DB	CB
- tool	ball, diameter 4 – 8 mm	ball,
- tool material	Ti3SiC2 (diamond)	Si3N4 (ceramic),
- burnished material	steel AISI 5140	steel 42CrMo4,
- surface hardness	57 $\pm$ 1 HRC	32 HRC,
- previous operation	turning	grinding,
- operating force	150 – 250 N	600 N,
- velocity	-	25 $\text{m} \cdot \text{min}^{-1}$,
- feed	0.068 – 0.102 $\text{mm} \cdot \text{rev}^{-1}$	0.075 $\text{mm} \cdot \text{rev}^{-1}$.



Table 19: Surface parameters after burnishing [6][15][7]

		Sa [μm]	Sq [μm]	Sz [μm]	Sp [μm]	Sv [μm]	Svk [μm]	Ssk [-]	Sku [-]	Sdr [%]	Sds [mm ⁻²]	Ssc [μm ⁻¹]
DB	min.	0.050	0.062	0.465	0.246	0.238	0.053	-0.800	2.470	0.002	304	0.0000851
	max.	0.185	0.235	1.980	1.510	1.500	0.249	0.055	4.440	0.027	807	0.002
CB		0.240	-	1.460	-	-	0.424	-0.533	-	0.041	1793	0.008

2.4 POSTPROCESSING OF ROUGH SURFACES DATA CHARACTERISTICS

2.4.1 NORMAL DISTRIBUTION TEST TOOL

Contact model by Greenwood & Tripp [11], which is incorporated in the implemented contact models in this doctoral thesis, presumes normal distribution of asperity heights. Therefore, the normal distribution test tool is listed in this text. Standard evaluation of statistics descriptive data then does not require testing of normal distribution. The reason for this is the number of statistical samples. If the number of values is higher than approximately 30, normal distribution testing is not needed. For normal distribution testing Chi-squared (χ^2), or Pearson's test is used. Fundamentally, it is a comparison of the differences between the observed distribution function and the normal distribution function.

Chi-squared criterion χ^2 is given by the equation [14]:

$$\chi^2 = \sum_{i=1}^{m_i} \left(\frac{t_{ci} - \tilde{t}_{ci}}{\tilde{t}_{ci}} \right)^2. \quad (9)$$

Where t_{ci} is the observed frequency count [-] in the particular class m_i , $\tilde{t}_{ci}$ is the theoretical frequency count [-] in the particular class m_i . The unknown variable in this case is the variable $\tilde{t}_{ci}$, its calculation is done by the following equation [14]:

$$\tilde{t}_{ci} = \left(\sum n_o \right) \left(\frac{x_i^+ - \bar{x}}{\sigma_s} - \frac{x_{i-1}^+ - \bar{x}}{\sigma_s} \right). \quad (10)$$

The amount of tested data is n_o , the mean value from values x is $\bar{x}$ and σ_s is the standard deviation. Quantification x^+ is gained from the division of examined data into frequency intervals. For example in interval $<1, 2>$ the considered value of x^+ is 2.

After the χ^2 number is computed, it must be compared with the critical value from the chi-squared distribution. When $\chi^2 < \chi^2_{crit}$, it means that the assumption of normal data distribution is fulfilled. If $\chi^2 > \chi^2_{crit}$, the normal distribution origin can be neglected.

In the following figures examples are shown of numerically generated, isotropic 3D rough surface together with results of normal distribution test. Surface fulfils conditions of normal distribution of asperity heights.

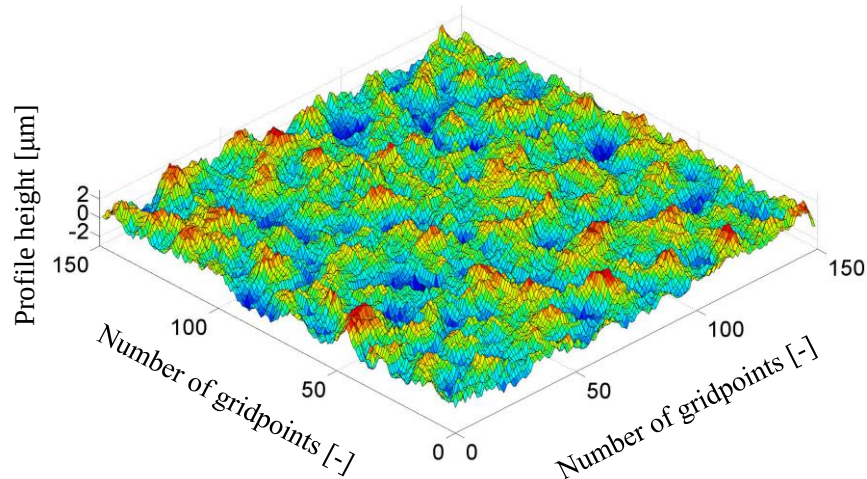


Fig. 32: Numerically generated, directionally independent rough surface (autocorrelation length = 8, size = $7.5 \cdot 10^{-6}$ m and RMS = $0.8 \mu\text{m}$)

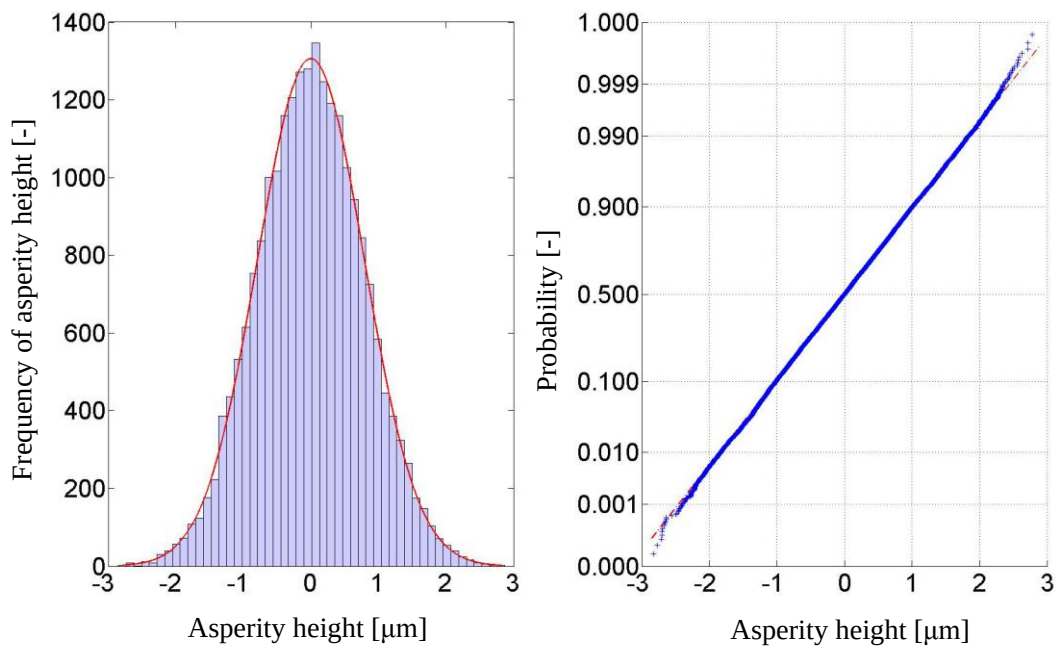


Fig. 33: Histogram and normal probability graph of rough surface

2.4.2 DETERMINATION OF DIRECTIONAL DEPENDENCE OF ROUGH SURFACES

Previous chapters, dealing with rough surfaces, mention directional dependence of rough surfaces or, the shape of Areal Autocorrelation Function (AACF). This AACF is widely used for directional dependence determination of rough surfaces patterns. The following equation was created on the basis of information from reference [46]. The original formula was



modified to show the shape of AACF without any other distortions. For the matrix shifting function *circshift*, from the MATLAB environment, was used.

$$AACF(\tau_i, \tau_j) = \sum_{l=1}^{m_y-j} \sum_{k=1}^{m_x-i} z(x_k, y_l) z(x_{k+i}, y_{l+j}) \quad (11)$$

Where: $\tau_i = i\Delta x, \tau_j = j\Delta y.$ (12)

$AACF(\tau_i, \tau_j)$ is the Areal Autocorrelation Function of the analysed rough surface, Δx is the differential length of the surface sample in direction x , Δy is the differential length of the surface sample in direction y , $i=0,1,\dots, m_x, j=0,1,\dots, m_y$, whereas m_x and m_y are the maximal values of autocorrelation lengths in directions x and y and z is the height of matrix of tested rough surface.

The AACF gives information about the correlation between the original surface and the same surface shifted in all possible plane directions – positive and also negative directions of axes x and y ($\pm\Delta x, \pm\Delta y$).

In the literature the directional dependence of rough surfaces is described by the so called Peklenik number γ_R [35]. This number is determined by the following formula [32]:

$$\gamma_R = \frac{\lambda_{x0.5}}{\lambda_{y0.5}}. \quad (13)$$

Here $\lambda_{x0.5}$ is the differential length of rough surface in direction of x axis, needed for the reduction of AACF by 50% of its original height. Similarly $\lambda_{y0.5}$ is the differential length of rough surface in direction of y axis, needed for the reduction of AACF by 50% of its original height.

Shapes of AACF for individual scanned and also numerically generated surfaces have already been listed together with corresponding examples of those surfaces. Therefore, these are not listed again in this subchapter.

Especially during the scanning process it is essential to keep in mind that under the scanning lens it is necessary to orient measured surfaces in the way that axes x and y of the resulting surface scan correspond with the orientation of real-life process. The following post-processing (rotation of scanned data) can cause loss of significant amount of data and/or can also cause errors. Random or incorrect orientation of scanned surfaces under the scanning lens can cause misleading results of analysis of the surface roughness dependence on hydrodynamic lubricating layer.



3 COMPUTATIONAL APPROACHES FOR CONTACT PRESSURE DETERMINATION

3.1 CONTACT MODEL BY GREENWOOD & TRIPP

As it was listed in chapter 1 of this thesis, the time effectivity of the calculation is very important and it is necessary to take it into consideration. Therefore, a time effective and widely used computational approach for contact pressure calculation is presented in this chapter. This approach was published in 1970 by authors Greenwood and Tripp [11] and is integrated among here implemented calculation models for contact pressure determination in the resulting product of this doctoral thesis – software tool F-CALC.

The basic equation for contact pressure determination has the following a form:

$$p_c(h) = \frac{8\pi}{5} (\eta_r \beta \sigma) K F_{5/2} \left(\frac{h}{\sigma} \right). \quad (14)$$

Where:

$$K = \frac{2\sqrt{2}}{3} (\eta_r \beta \sigma) E' \sqrt{\left(\frac{\sigma}{\beta} \right)}, \quad (15)$$

$$\sigma = \sqrt{Sq_1^2 + Sq_2^2}, \quad (16)$$

$$\frac{1}{E'} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}. \quad (17)$$

Where η_r is the surface density of roughness peaks [m^{-2}], β is the average radius of curvature of rough surface asperities [m], σ is the combined RMS of both surfaces [m], ν is the Poisson's constant [-], Sq is the RMS of 3D rough profile [m], E' is the combined Young's modulus [Pa], $E_{1(2)}$ is the Young's modulus for each material of the contact pair [Pa] and $F_{5/2}$ is the function of contact pressure increase [-].

Function $F_{5/2}$ is given by the table in the reference [11]. From the equation (14) it is possible to see that this variable depends on the (h/σ) ratio. Therefore, it is later necessary to determine values of this function for all (h/σ) ratios which occur during simulation.

One possible way to obtain values of $F_{5/2}$ is via approximation of this function by regression function. This, however, turned out to be a less suitable option. In respect to the nonlinearity of this function small inaccuracies may be caused by approximation, which could result in relatively high inaccuracies in the results of the contact pressure.

Another way to obtain values of $F_{5/2}$ function is to use a kind of interpolation spline. These functions are available in programing environment MATLAB as well as in programing environment FORTRAN. However, both cases are highly time consuming. For this reason these options are unsuitable.



Therefore, the best option, for this purpose, seems to be interpolating polynomial of $(n_{F5/2}-1)$ -degree. While $n_{F5/2}$ is the overall number of the functions of contact pressure increase. The table is given by nine values and therefore the eight-degree interpolating polynomial was applied. Coefficients of this polynomial are given by the function *polyfit* from the programming environment MATLAB. To keep the maximal possible accuracy these coefficients were exported in double precision format. Coefficients are listed in the following table.

Table 20: Coefficients of interpolating polynomial

Coefficient number 1	Coefficient number 2	Coefficient number 3
$2.2857142857010057 \cdot 10^{-5}$	$-4.2438095237826926 \cdot 10^{-4}$	$3.873777777559813 \cdot 10^{-3}$
Coefficient number 4	Coefficient number 5	Coefficient number 6
$-2.4581333333240883 \cdot 10^{-2}$	$1.1768111111088903 \cdot 10^{-1}$	$-4.0137633333302764 \cdot 10^{-1}$
Coefficient number 7	Coefficient number 8	Coefficient number 9
$8.8829725396802406 \cdot 10^{-1}$	$-1.1295729523808720 \cdot 10^0$	$6.2663999999999331 \cdot 10^{-1}$

Equation for calculation of the $F_{5/2}$ depending on value of (h/σ) ratio is then:

$$\begin{aligned}
 F_{5/2}(h/\sigma) = & Coef(1) \cdot (h/\sigma)^8 + Coef(2) \cdot (h/\sigma)^7 + Coef(3) \cdot (h/\sigma)^6 + \\
 & + Coef(4) \cdot (h/\sigma)^5 + Coef(5) \cdot (h/\sigma)^4 + Coef(6) \cdot (h/\sigma)^3 + \\
 & + Coef(7) \cdot (h/\sigma)^2 + Coef(8) \cdot (h/\sigma)^1 + Coef(9)
 \end{aligned} \quad (18)$$

Function $F_{5/2}(h/\sigma)$ is in the original information source [11] defined only for values of (h/σ) ratio in the range between 0 and 4. For values of (h/σ) ratio higher than 4 the situation is not difficult. Function $F_{5/2}(h/\sigma)$ here has zero values (surfaces are separated enough from each other and contact does not occur between them) and therefore also the contact pressure is zero. In the case when the EHD approach is not considered the (h/σ) ratio could decrease under the value of 0 (penetration of contact surfaces). Therefore also this situation has to be on the safe side considering the numerical solution. Function $F_{5/2}(h/\sigma)$ is then from the point $(h/\sigma) = 0$ tangentially extrapolated by a linear function. Equation describing this situation has the following shape:

$$F_{5/2}(h/\sigma) = -0.99888 \cdot (h/\sigma) + Coef(9). \quad (19)$$

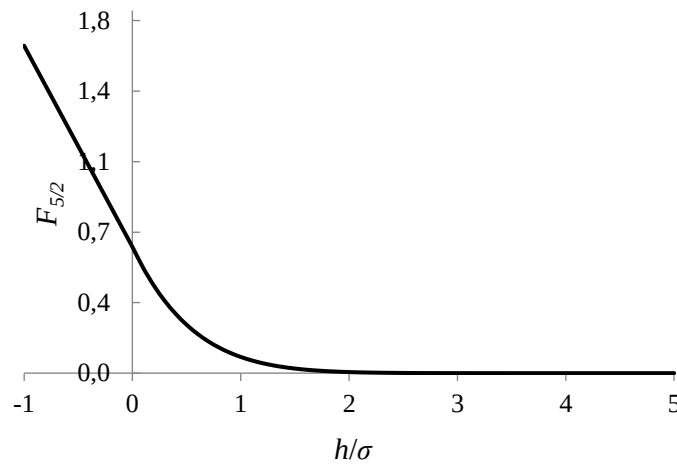


Fig. 34: Function of contact pressure increase depending on the surfaces separation

The final shape of the total pressure acting on the bearing shell (or bearing pin) is given by the sum of the hydrodynamic and contact pressure (see next equation).

$$P_{Total} = P_{Hydrodynamic} + P_{Contact} \quad (20)$$

This equation is discussed in the subsequent text because the separate solution of these regimes and their additional sum can cause inaccuracies in the solution of slide bearing behaviour. The fact that both lubricating regimes (pure hydrodynamic and mixed) are in close relation is aptly demonstrated by the Stribeck curve.

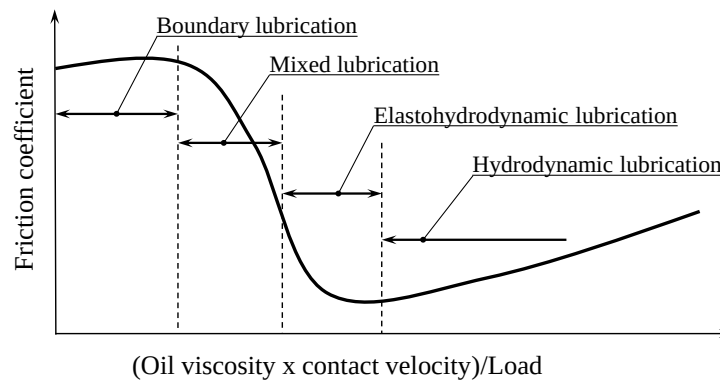


Fig. 35: Stribeck curve

Hydrodynamic lubricating layer is in the proximity of surfaces affected by the flow of lubricant between surface asperities. Via detailed solution of this flow of lubricant between asperities the connection of pure hydrodynamic and mixed lubrication regime is given. This connection is also solved for all other contact models.

However, before the description of other computational models for contact pressure calculation, one fundamental information should be mentioned about the Greenwood & Tripp [11] contact model. This reference lists relatively important information about the term $(\eta_r \beta \sigma)$ of equations (14) and (15). Variable K , given by equation (15) also appears in equation (14). The impact of the term $(\eta_r \beta \sigma)$ is then multiplied and its size has significant influence on the



resulting value of the contact pressure. In the information source [11], with reference to [5], is listed that the value of term $(\eta_r\beta\sigma)$ has to be in the range of 0.03 to 0.05. If variable η_r (surface density of asperities of rough surface) has unit $[\mu\text{m}^{-2}]$, variable β (average radius of asperity curvature) unit $[\mu\text{m}]$ and variable σ (combined RMS of both surfaces) unit $[\mu\text{m}]$, then the term $(\eta_r\beta\sigma)$ is dimensionless. By the analysis of two surface samples using the optical profilometer surface parameters were obtained, which are described in the previous text. It was the ZETOR piston liner and the experimental piston. According to information from reference [13] variable Ssc (mean arithmetic asperity radius of curvature) $[1/\mu\text{m}^2]$ was simply transferred to variable β $[\mu\text{m}^2]$. Measured variable Sds (surface density of asperities) $[1/\mu\text{m}^2]$ was directly equal to variable η_r $[1/\mu\text{m}^2]$. Values of Ssc , Sds , and Sq were averaged from ten measurements. Overview of these averaged values is in the following table together with the final value of the $(\eta_r\beta\sigma)$ parameter.

Table 21: Selected surface parameters of piston and piston liner

Piston			Piston liner		
Sds [1/μm ²]	Ssc [1/μm]	Sq [μm]	Sds [1/μm ²]	Ssc [1/μm]	Sq [μm]
0.002601	0.123699	0.742373	0.003486	0.294883	1.245464
η_r [1/μm ²]	β [μm]	Sq [μm]	η_r [1/μm ²]	β [μm]	Sq [μm]
0.002601	8.084168	0.742373	0.003486	3.391175	1.245464
Average values					
η_r [1/μm ²]		β [μm]		σ [μm]	
0.003043		5.737672		1.449930	
η_r [1/m ²]		β [m]		σ [m]	
3.043178·10 ⁹		5.737672·10 ⁻⁶		1.449930·10 ⁻⁶	
$(\eta_r\beta\sigma)$					
0.025317					

In this case the parameter $(\eta_r\beta\sigma)$ is very close to the presumptive range of 0.03 to 0.05. The control of the value of this parameter is highly recommended during contact pressure calculation. If higher deviation from this parameter occurs, one cannot expect relevant results from contact pressure calculation. In the case there is a need to determine variables η_r , β , σ from files with surface data (or from numerically generated surfaces), a specialised software tool is necessary to be developed. The determination of these variables is described in the following chapter.



3.1.1 STATISTICAL DESCRIPTIVE DATA OF ROUGH SURFACES FOR CONTACT MODEL BY GREENWOOD & TRIPP

The determination of **density of surface asperities** η_r (or summit density), required only into one contact model – Greenwood & Tripp – is described in the norm ISO 25178-2, however in this source marked as variable Sds . This norm refers to source [46] and considering the comparable results, further information was searched for in the operation guide for the optical profilometer ContourGT-X from the BRUKER company, which was used for 3D surface profile scanning.

$$Sds = \frac{\text{number of summits}}{\text{analysed area}} \quad (21)$$

The formula itself is quite simple. However, small complications occur during closer determination of what exactly the phrase summit means. The summit is considered to be a given point of surface roughness mesh grid, which is higher than its all eight neighbouring points. Summits are then local maximums in the rough surface mesh grid. Situation is illustrated in the following figure.

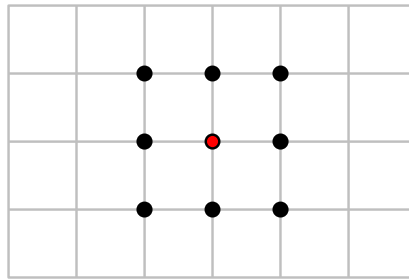


Fig. 36: Surface summit and its eight neighbours

The next condition to declare the surface roughness peak as a summit is that the summit has to lie above the threshold which is 5% of the Sz value. As it is described in reference [46], the variable Sz is determined as a mean value from the five highest and five lowest points, chosen according to the previous rule. For this purpose, this rule was applied separately on peaks (point has to lie higher than its neighbouring eight points – κ_{vi}) and valleys (point has to lie lower than his neighbouring eight points – κ_{ui}). This way, the maximal possible span between analysed values was achieved.

$$Sz = \frac{\sum_{i=1}^5 |\kappa_{ui}| + \sum_{i=1}^5 |\kappa_{vi}|}{5} \quad (22)$$

The last applied rule, which has to be fulfilled, to determine a peak as a summit is that peaks, chosen by previous rules, cannot be separated by distance shorter than 1% from the side size of analysed surface sample. By this rule, again, the number of summits is significantly reduced. To execute the last rule, another function was made, which again analyses the matrix of summits gained by previous two rules. The situation described in this paragraph is supported by the following figure.

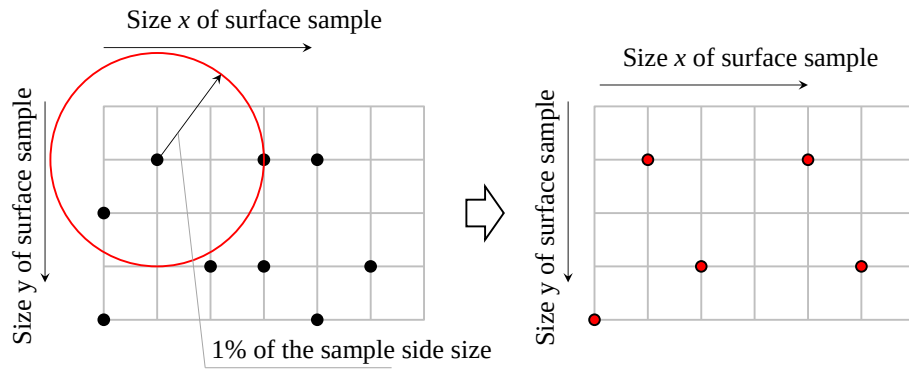


Fig. 37: The final peaks filtration

The second parameter used, is the **average radius of asperity curvature** β . To obtain values of this parameter all the references proposed in the introductory paragraph of this chapter were used again. Parameter β is derived from the surface parameter S_{sc} , while equation which gives their relationship has the following shape:

$$\beta = \frac{1}{S_{sc}}. \quad (23)$$

$$S_{sc} = -\frac{1}{2} \cdot \frac{1}{n_v} \sum_1^{n_v} \left(\frac{\partial^2 z_r(x, y)}{\partial x^2} + \frac{\partial^2 z_r(x, y)}{\partial y^2} \right) \quad (24)$$

Numerical derivations are, for this case, calculated for all key points of the Finite Difference Method (FDM) mesh of surface roughness. Only values of numerical derivations are incorporated into equation (24), in which the FDM surface has mesh summits (given by the three rules mentioned above).

The method of numerical derivations calculation is changed according to the position of the key point in the data matrix (for determination of the second derivative of the node, which lies on the position [1, 1], two previous nodes do not exist). Suitably, formulae for forward, central, and backward derivation are then used.

$$\dot{f}(x_i) = \frac{f(x_{i+1}) - f(x_i)}{\Delta x}, \quad \ddot{f}(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) - f(x_i)}{\Delta x^2}, \dots \quad (25)$$

Where $\dot{f}(x_i)$ is the first numerical derivative of analysed function of rough profile in node x_i , $\ddot{f}(x_i)$ is the second derivative of analysed function in node x_i .

The last one from the three determined surface parameters is the **combined RMS of both surfaces** σ . The equation for its calculation is mentioned above – equation (16).

$$\sigma = \sqrt{Sq_1^2 + Sq_2^2} \quad (26)$$



$$Sq_{1(2)} = \sqrt{\frac{1}{MN} \sum_{j=1}^N \sum_{i=1}^M z_{r1(2)}^2(x_i, y_j)} \quad (27)$$

This way obtained parameters of rough surface, together with material properties and loading conditions, create inputs for complete computational model for contact pressure calculation according to authors Greenwood & Tripp in [11]. The presumption of asperity heights normal distribution may be regarded as a main disadvantage of this approach. This presumption is logical with regard to the fact, that the rough surface is completely described just via three parameters η_r , β and σ . Therefore, a description of next, more detailed, computational approaches for contact pressure calculation is listed in the following paragraphs. All described contact models are implemented into the software tool, which is the main product of this doctoral thesis.

3.2 CONTACT MODEL BY HERTZ

A quite evident reason led to the implementation of this computational model – it is one of the most widely known and most widely used contact models, which should not be omitted from the list of other computational models of contact pressure.

In the case of analysis between isotropic surfaces, it is possible to use only two-dimensional model of surface profile. In the case of anisotropic surface profile, it is essential to use three-dimensional surface profile model to capture also directional dependence of surface pattern. Description of such contact analysis can be seen in reference [52] – for anisotropic surface, or in [44] – for isotropic surface roughness pattern. In both cases Hertz theory was applied on the contact between three-dimensional fractal rough surfaces, generated by using Weierstrass-Mandelbrot function. In the case of analysis of 3D surfaces, it is necessary to accept some simplifying assumptions. Contact bodies are considered isotropic, elastic, perfectly smooth and the contact area is considered small and flat in comparison with the radius of curvature of undeformed contact bodies (in our case bearing shell and pin).

Reference [45] gives a comprehensive overview of several applications of Hertz theory on the contact between bodies of different shapes. In our case, a good idea seems to be to replace the real shape of surface asperities with the ideally spherical shape and to solve the Hertz contact as a contact of two spherical bodies. Individual formulae for Hertz contact pressure calculation follow, together with explanation of all variables, and description of all factual challenges from the produced software tool.

$$W = \sqrt{\frac{\delta_R^3 E' R'}{1.0397^3}} \quad (28)$$

Here the variable W represents a contact force [N], δ_R is the deflection (penetration) of surfaces [m], E' is the combined Young's modulus [Pa], and R' is the combined radius of curvature [m]. The radius of contact area a [m] is given by the formula:

$$a = \left(\frac{3WR'}{E'} \right)^{1/3} \quad (29)$$



$$P_{\max} = \frac{3W}{2\pi a^2}, P_{av} = \frac{W}{\pi a^2}. \quad (30)$$

Formulae for the maximal ($P_{\max}$) and the average (P_{av}) contact pressure [Pa] are given by the equation above.

Inputs into this computational model are then: the deflection (penetration) of contact surfaces δ_R [m], material characteristics combined Young's modulus E' [Pa], and combined radius of curvature R' [m]. Here we come across another area which needs more detailed explanation – how to get variables δ_R , E' and R' during calculation.

Determination of variable δ_R is derived from the determination of so called micro-lubrication height.

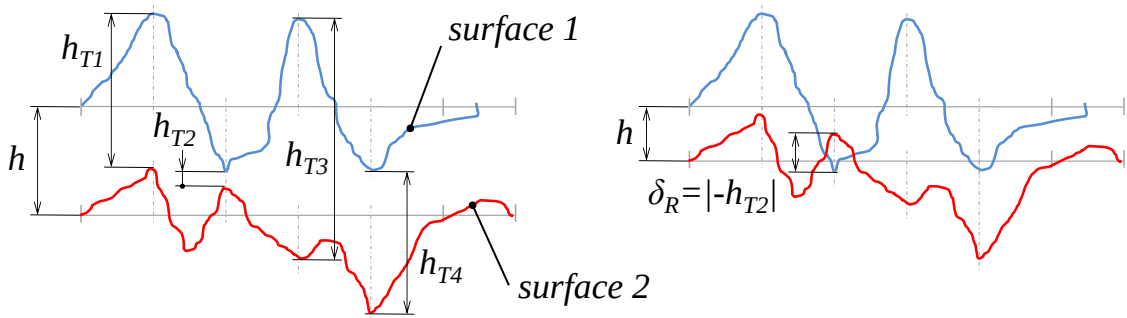


Fig. 38: Determination of the δ_R variable of rough surfaces

In Fig. 38, on the left side, three cases are shown which can occur during micro-lubrication height determination. In the first case (height h_{T1}) both surface peaks are above the zero plane. In this place lubrication layer has a value of:

$$h_{T1} = z_{r1} + h - z_{r2}. \quad (31)$$

Here $z_{1(2)}$ are heights of peaks of surface 1(2). For another three cases the formulae have following shapes:

$$h_{T2} = h - |z_{r1}| - z_{r2}, \quad (32)$$

$$h_{T3} = z_{r1} + h + |z_{r2}|, \quad (33)$$

$$h_{T4} = h - |z_{r1}| + |z_{r2}|. \quad (34)$$

Fig. 38, on the right side, shows the situation when both surfaces are so close to each other that deflection occurs (lubrication height has negative values). In this case, the absolute value of this negative lubrication height is saved in the variable δ_R .

The calculation of combined Young's modulus is described above by equation (7). Therefore, what remains is to specify the calculation of combined radius of asperity curvatures R' . This reduced radius is the combined radius of both contact bodies in both directions x and y .



$$\frac{1}{R'} = \frac{1}{R_x} + \frac{1}{R_y} = \frac{1}{R_{1x}} + \frac{1}{R_{1y}} + \frac{1}{R_{2x}} + \frac{1}{R_{2y}} \quad (35)$$

Here R_x and R_y are reduced radii of contact bodies in directions x and y . R_{1x} , R_{1y} , R_{2x} and R_{2y} are radii of curvature of contact body 1(2) in directions x and y .

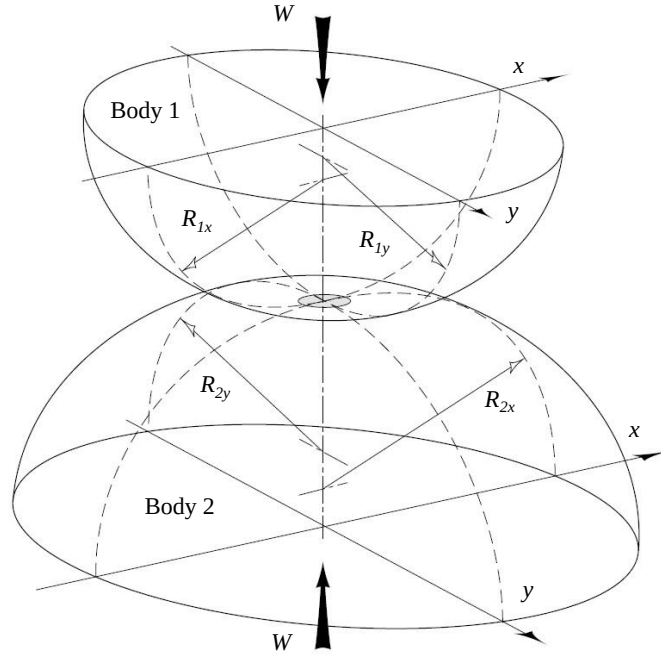


Fig. 39: Contact bodies terminology [45]

The description of each radius of curvature calculation is presented in the following part. For all contact models, except the model by Greenwood & Tripp, an identical radii calculation method is used. The calculation is based on the determination of the radius of curvature of the osculation curve. This radius is calculated for every node of surface roughness profile for the direction x and y . The radius of curvature of the contact body 1 in direction x is calculated by the following equation:

$$R_{1x} = \frac{\left(1 + (z'_r(x))^2\right)^{3/2}}{z''_r(x)}. \quad (36)$$

The contact pressure in the case of this computational approach is calculated similarly as in the case of contact model by Greenwood & Tripp, determined only for selected nodes of the FDM mesh. By the fine tuning of the computational model and by employing information from reference [44] the following conditions were determined, which have to be fulfilled to label a node of the FDM as a summit:

- it has to be higher than its both neighbouring nodes in the data array,
- the turning point of the function has to be in the given node (the positive gradient is changed to the negative gradient),
- function has to be concave in this node, and



- the node has to be above the zero plane of the surface.

The first condition does not require any further comments. The second condition uses the result of first derivative of the function, where the sign of this derivative is monitored. In the case of the third condition, the sign of the second derivative of the surface roughness profile function in the given node is checked. And finally, the last condition – the node is above the zero plane of the surface when its value is higher than the mean value calculated from all nodes of the FDM mesh of the surface profile. When all these four conditions are met, the node is considered as a summit. From these summits the density of asperities and the average radius of asperity curvature are derived (both are just informative values – these values are not required as inputs into contact pressure calculation).

During data postprocessing from surface scanning process, a situation which has to be mentioned occurred. Sometimes is not possible to scan some areas of the surface roughness profile (particularly because of imperfect lighting of surface valleys, especially of surfaces with $Sa > ca. 0.8$). Then, in the data matrix there is an empty value, or *NaN* is written there. These values have to be replaced by some real values. The simplest way is to replace these values by a constant value – for example the mean asperity height. But if there are more identical values close to each other, the radius of curvature reaches extreme values in this area. This event can markedly distort the value of average radius of curvature of the surface roughness profile. A possible way how to fix this problem is to multiply the “problematic” values by, for example, random numbers with given mean value and standard deviation. But the random numbers generator from the MATLAB environment does not work without problems (especially in terms of time efficiency). For that reason three sigma rules on the matrix with radii of curvature was applied. If some peak has radius of curvature higher than triplication of the standard deviation, calculated for the matrix of radii of curvature, then this point is not counted among the points for average radius calculation.

Equation for standard deviation calculation can be written as:

$$\text{Stand.dev.} = \sqrt{\left(\frac{1}{N} \sum_{i=1}^N \text{radius}_i^2 \right) - \text{avg.radius}^2}. \quad (37)$$

3.3 CONTACT MODEL BY LAGEMANN

Volker Lagemann in his doctoral thesis [19] described slightly modified equations for the Hertzian contact pressure calculation. These equations were implemented into the developed software tool just for the completeness and to illustrate that also equations for contact pressure can have several modifications.

The maximal contact pressure is given by equation:

$$P_{\max} = \frac{1}{\pi} \sqrt{\frac{6E'^2 W}{R'^2}}. \quad (38)$$

Combined Young's modulus and combined radius of curvature of contact bodies are determined by using same equations as in the case of previously mentioned Hertzian pressure. The radius of contact area is different and it is given by following equation:



$$a = \left(\frac{3WR'}{4E'} \right)^{1/3} . \quad (39)$$

The normal contact force is also different and it is solved by using equation:

$$W = \sqrt{\frac{16\delta_R^3 E'^2 R'}{9}} . \quad (40)$$

Depending on the distance of the point from the contact point the average contact pressure is calculated as:

$$P_{av} = P_{\max} \sqrt{1 + \frac{\text{distance}^2}{a^2}} . \quad (41)$$

Results of contact analyses were during the software development process confronted with results from similar analyses, provided by our industrial partners. Values of contact pressure from one commercially available software tool were significantly lower than values of contact pressure given by Hertzian contact model. Gradually was learnt that this software tool also considers different states of material – not only elastic behaviour of contact bodies as it is in the case of Hertzian model. Therefore, was implemented one contact model taking into account all possible kinds of material behaviour, where the simplicity and time effectivity were preserved.

3.4 CONTACT MODEL BY PASARIBU & SHIPPER

This computational model, described in reference [31], takes into account all three states of material behaviour – fully elastic, elasto-plastic, and fully plastic. In comparison with all above mentioned contact models here is one additional input variable – the hardness of the softer material. The transition between individual behaviours of material is driven by the value of deflection (penetration) of contact bodies. Boundary values of deflection (ω_c [-]) are given by the following equations:

$$\omega_{c1} = 0.89 R' \left(\frac{H_m}{E'} \right), \quad (42)$$

$$\omega_{c2} = 54 \omega_{c1} . \quad (43)$$

If the first condition $\delta_R < \omega_{c1}$ is valid, the materials are in the area of elastic behaviour. In the case when $\omega_{c1} < \delta_R < \omega_{c2}$, materials behave elasto-plastically. And if $\omega_{c2} < \delta_R$, materials are in the plastic state.

In the case of fully elastic behaviour the calculation conforms to the following formulae:

$$A_{el} = \pi R' \delta_R , \quad (44)$$



$$F_{el} = \frac{4}{3} E' R'^{0.5} \delta_R^{1.5}. \quad (45)$$

If the second condition is valid (materials are in elasto-plastic state), next equations are used for calculation:

$$A_{ep} = \pi R' \delta_R \left[1 - 2 \left(\frac{\delta_R - \omega_{c1}}{\omega_{c2} - \omega_{c1}} \right)^3 + 3 \left(\frac{\delta_R - \omega_{c1}}{\omega_{c2} - \omega_{c1}} \right)^2 \right], \quad (46)$$

$$F_{ep} = \left[H_m - 0.6 H_m \frac{\ln \omega_{c2} - \ln \delta_R}{\ln \omega_{c2} - \ln \omega_{c1}} \right] A_{ep}. \quad (47)$$

And finally, equations for fully plastic behaviour have shape:

$$A_p = 2\pi R' \delta_R, \quad (48)$$

$$F_p = H_m A_p. \quad (49)$$

Where A_{el} , A_{ep} and A_p are contact areas [m^2] for each states of materials, H_m is the hardness of softer material [Pa], and F_{el} , F_{ep} and F_p are contact forces [N].

As in the case of all above mentioned contact models, the integral contact pressure is calculated as the overall sum of contact forces and applied on the whole area of analysed surface element (sample).

The comparison of individual computational models is presented in the chapter dealing with the validation of contact area/pressure.

Especially for detailed calculation of hydrodynamic pressure of slide bearing it is essential to accurately determine the contact area. If in some nodes of FDM surface roughness mesh is the contact pressure, this node has no space to load a hydrodynamic pressure profile. And therefore the area, loading the hydrodynamic pressure, should be reduced by the contact area. Initially one may say that for this purpose the contact area, whose radius is calculated in both variations of Hertzian contact pressure, can be used. But if the sum of all these micro-areas is performed, the overall contact pressure can reach higher value than the value of the whole analysed surface sample. Therefore, for the export of contact area ratio not exactly the contact area calculation is used but the nodes in contact are summed over the whole surface sample. For the evaluation of this presumption, for example, an experimental evaluation of contact area can be used, described in the following chapter.



4 EXPERIMENTAL EVALUATION OF CONTACT AREA

Measuring the contact area of two bodies in contact during dynamic experiment is a relatively complicated issue. Especially if it is such a complex task as the behaviour of slide bearing is.

Very sophisticated slide bearing stand is described by authors Valkonen, Juhanko and Kuosmanen in reference [47]. Authors here focus on the measuring of pressure in the hydrodynamic lubricating layer. But for every new type of tested bearing new segments are required (prospectively theirs fundamental modifications). This significantly affects the price of the whole device and decreases its versatility. However, a similar device could function as a proper stand for the verification of hydrodynamic pressure in slide bearing oil film layer.

If we focus on the prime need, to verify the size of contact area, methodology from references [38], [12], [24], and [36] seems to be more suitable. The principle of electrical contact resistivity measurement is used here. When the thickness of oil film layer is measured using this method, the resistivity is relatively high. If this principle is applied on the dry contact between the sample piston and piston liner, electrical current goes through a large number of parallelly connected resistors (represented by asperities). Therefore, in this case, the overall value of resistivity is very low. The value of contact resistivity is given by the number of contact spots.

From the MBS simulation of the contact between piston and piston liner using results from the contact pressure database, determined by simple analytical formulae, (executed via user-written FORTRAN subroutine), the contact area size is obtained. This simulation is done for all the above mentioned contact models. Similar detailed determination of contact area requires also detailed information about contact surfaces. These data is obtained by 3D scanning process. Resistivity of the whole measuring circuit is highly affected by other factors. These factors are: oxidation of both surfaces, resistivity of each part (piston and piston liner), resistance of conductors, or resistances of contacts between connectors and conductors, etc.

4.1 MBS MODEL

From the above described software tool a database of contact pressure depending on the analysed surfaces separation was created for this MBS simulation. Shapes of individual curves are shown in the following figure. Because of the very low values of contact pressure given by computational model by Greenwood & Tripp [11] this curve is plotted on the minor axis, in the graph presented. Significantly lower contact pressure values of this model are also stated in the doctoral thesis of Lagemann [19].

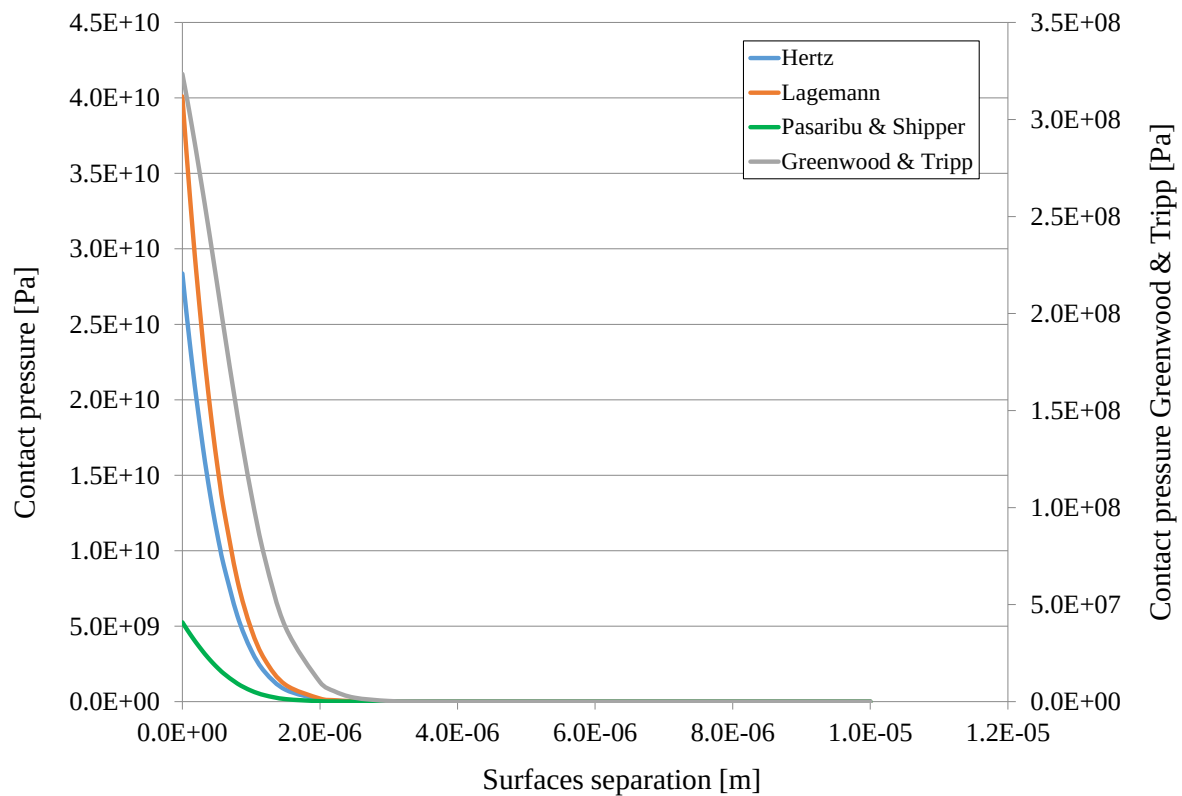


Fig. 40: Contact pressure depending on surfaces separation

Taking into account the presumed loads of contact pair, only elastic deformations of both parts are presumed. Therefore, for the MBS simulations only rigid bodies were used. For the MBS simulation a commercially available software tool – ADAMS – was used. Simple diagram of MBS simulation functionality is shown in the next figure. MBS simulation works as an actual experiment.

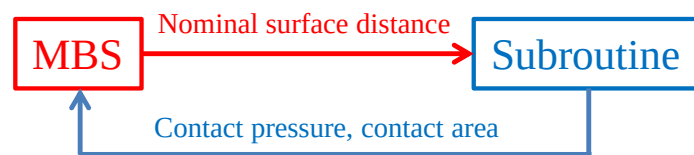


Fig. 41: Diagram of MBS simulation functionality

Each node of the computational mesh represents an area of one element. If one element is represented by one node, the nominal distance between two elements is a constant over the whole area of element. These presumptions, to keep sufficient accuracy of simulation results, require relatively dense computational mesh. For this purpose variable mesh density was applied, taking into consideration the presumed areas of contact. This approach allows to maintain high precision of calculation together with sufficient time efficiency of the developed algorithm. To suppress piston oscillation a damping force was applied. The final force acting in normal direction is given by the following formula:

$$F_{c_MBS} = P_c A_c - B_{MBS} v_{MBS} \cdot \quad (50)$$



Where F_{c_MBS} is the contact force [N], P_c is the contact pressure [Pa], A_c is the contact area [m²], B_{MBS} is damping [N·s·m⁻¹], and v is the relative velocity of surfaces [m·s⁻¹]. This model then simulates a static equilibrium between piston and piston liner without any relative movement between piston and piston liner in the direction of piston axis.

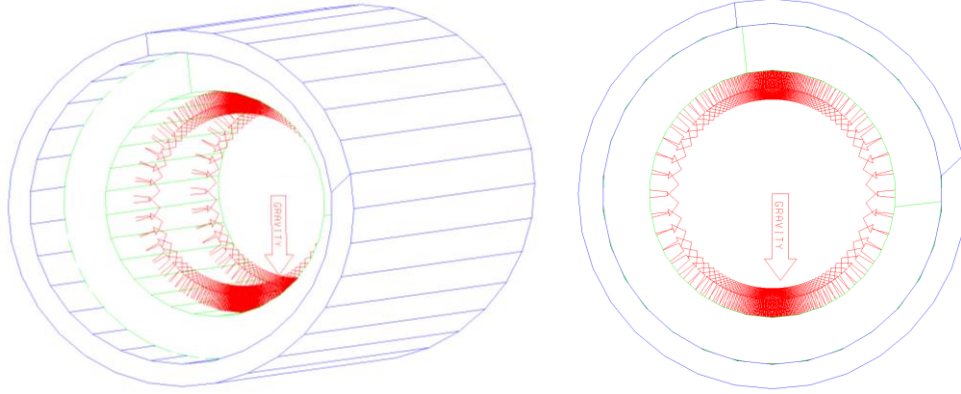


Fig. 42: Variable density of computational mesh

4.2 DESIGN OF MEASURING CIRCUIT

By using a simple equation, described in the reference [24], it is possible to preliminarily suggest values of voltage and electric current in the measuring circuit.

$$R_T = \frac{\zeta h_R}{A_R} \quad (51)$$

Where R_T is the contact resistance between piston and piston liner [Ω], A_R is the contact pressure [m²], h_R is the electric contact height [m], and ζ is the material resistance [Ω m].

In case when there is no additional acting force on the piston, except the gravity, the contact area can reach, for the case of contact model by Pasaribu & Shipper [31], value 9.81 mm². Resistance of steel is $5.5 \cdot 10^{-7}$ Ω m and as contact height in this case considered is the sum of S_q values of both surfaces (piston and piston liner), see Table 21. By substituting of all above mentioned values into equation (51) the given resistance is $1.11 \cdot 10^{-7}$ Ω .

The overall (simplified) resistance in this circuit is given by the sum of these contact resistances, the resistance of conductors, other contact resistances and the resistance of each body. This resistance is possible to determine by the use of equation (51). A contact height h_R is here substituted by the height of piston liner (piston) wall. Then the resistance of piston is $2.02 \cdot 10^{-5}$ Ω . The resistance of piston liner is $4.89 \cdot 10^{-5}$ Ω . The overall resistance is $6.92 \cdot 10^{-5}$ Ω if other resistances are neglected. If the voltage entering the measuring circuit is 12V, the electric current is $1.73 \cdot 10^5$ A.

$$I = U_R / (R_T + R_{piston} + R_{piston_liner}) \quad (52)$$

It is a very high value of electric current. For this reason, it is necessary to integrate an additional resistor of a known value (in our case 100 Ω), which causes the decrease in the



value of electric current going through the electric circuit. Electric current, reduced this way, then has a value of $1.2 \cdot 10^{-1}$ A (120 mA), which is an acceptable value (the electric current source is limited to a value of 1A). Furthermore, by using a formula:

$$U_c = R_T \cdot I. \quad (53)$$

The value of voltage in the piston/piston liner contact $U_c = 1.34 \cdot 10^{-8}$ V. To record this value (13.4 nV) a very precise measuring device (nanovoltmeter together with stable source of electric current) is required. The design of both main parts does not allow to separately measure only the transitional resistance. It is possible to measure only the resistance (or voltage) of the whole circuit (resistance of all bodies and contacts in the electrical circuit).

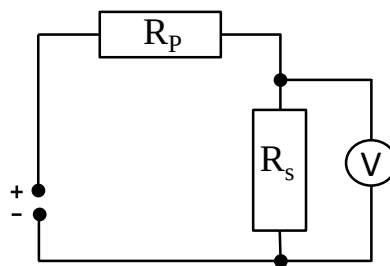


Fig. 43: Primarily designed electrical scheme:

R_p – added resistance

R_s – associated resistor (resistance of the contacts + resistance of conductors and bodies)

The intentionally chosen design of the experimental piston allows to insert a weight inside the piston body. Thus, it is possible to gradually change the acting force between piston and piston liner and to observe the contact area changes.

When the measuring circuit was finished, greater (than predicted) values of resistivity were measured. Therefore, it was possible to remove the additional resistor.



4.3 RESULTS OF ELECTRIC CONTACT RESISTANCE MEASUREMENT

Values of measured dependence of acting force between piston and piston liner are given in the following table. (Measurement 1 – Measurement 6).

Table 22: Measured values of electrical contact resistance

Weight [kg]	R_T [mΩ]						
	M1	M2	M3	M4	M5	M6	Avg
5.4400	7.1124	9.5167	6.6582	7.8624	8.7874	8.6104	8.0913
7.4400	6.5575	9.1613	5.9213	7.2133	8.3133	7.9753	7.5237
10.5260	4.4391	6.4177	5.8974	5.5848	6.3048	7.0488	5.9488
13.6160	3.8703	5.0967	4.6878	4.5516	5.2016	5.1416	4.7583
16.7080	3.3107	4.2789	3.6717	3.7537	4.4537	4.3717	3.9734
19.7950	2.7483	3.6219	3.3810	3.2504	3.5504	3.8704	3.4037
22.8820	2.1726	2.3192	2.0260	2.1726	2.4726	2.5866	2.2916
25.9660	1.7723	1.4934	2.0512	1.7723	1.9723	2.0123	1.8456

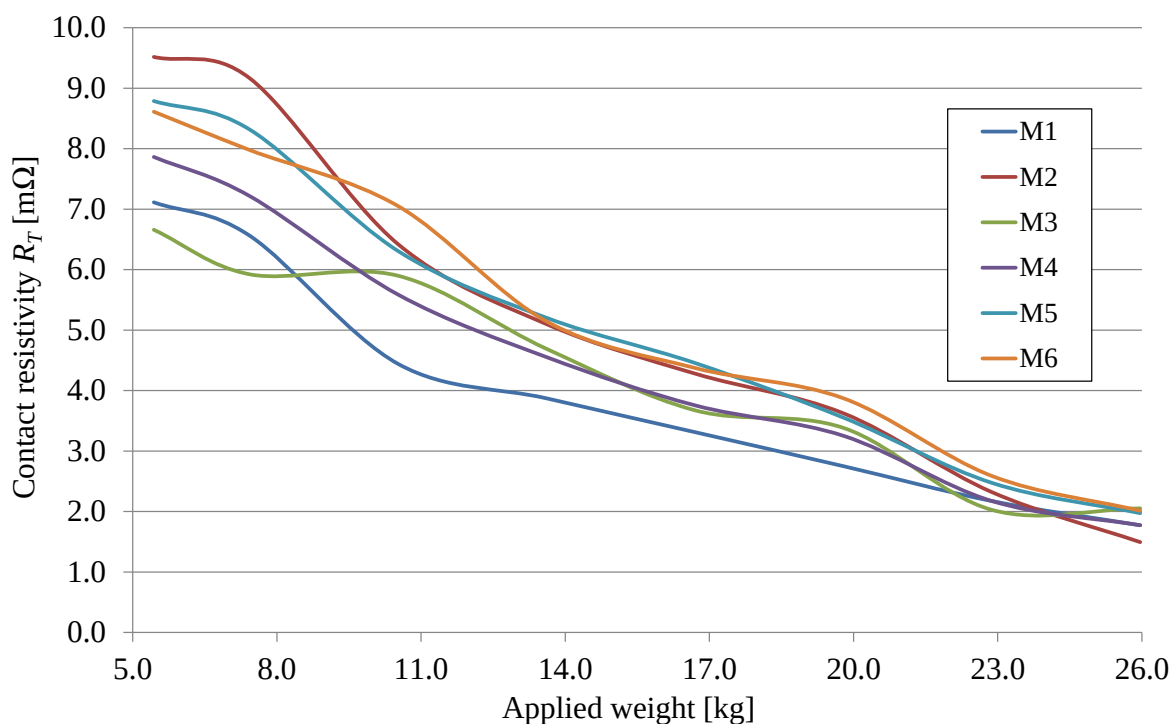


Fig. 44: Contact resistance on applied weight dependence

Measuring was done with the device Keythley 6221/2182A Delta mode system with AC and DC current source and nanovoltmeter.



Fig. 45: Current source and nanovoltmeter Keythley [2]

From the measured values, it is possible to observe great difference when compared to the predicted values. These differences are caused (as was partially mentioned above) by:

- different resistances of both materials,
- different resistances of conductors,
- different resistances of transitional resistors,
- different transitional resistance of contact (oxidation/contamination of surface layers),
- and others.

The contamination of both surfaces (by coolant, lubricant, etc.) could have been eliminated, for example, by ultrasonic cleaner. Exact determination (elimination) of other resistances is very complicated.

The measuring was done for different weights, and therefore, the obtained results show the difference in the contact area after gradual applying of the weights. These differences (possibly ratios to the first value) are comparable, despite the great differences in comparison with the results of the numerical simulation.

The following graph shows the results of the numerical simulation of contact between piston and piston liner. Applied weights were similar to the weights of the actual experiment.

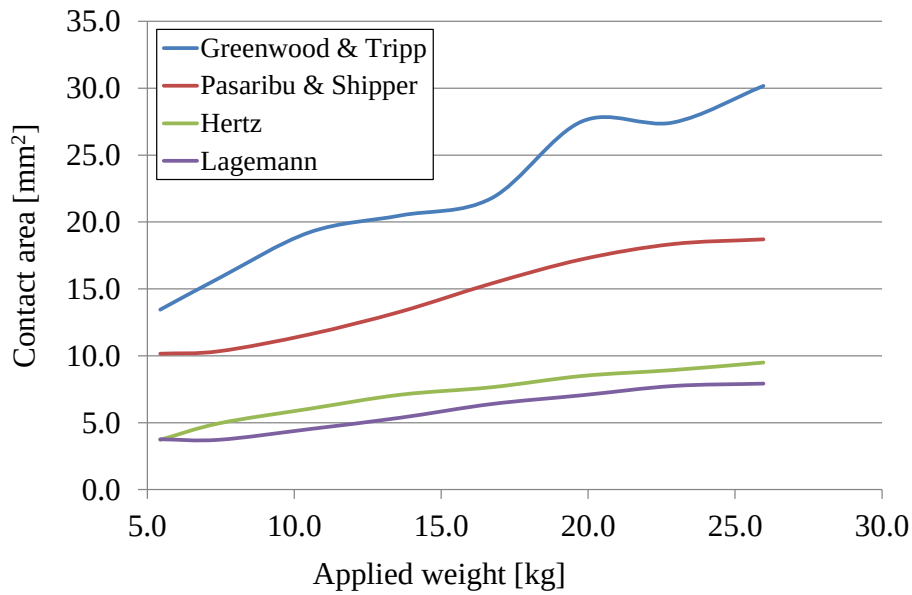


Fig. 46: Results of the piston/piston liner contact MBS simulation

From the shape of individual lines, it is possible to observe the expected differences between the individual computational models. The computational model by Greenwood & Tripp [11] gives the lowest contact pressure from all compared approaches. This causes the smallest push away of the piston, and therefore, logically the largest contact area. On the other hand, the computational approach presented by Lagemann [19], gives the greatest value of contact pressure from compared models, and therefore, the piston is pushed away the most, which causes the smallest contact area from all compared contact models.

For conversion of the contact area to electrical contact resistance the newly used formula is presented in reference [8]. Here authors derived more sophisticated formula for electrical contact resistance calculation in comparison with reference [24]. Inputs into this model consist of surface density of asperities on the contact area n_{rs} [-] and average radius of the micro-contact area r_s [m].

$$R_T = \frac{\zeta}{n_{rs} r_s} \quad (54)$$

To simplify the problem, here is incorporated a presumption that the average radius of micro-contact area equals to the average radius of asperity curvature β [m]. The average number of asperities on the contact area is given by formula [8]:

$$n_{rs} = \frac{A_R}{\pi r_s^2}. \quad (55)$$

After the substitution of formula (55) into formula (54), the following shape for the transitional resistance determination is then given:



$$R_T = \frac{\zeta}{r_s} \frac{1}{\frac{A_R}{\pi r_s^2}} = \frac{\zeta \pi \beta}{1} \frac{1}{A_R}. \quad (56)$$

The conversion of contact area to the transitional resistance can be done by the above mentioned formula. The following graph presents curves representing ratios of each transitional resistance to its first value. It allows the comparison of measured curves with the calculated curves. This is the reason why all curves start from the same value, the value of 1.

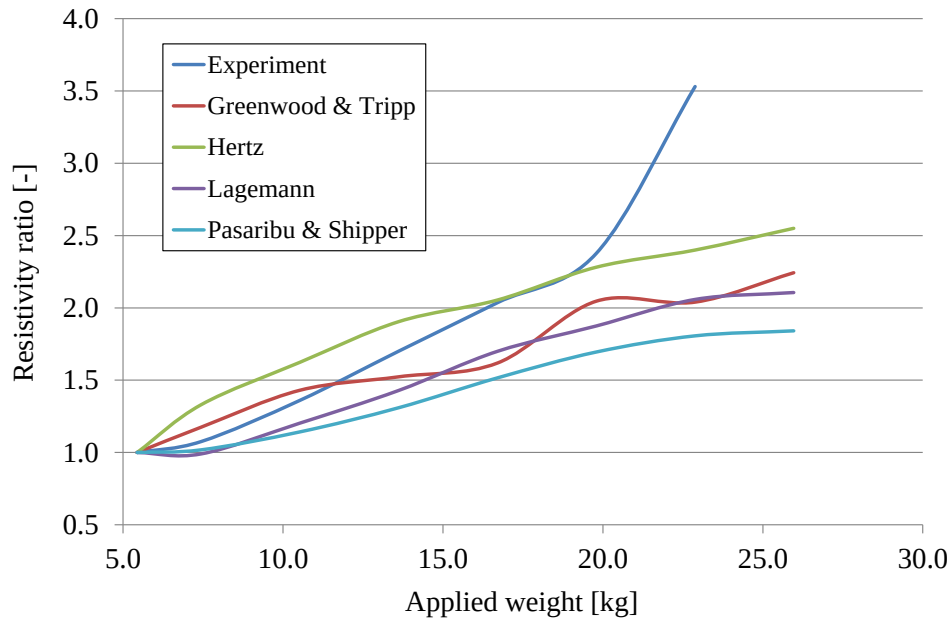


Fig. 47: Comparison of results from MBS simulation with results from experiment

By looking at the graph, it is possible to state that this way of contact area measuring is useable and the functionality of this principle was validated.

Simultaneously, it is necessary to think about that fact that this graph does not provide unequivocal information about which of the analytical approaches is the closest to results from measuring – this is a logical assumption. The maximum force (weight) applied on the piston was about 255 N (26 kg).

It is possible to state that results are in the area of low elastic deformations. All of the implemented contact models give very similar results in this area (three out of four are based on the Hertzian theory for the elastic behaviour).

To obtain results which would be better in revealing the differences between individual computational models, the use of simpler test-samples would be necessary (geometrically), and would have to apply forces causing deformations from the whole range (from elastic to the plastic deformations). For this purpose, for example, the hydraulic press seems to be suitable.

The complexity of the whole problem shows that the evaluation of contact area, using similar experiment that is outline here, would cover a topic for a single doctoral thesis. The result



of the functionality of this principle described here, together with the work done so far, can be used as a base for further work in this area.

The complete experimental device is equipped with highly sensitive sensors of distance, complex lubricating condition, and, for example, also with a crank mechanism to create straightforward reciprocating movement of the piston in the piston liner. This experimental device is going to be further developed independently from this thesis. It is going to be used for the experimental evaluation of computational model of lubrication height between piston and piston liner.

Taking into consideration the need to choose from analytical contact models, further methodology was suggested for this purpose. It is a possibility to use the commercially available, well-known software tool ANSYS.



5 EVALUATION (SELECTION) OF AN APPROPRIATE MODEL FOR CONTACT PRESSURE CALCULATION

As well as the experimental validation of contact area calculation, also the experimental validation of contact pressure calculation is very complicated. For that reason, the possibility to use the above mentioned software tool ANSYS was chosen.

The only input for the Finite Element Method (FEM) modelling were matrixes of values describing roughness in one axis (height) and values indicating the distance between individual points (distance). The number of these points varied depending on the type of task the model was subjected to. If the trends in different parameters setups were tested, the grid with fewer points (50x50) could be modelled sufficiently, but in the final calculations grids with a higher number of points were used (400x400) for greater accuracy.

5.1 SURFACE MESH

Firstly, it is necessary to create closed volume from which the mesh is later generated. Here is possible to apply several methods to create surface between keypoints. In the case of very dense mesh, which is common for optical scanning of rough surfaces, the surface elements are created directly from keypoints. In each step four keypoints are chosen (according to their position or number) and by one command the surface is created.

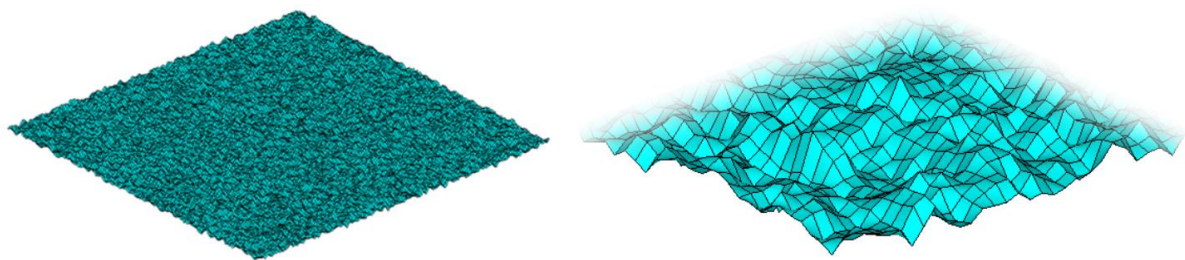


Fig. 48: Surface of FEM model (400x400), created from keypoints (detailed view on the right side)

It is necessary to have five sub-areas to close the volume. Those were created also by using keypoints.

5.2 VOLUME MESH

The element size is chosen with regard to two antithetical demands: sufficiently dense mesh on the surface and minimized number of elements for time efficiency of the calculation. This was solved in the case of the mesh in the MBS model by variable element size.

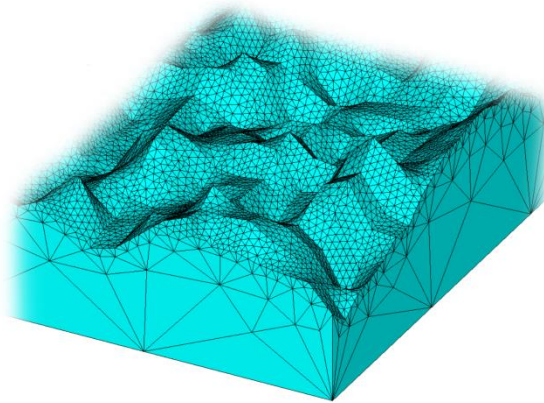


Fig. 49: Variable element size of the FEM model of rough surface

5.3 MATERIAL MODEL

Different materials (material properties) were adjusted for each body with surface roughness, but the same material model was used for both: multilinear isotropic hardening. Multilinear model was necessary for better capturing of the stress-strain curve, especially in areas with a larger deformation where the stress-strain curve is approximated by horizontal line. Isotropic model was chosen because of the better tendencies towards the convergence even if from the point of view of theoretical premises is for roles with higher values of nodes displacement kinematic hardening model more convenient.

5.4 BOUNDARY CONDITIONS

All six degrees of freedom were removed from the bottom of one body to prevent it from any displacement and rotation.

The side walls of the bodies can move only in z-axis direction and rotate as well. This setting should correspond to the fact that in reality these bodies are only partial elements of different components and their sides are in reality followed by another material.

Pushing of surfaces to each other was simulated by moving the upper side of one body towards the other, see the following figure.

For each computed surface about twenty loading states were solved with different distances from the zero planes. It enabled a sufficiently accurate evaluation of the behaviour of the rough surfaces with different degrees of deformation of individual peaks of roughness.

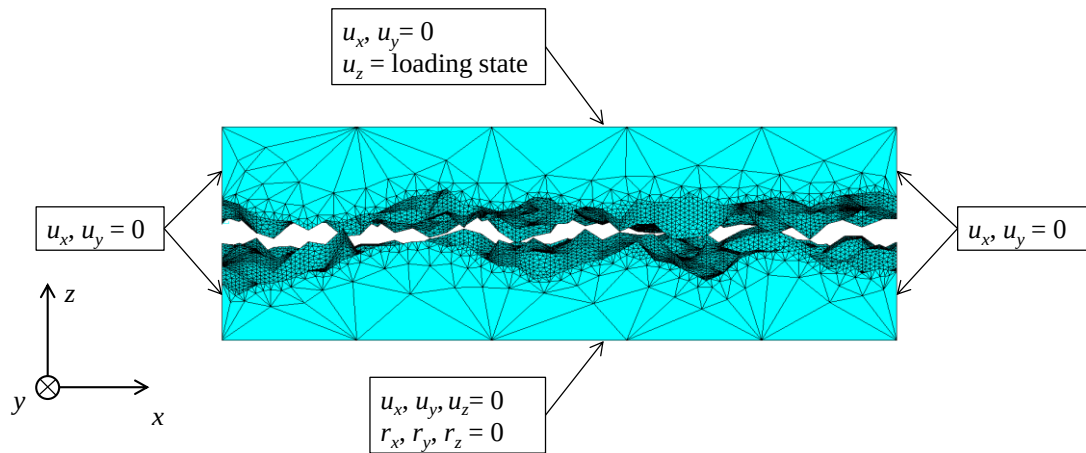


Fig. 50: Applied boundary conditions

5.5 VOLUME DEFORMATIONS CORRECTIONS

The loading state for comparison with the analytical computational method was defined as the distance between zero planes of rough surfaces. Zero plane is defined as a place where the mean average height of all surface roughness asperity is equal to zero.

This loading state cannot be adjusted precisely in FEM solver because the displacement conditions placed directly on the rough surface prevented any further calculation. Setting of the loading state by shifting the lower edge of the model is problematic because it brings the problem of the whole body model compression into the calculation. Therefore, corrections were necessary.

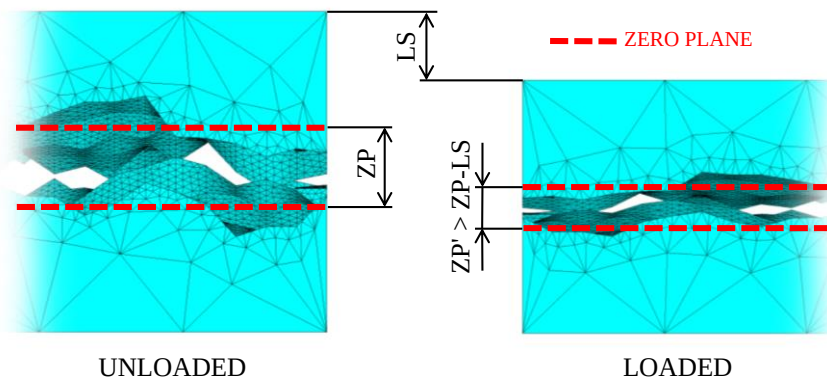


Fig. 51: Volume deformations corrections

Corrections were calculated from the deformation of all nodes of the rough surface mesh. Each correction was calculated as the difference between the zero plane distance (load state parameter) and resulting node-shift after load application. Results from the FEM computational model were then associated with the results from analytical model with the same distance of zero planes after application of load.



5.6 CONVERGENCE CHALLENGES

Calculation of contact problem of real rough surfaces proved to be very problematic in terms of convergence. The reason is that the individual gradients of roughness's raising are relatively steep and are in contact with the opposite surface only on a very small area. Load state is, therefore, composed of many "point load" states. This way the loading itself is usually referred to as divergent.

One of the most important variables influencing the convergence is contact stiffness. Greater values of normal contact stiffness reduce the penetration and thus the numerical solution approximates more precisely the real state of the loading. On the other hand, the higher the normal contact stiffness the more convergence difficulties appear. In case of convergence troubles it is necessary to decrease the contact stiffness which leads to greater penetration distance, i.e. fictional loading state situations, and therefore to inaccurate results. These inaccurate results appear especially in areas with maximum pressure values, i.e. in our case in nodes with point loading state and its immediate vicinity. But in terms of physical principles a balance of both bodies must be achieved. The sum of total contact pressures at all contact nodes is always the same, regardless the contact stiffness. That is why more extensive areas of roughness surface were compared with analytical model, not only results in single points. The presence of at least several local minimums and maximums was the condition for the comparison process.

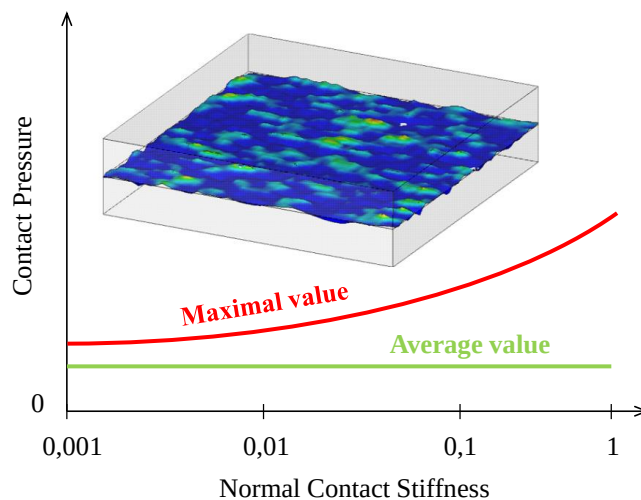


Fig. 52: Normal contact stiffness

5.7 RESULTS COMPARISON

5.7.1 ANALYSIS Number 1

The closest to the results from FEM analysis is in the first case model by authors Greenwood & Tripp. A pair of surfaces was analysed here, with surface parameter ($\eta, \beta\sigma$) very close to the originally presumed range from 0.03 to 0.05. Computational algorithms by Hertz and Lagemann were printed to the minor axis of the graph because of completely different (higher) results.

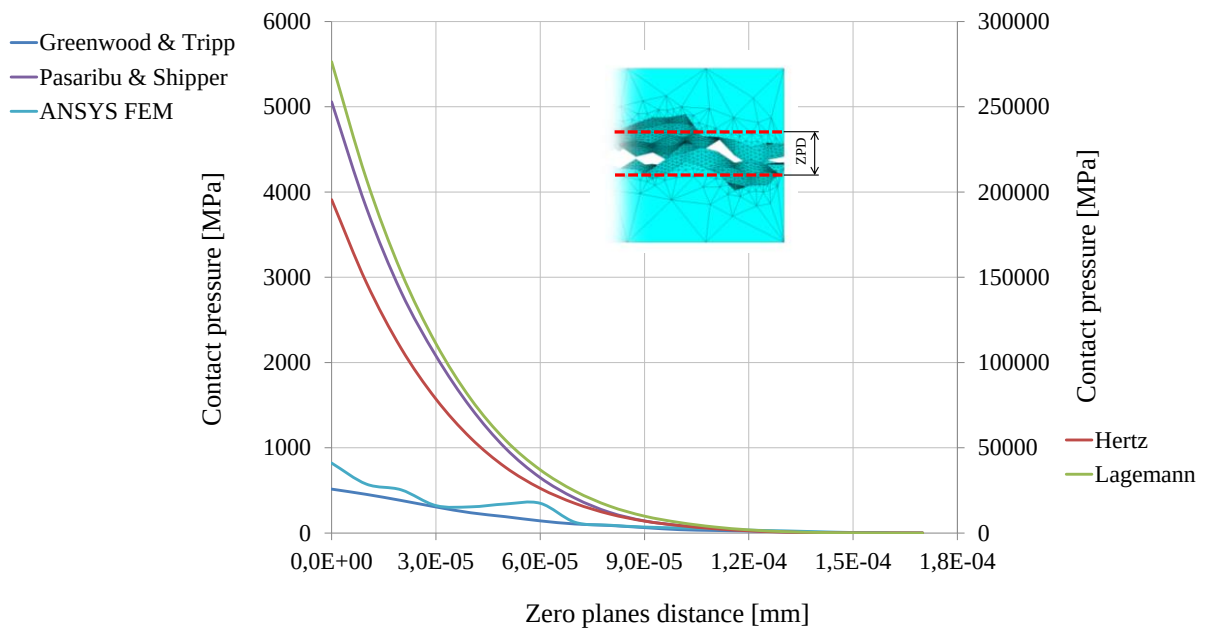


Fig. 53: Comparison of results from the FEM simulation and results given by analytical formulae for surfaces pair with parameter $(\eta, \beta\sigma)$ close to the required range

5.7.2 ANALYSIS NUMBER 2

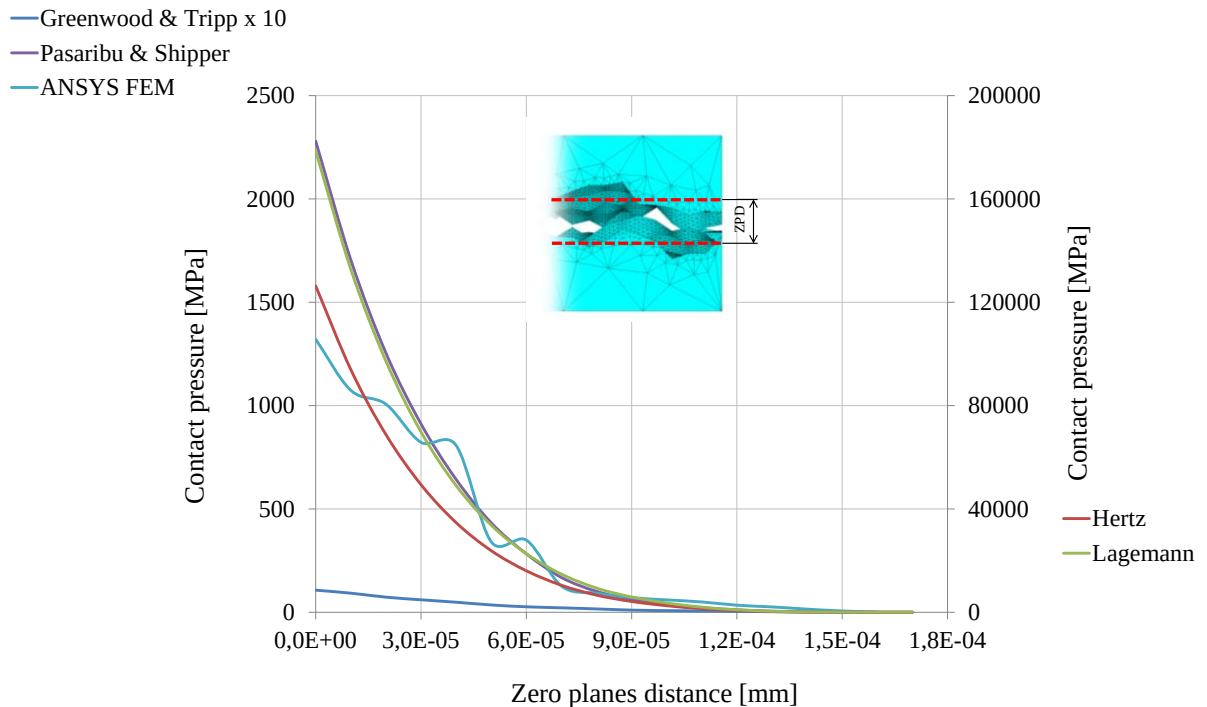


Fig. 54: Comparison of results from the FEM simulation and results given by analytical formulae for general surfaces pair



The closest to the results of FEM analysis is the model by authors Pasaribu & Shipper. This computational model considers all possible states of material behaviour, as well as the FEM model does in ANSYS. The hardness of the softer material is in this case an essential variable.

On the other hand, the computational model by Greenwood & Tripp gives the lowest values of contact pressures. For this reason, plotting the values had to be multiplied by 10. The reason is the deviation of the parameter ($\eta, \beta\sigma$) which should be, based on information source [5], in the range between 0.03 and 0.05. If the tested surfaces are not within this range, we can expect biased results.

For the results of computational models by Lagemann and Hertz the minor axis of the graph had to be selected again.

The purpose of this analysis using FEM was, by all means, fulfilled. From the selected computational models the best two which will be used for the fast solution of contact problems of real rough surfaces of machinery parts were determined. In case of both chosen computational models (Pasaribu & Shipper and Greenwood & Tripp) is necessary to mind correct input values and to check partial results after surface analyses.

Detailed solution using FEM in the MBS software environment is not applicable to online contact task solutions most importantly due to its high demands and time-inefficiency.



6 FLOW FACTORS CALCULATION

Flow factors depend (except other marginal conditions) on the separation of rough surfaces, measured from the mean planes of both surfaces, between which the lubricant flows. The calculation is based on the numerical solution of the Reynolds equation. Determined is the flow between two rough surfaces which are separated from each other by the space h_T , and the flow between two smooth surfaces separated from each other by the nominal oil film height h . Both flows are derived from the hydrodynamic pressure loss in both cases. The comparison of flow for the case of smooth and rough surfaces gives the value of flow factor for given value of surfaces separation. Rough surfaces are there just because of the micro-scale determination of the h_T matrix (the matrix with local changes of surfaces distance).

Flow factors are determined for a pair of rough surfaces (bearing shell and bearing pin). The directional dependence (AACF) is derived from the matrix with so called micro-lubricating layer height. On this matrix directional dependencies of both surfaces appear.

Solution of hydrodynamic power loss is described in the following, separate chapter.

Flow factors are calculated for several different, but statistically equivalent, rough surfaces, and the resulting value is the mean value from all these particular results. The number of these statistically same surfaces is given by number of scans (in case of scanned surfaces), or (in case of numerically generated surfaces) this value is set by the user.

6.1 HYDRODYNAMIC PRESSURE LOSS DUE TO OIL FLOW THROUGH MICRO-LUBRICATION HEIGHT

Except the boundary conditions, for calculation of flow between two rough surfaces their local distance is important. It is so called micro-lubrication distance (height). Its determination was described in detail in the chapter 3.2. Nodes where this micro-lubrication height has negative values are saved and used further for contact pressure calculation and for more precise solution of slide bearing dynamics. In these points almost no oil flows. These nodes do not help with carrying the hydrodynamic load.

As mentioned above, solution of this hydrodynamic pressure loss is based on the numerical solution of the Reynolds equation. The following pressure boundary conditions are applied:

- input pressure between surfaces (set by user),
- output pressure from surfaces (set by user),
- pressure along the sides of surface (distributed uniformly according to two previous values).

The next boundary condition (for calculation of pressure flow factor) is the boundary condition of the relative speed of the analysed surface pair. This is again set by the user. For shear flow factor the relative movement of surfaces is forbidden. The last boundary condition is the oil viscosity (set by user).

In the surrounding area of the surface element edges the hydrodynamic pressure is affected by the applied boundary conditions. Therefore, the oil flow is determined just from the selected part of hydrodynamic pressure profile. These boundaries are again set by user.

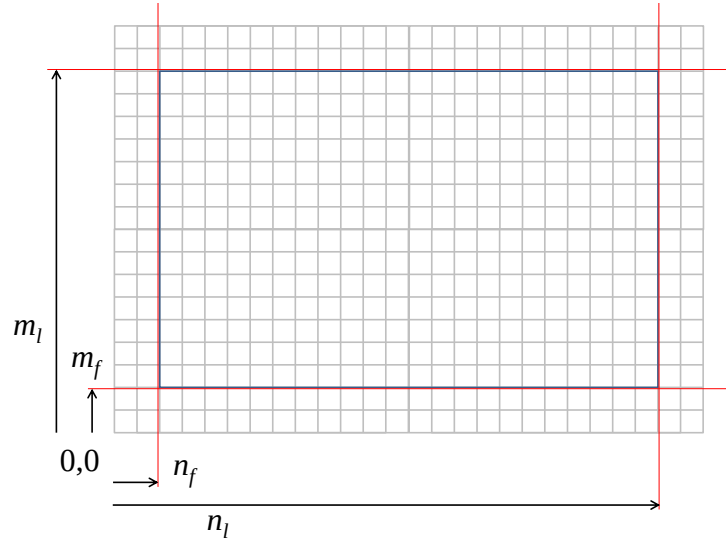


Fig. 55: Neglecting FDM mesh nodes affected by boundary conditions

In the previous picture n means the number of columns, m denotes the number of rows and index f means “first” and l means “last”. m_f is then the first row of used (selected) hydrodynamic pressure matrix and m_l is the last one. The same situation is in case of columns.

The Reynolds equation, together with dimensionless variables, has shape:

$$\frac{\partial}{\partial x} \left(h_T^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(h_T^3 \frac{\partial p}{\partial y} \right) = 6\eta U \frac{\partial h_T}{\partial x}, \quad (57)$$

$$H = \frac{h_T}{\sigma}, \quad X = \frac{x}{\sigma}, \quad Y = \frac{y}{\sigma}, \quad P = p\sigma, \quad \bar{\eta} = \frac{\eta}{\eta_0}. \quad (58)$$

Here: h_T denotes the local oil film thickness [m], x, y are the coordinates, p is the pressure [Pa], η is the oil dynamic viscosity [Pa·s], σ is the combined roughness (RMS) of both surfaces [m], U is the combined circumferential speed of surfaces [m·s⁻¹], and H, X, Y, P and $\bar{\eta}$ are the dimensionless variables.

After discretization the final equation for the description of hydrodynamic pressure can be in the following shape.

$$P_{(i,j)} = \frac{6\bar{\eta}U \frac{H_{(i+\frac{1}{2},j)} - H_{(i-\frac{1}{2},j)}}{\Delta X} - \frac{H_{(i-\frac{1}{2},j)}^3 P_{(i-1,j)} + H_{(i+\frac{1}{2},j)}^3 P_{(i+1,j)}}{(\Delta X)^2} - \frac{H_{(i,j+\frac{1}{2})}^3 P_{(i,j+1)} + H_{(i,j-\frac{1}{2})}^3 P_{(i,j-1)}}{(\Delta Y)^2}}{\frac{\left(-H_{(i+\frac{1}{2},j)}^3 - H_{(i-\frac{1}{2},j)}^3 \right)}{(\Delta X)^2} + \frac{\left(-H_{(i,j+\frac{1}{2})}^3 - H_{(i,j-\frac{1}{2})}^3 \right)}{(\Delta Y)^2}} \quad (59)$$



The Successive Over Relaxation (SOR) method is used for speeding up of the iterative solution of Reynolds equation [39]. Classic Gauss-Seidel solution is modified by parameter ω_{SOR} . This parameter is for the first iteration set to value 1.

$$\omega_{SOR}(1^{st} \text{ iter}) = 1 \quad (60)$$

For the second and all the other iterations are given by following equation:

$$\omega_{SOR}(2^{nd} \text{ iter}) = \frac{1}{1 - 0.5\rho_{Jacobi}^2}, \quad (61)$$

$$\omega_{SOR}(2^{rd} \text{ and next iter.}) = \frac{2}{1 - 0.25 \cdot \rho_{Jacobi}^2 \cdot \omega_{SOR}(prev.)}. \quad (62)$$

And ρ_{Jacobi} is given by equation:

$$\rho_{Jacobi} = \frac{\cos\left(\frac{2\pi}{m}\right) + \left(\frac{\Delta x}{\Delta y}\right)^2 \cdot \cos\left(\frac{2\pi}{n}\right)}{1 + \left(\frac{\Delta x}{\Delta y}\right)^2}. \quad (63)$$

Here m and n are number of rows and columns of the hydrodynamic pressure matrix [-] and Δx and Δy are differential lengths of the elements in the FDM mesh [-].

Then the equation for hydrodynamic pressure profile calculation has the following shape:

$$P_{SOR} = P_{prev.}(1 - \omega_{SOR}) + \omega_{SOR}P. \quad (64)$$

Here P_{SOR} is the hydrodynamic pressure calculated by using the SOR method, $P_{prev.}$ is the hydrodynamic pressure calculated in the previous step and pressure P is calculated according to equation (59).

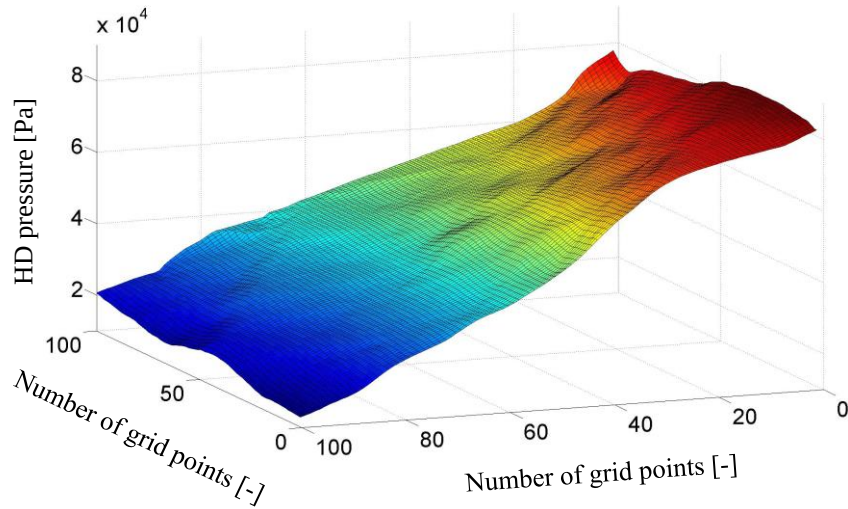


Fig. 56: Example of the hydrodynamic pressure profile for pressure flow factor ϕ_x calculation

6.2 OIL FLOW DETERMINATION

As mentioned above, to calculate flow factors it is necessary to determine oil flow between the pair of rough and pair of smooth surfaces. Equations for oil flow are taken from reference [22].

$$q_{rough_x} = -\frac{h_T^3}{12\eta} \frac{\partial p}{\partial x} + \frac{Uh_T}{2} \quad (65)$$

$$q_{rough_y} = -\frac{h_T^3}{12\eta} \frac{\partial p}{\partial y} + \frac{Uh_T}{2} \quad (66)$$

$$q_{smooth_x} = -\frac{h^3}{12\eta} \frac{\partial p}{\partial x} \quad (67)$$

$$q_{smooth_y} = -\frac{h^3}{12\eta} \frac{\partial p}{\partial y} \quad (68)$$

Here h_T determines the local oil film height (local distance between surfaces), [m], x , y are coordinates, p is pressure [Pa], η is the dynamic viscosity of oil film [Pa·s], U is combined circumferential speed of surfaces [m·s⁻¹], and h is the nominal oil film thickness [m].



6.3 FLOW FACTORS

Pressure flow factor in direction x [22]:

$$\phi_x = \frac{q_{rough_x}}{q_{smooth_x}}. \quad (69)$$

Pressure flow factor in direction y [22]:

$$\phi_y = \frac{q_{rough_y}}{q_{smooth_y}}. \quad (70)$$

Shear flow factor is determined just in the direction of relative movement of both surfaces. In case of bearing it is the circumferential direction. Therefore the shear flow factor is determined for the x direction [49]:

$$\phi_s = \frac{\frac{2}{L_x L_y} \int_0^{L_y} \int_0^{L_x} \left(-\frac{h_T^3}{12\eta} \frac{\partial p}{\partial x} \right) dx dy}{U \sigma}. \quad (71)$$

Here L_x and L_y are sizes of the analysed surface sample [m] and σ is the combined RMS of both surfaces [m].



7 SHEAR STRESS FACTORS CALCULATION

Shear stress factors are useful for more precise calculation of the mean hydrodynamic shear stress during the slide bearing dynamics calculation. As well as in the case of pressure flow factors these factors are also calculated for several different, but statistically equivalent, surfaces and subsequently averaged.

Standard formula for shear stress calculation [49]:

$$\tau = \frac{\eta U}{h} \pm \frac{h}{2} \frac{\partial p}{\partial x}, \quad (72)$$

is replaced by more complicated formula with shear stress factors [33]:

$$\bar{\tau} = \frac{\eta U}{h} \left[\left(\frac{h}{\bar{h}_T} \right) \pm \phi_{fs} \right] \pm \phi_{fp} \left(\frac{h}{2} \frac{\partial p}{\partial x} \right). \quad (73)$$

Here h is the nominal oil film thickness [m], $\bar{h}_T$ is the mean value of local surfaces distances [m] and ϕ_{fp} and ϕ_{fs} are shear stress factors [-]. τ and $\bar{\tau}$ are the original and modified shear stress in the lubricating layer [Pa].

The value of $\bar{h}_T$ variable is derived from the rough surfaces analysis.

Shear stress factors are calculated according to the following formulae [49]:

$$\phi_{fp} = \frac{\frac{1}{L_x L_y} \int_0^{L_y} \int_0^{L_x} \left(-\bar{h}_T \frac{\partial p}{\partial x} \right) dx dy}{-h \frac{\partial \bar{p}}{\partial x}}, \quad (74)$$

$$\phi_{fs} = \frac{\frac{1}{L_x L_y} \int_0^{L_y} \int_0^{L_x} \left(\frac{\bar{h}_T}{2} \frac{\partial p}{\partial x} \right) dx dy}{U \sigma}. \quad (75)$$

$\frac{\partial \bar{p}}{\partial x}$ determines the change of pressure between smooth surfaces whereas $\frac{\partial p}{\partial x}$ is the change of pressure between rough surfaces.

In case when values of $\bar{h}_T$ are not available from the surfaces analysis, it is possible to use an empirical formula from reference [33]:

$$\bar{h}_T = \frac{3\sigma}{256} \left\{ 35 + Z_{Patir} \left(128 + Z_{Patir} \left(140 + Z_{Patir}^2 \left(-70 + Z_{Patir}^2 \left(28 - 5Z_{Patir}^2 \right) \right) \right) \right) \right\}. \quad (76)$$



Where:

$$Z_{Patir} = \frac{\frac{h}{\sigma}}{3}. \quad (77)$$

Equation (76) is valid for the case when $h/\sigma < 3$. In the opposite case $\bar{h}_T = h$.



8 UTILIZATION OF ANALYTICAL FUNCTIONS – PATIR & CHENG

If advanced information about rough surfaces is not available and it is still required to include into calculation the impact of surfaces roughness structures, it is possible to use simple analytical equations, described in this chapter.

The computational approach describing the connection between the hydrodynamic lubrication regime and the mixed lubrication regime (in our case it is the impact of rough surface profile pattern on the hydrodynamic pressure profile in slide bearing), was published in 1978 by authors Nadir Patir and H. S. Cheng [32]. The above mentioned impact is also included here by the incorporation of flow factors into the Reynolds equation. The resulting shape of this equation is:

$$\frac{\partial}{\partial x} \left(\phi_x \frac{\rho h^3}{12\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\phi_y \frac{\rho h^3}{12\eta} \frac{\partial p}{\partial y} \right) = \frac{U_1 + U_2}{2} \frac{\partial(\rho h_T)}{\partial x} + \frac{U_1 - U_2}{2} \sigma \frac{\partial(\rho \phi_s)}{\partial x} + \frac{\partial(\rho h_T)}{\partial t}. \quad (78)$$

Here h denotes the nominal oil film thickness [m], h_T is the local oil film thickness [m], x and y are coordinates, t is time [s], p is pressure [Pa], ρ is oil density [$\text{kg}\cdot\text{m}^{-3}$], η is dynamic viscosity of oil [$\text{Pa}\cdot\text{s}$], σ is combined RMS of both surfaces [m], U_1 and U_2 are circumferential velocities of surface 1 and 2 [$\text{m}\cdot\text{s}^{-1}$] and ϕ_x, ϕ_y, ϕ_s are flow factors [-]. The situation is explained in the following figure.

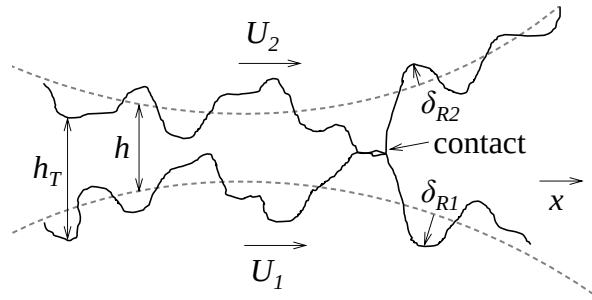


Fig. 57: Oil film thickness [32]

Local oil film thickness is then defined as:

$$h_T = h + \delta_{R1} + \delta_{R2}, \quad (79)$$

where δ_{R1} and δ_{R2} are amplitudes of rough surfaces measured from mean planes.

Equation (78) is valid for isotropic and also for anisotropic surfaces. Factors are in this case given by empirical formulae (for better time-effectivity) as a function of h/σ ratio and Peklenik number [33].



8.1 PRESSURE FLOW FACTORS

Equation for isotropic surface has the form:

$$\phi_{x(y)} = 1 - 0.9e^{-0.56(h/\sigma)}, \quad (80)$$

where e is the Euler's number.

Graph characterising this dependence is presented in the following picture.

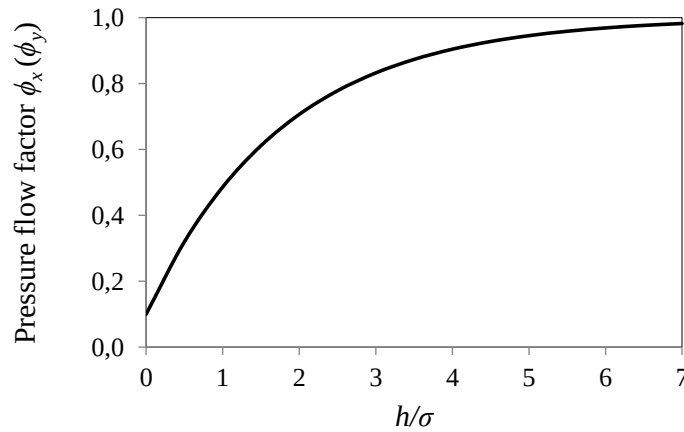


Fig. 58: Determination of pressure flow factors $\phi_x(\phi_y)$ for isotropic surfaces

If the analysed surfaces are not isotropic, equations describing flow factors are slightly different.

$$\phi_x = 1 - Ce^{-r'(h/\sigma)} \quad \text{for } \gamma_R \leq 1 \quad (81)$$

$$\phi_x = 1 + C(h/\sigma)^{-r'} \quad \text{for } \gamma_R > 1 \quad (82)$$

The following figure is the graphical interpretation of previous formulae, where:

$$\phi_y(h/\sigma, \gamma_R) = \phi_x(h/\sigma, 1/\gamma_R). \quad (83)$$

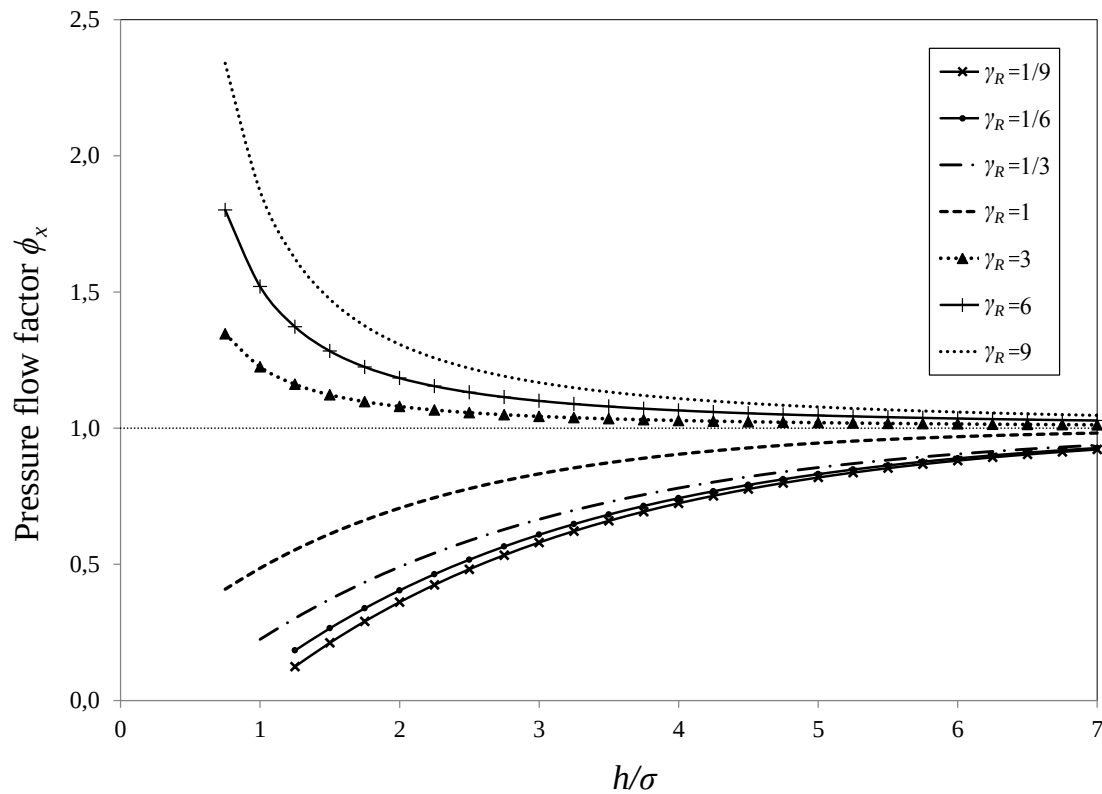


Fig. 59: Effect of directional dependencies of rough surfaces on pressure flow factors ϕ_x

Table 23: Parameters for pressure flow factors calculation [33]

γ_R	C	r'	Range
1/9	1.48	0.42	$h/\sigma > 1,00$
1/6	1.38	0.42	$h/\sigma > 1,00$
1/3	1.18	0.42	$h/\sigma > 0,75$
1	0.90	0.56	$h/\sigma > 0,50$
3	0.23	1.50	$h/\sigma > 0,50$
6	0.52	1.50	$h/\sigma > 0,50$
9	0.87	1.50	$h/\sigma > 0,50$

8.2 SHEAR FLOW FACTORS

Similarly as in the case of previous calculations, these factors are also possible to formularize via simple analytical formulae, taken from reference [33].

$$\phi_s = A_1 \cdot (h/\sigma)^{\alpha_1} e^{-\alpha_2 \cdot (h/\sigma) + \alpha_3 \cdot (h/\sigma)^2} \text{ for } h/\sigma \leq 5 \quad (84)$$

$$\phi_s = A_2 e^{-0,25 \cdot (h/\sigma)} \text{ for } h/\sigma > 5 \quad (85)$$



Values of individual variables are introduced in the following table.

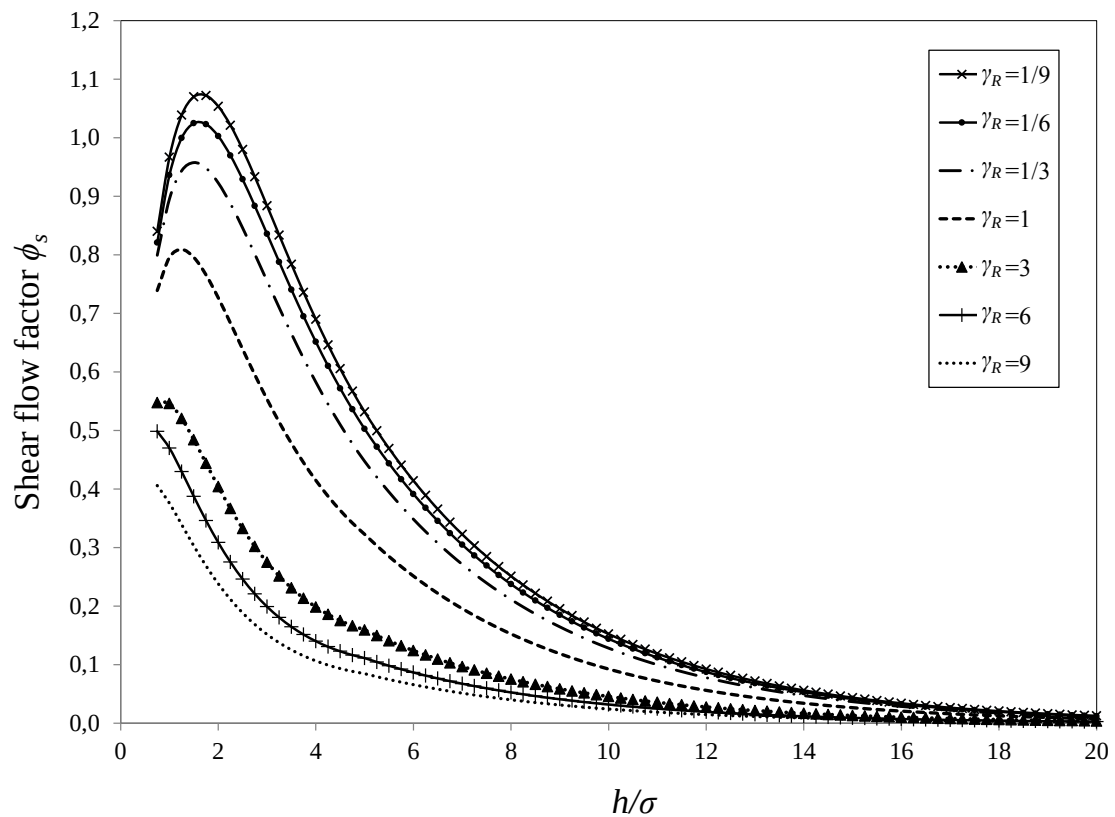


Fig. 60: Impact of surfaces directional dependencies on flow factors ϕ_s

Table 24: Parameters for analytical calculation of flow factors ϕ_s [33]

γ_R	A_1	α_1	α_2	α_3	A_2
1/9	2.046	1.12	0.78	0.03	1.856
1/6	1.962	1.08	0.77	0.03	1.754
1/3	1.858	1.01	0.76	0.03	1.561
1	1.899	0.98	0.92	0.05	1.126
3	1.560	0.85	1.13	0.08	0.556
6	1.290	0.62	1.09	0.08	0.388
9	1.011	0.54	1.07	0.08	0.295

8.3 SHEAR STRESS FACTORS

For more precise calculation of hydrodynamic shear stress (see equation (73)) shear stress factors are used. Analytical formulae for the determination of shear stress factors are from reference [33].

$$\phi_{fp} = 1 - D_R e^{-s_R (h/\sigma)} \quad (86)$$

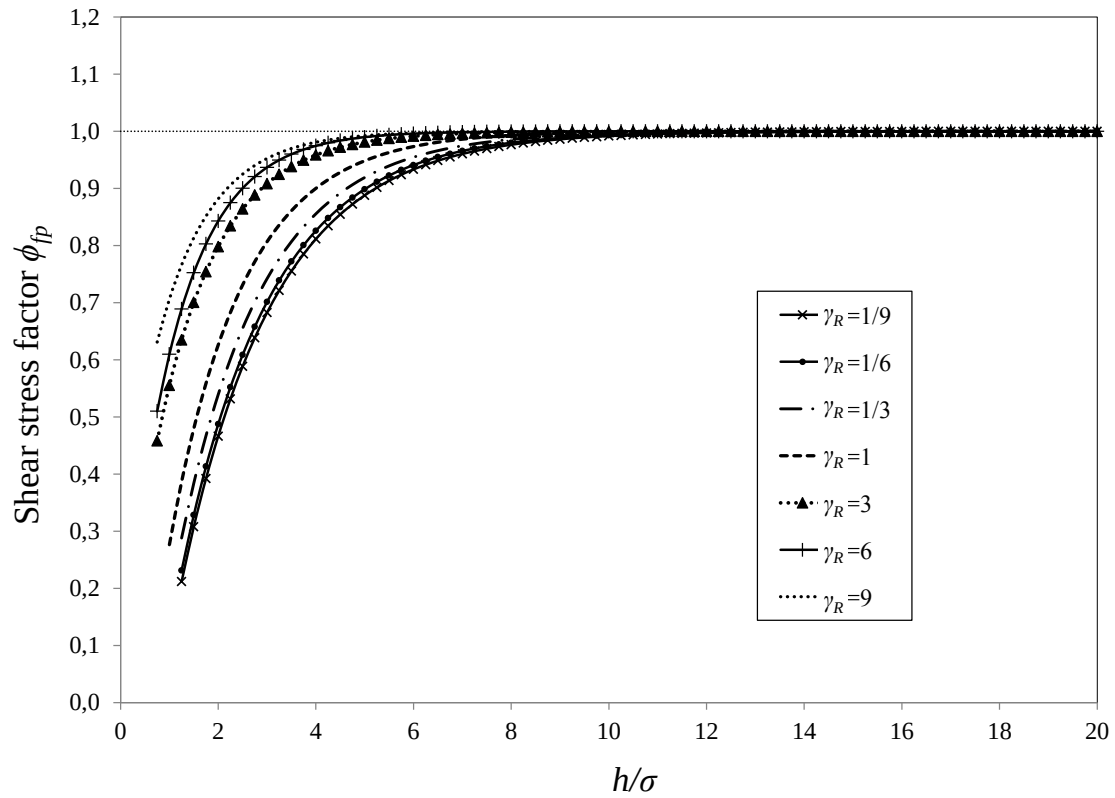


Fig. 61: Impact of surfaces directional dependencies on shear stress factors ϕ_{fp}

Table 25: Parameters for shear stress factors ϕ_{fp} [33]

γ_R	D_R	s_R	Range
1/9	1.51	0.52	$h/\sigma > 1,00$
1/6	1.51	0.54	$h/\sigma > 1,00$
1/3	1.47	0.58	$h/\sigma > 1,00$
1	1.40	0.66	$h/\sigma > 0,75$
3	0.98	0.79	$h/\sigma > 0,50$
6	0.97	0.91	$h/\sigma > 0,50$
9	0.73	0.91	$h/\sigma > 0,50$

$$\phi_{fs} = A_3 \cdot (h/\sigma)^{\alpha_4} e^{-\alpha_5 \cdot (h/\sigma) + \alpha_6 \cdot (h/\sigma)^2} \quad (87)$$

Individual input variables from previous equation are again given in following table. Equation is valid only for the range $0.5 < (h/\sigma) < 7$.

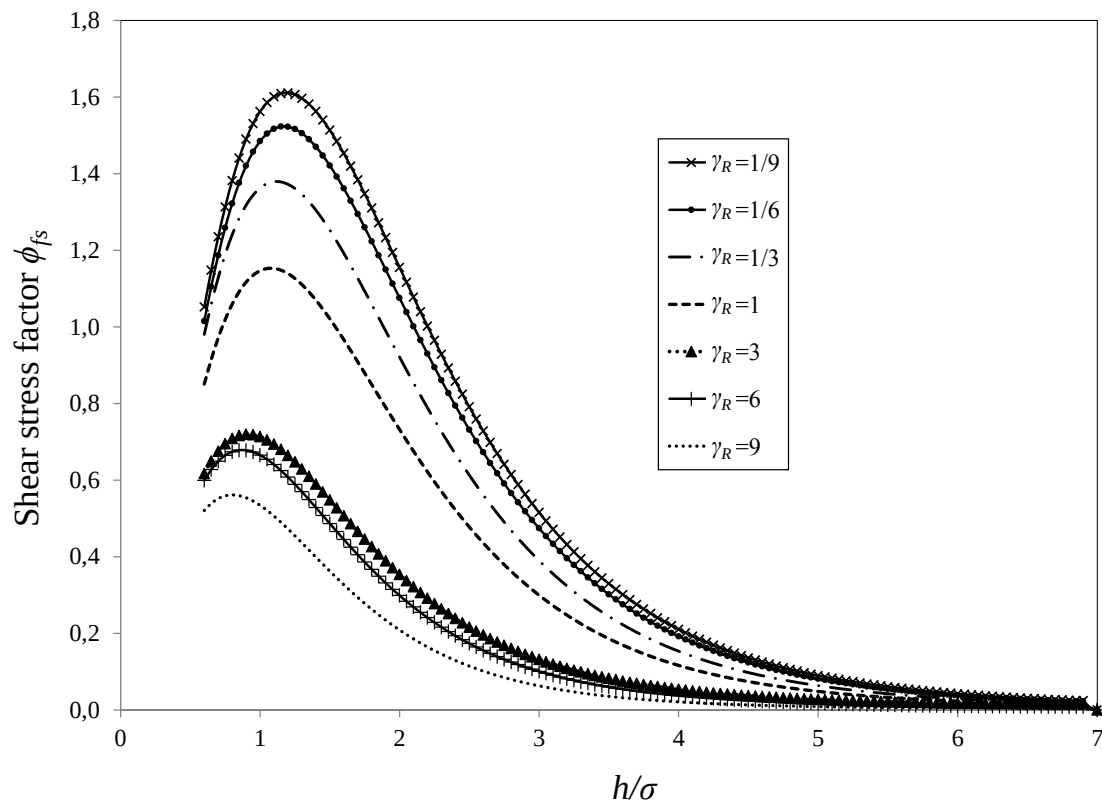


Fig. 62: Impact of surfaces directional dependencies on shear stress factors ϕ_{fs}

Table 26: Parameters for shear stress factors ϕ_{fs} [33]

γ_R	A_3	α_4	α_5	α_6
1/9	14.10	2.45	2.30	0.10
1/6	13.40	2.42	2.30	0.10
1/3	12.30	2.32	2.30	0.10
1	11.10	2.31	2.38	0.11
3	9.80	2.25	2.80	0.18
6	10.10	2.25	2.90	0.18
9	8.70	2.15	2.97	0.18

For better illustration what the value γ_R means, the following figure is included.

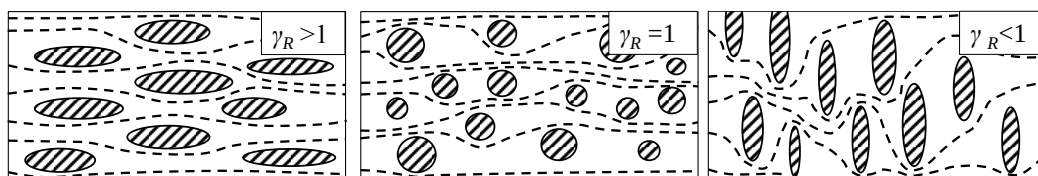


Fig. 63: Examples of typical contact areas of longitudinally oriented ($\gamma_R > 1$), isotropic ($\gamma_R = 1$) and transversely oriented ($\gamma_R < 1$) surfaces [32]



9 COMPARISON OF RESULTS FROM NUMERICAL SIMULATION AND FROM ANALYTICAL FORMULAE

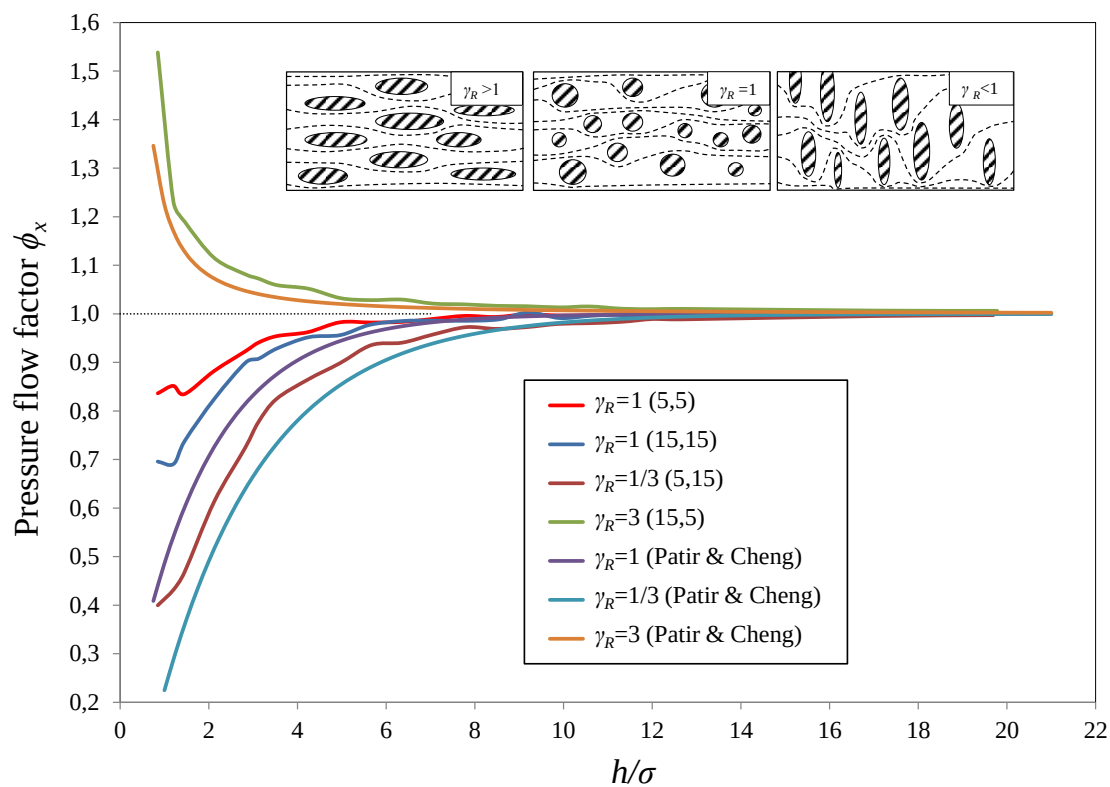
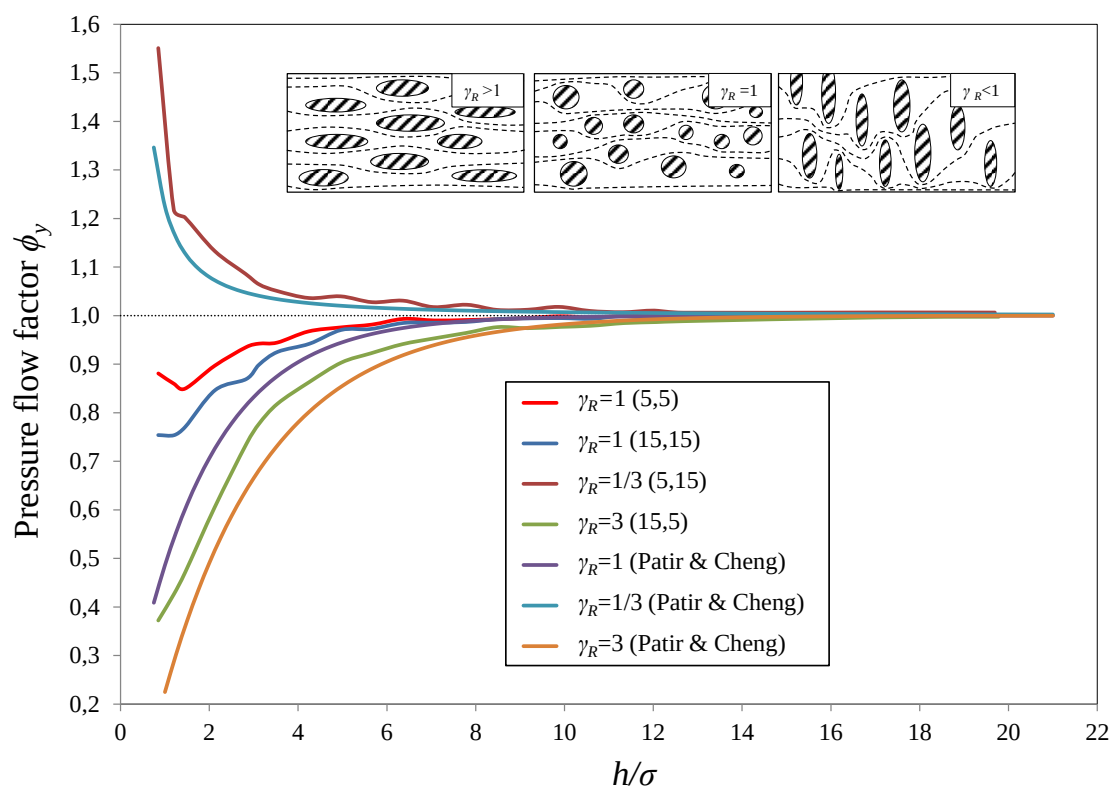
The comparison of the results from simple analytical equations [33] and from the numerical simulation, described in this doctoral thesis, can be used as a partial validation of the main results of this thesis.

A few graphs with comparison of results for differently oriented surface pattern follow. Displayed are results from analyses of isotropic and anisotropic contact surfaces. In the case of isotropic surfaces, it is proved that results depend not only on directional dependence itself but also on each value of autocorrelation length in each direction (frequency of surface asperities). In the case of simple analytical equations, it is not possible to include this information in the flow and shear stress factors determination. The precision of the numerically generated results is very high. It is possible to state that via different variations of density of surface asperities it is possible to reach an absolute concordance between the results from simple analytical formulae and from precise numerical solution. However, this is not the goal of this thesis. The aim of this precise numerical generation of all factors is the ability to calculate all these factors as accurately as possible and thus be able to predict the behaviour of hydrodynamic lubricating layer depending on the surface pattern of each acting part.

All the graphs presented below were generated, except for setting of the directional dependence of surfaces, with the following settings (boundary conditions):

- relative velocity of surfaces: $U=1.5$ m/s,
- oil viscosity: 0.0115 Ns/m²,
- input pressure between surfaces: $1 \cdot 10^5$ Pa,
- output pressure: 0.0 Pa,
- length of analysed surface sample side: $50 \cdot 10^{-6}$ m,
- sampling of surface sample: 299×299 ,
- RMS of both surfaces $0.5 \cdot 10^{-6}$ m,
- Young's modulus for material 1: $1 \cdot 10^{11}$ Pa,
- Young's modulus for material 2: $2.11 \cdot 10^{11}$ Pa,
- Poisson's constant for both materials: 0.3 ,
- error for iterative solution of hydrodynamic pressure: $1.0 \cdot 10^{-6}$,
- number of generated, statistically equivalent, surfaces: 9 ,
- hardness of softer material: $241.0 \cdot 10^6$ Pa,
- number of neglected rows and columns along both sides of hydrodynamic pressure matrix for oil flow calculation: 25 .

In captions for each graph are, in cases of directionally dependent surfaces, listed (in brackets) the autocorrelation lengths for direction x and y from which the final directional dependence of each surface is derived.

Fig. 64: Comparison of pressure flow factors ϕ_x resultsFig. 65: Comparison of pressure flow factors ϕ_y results

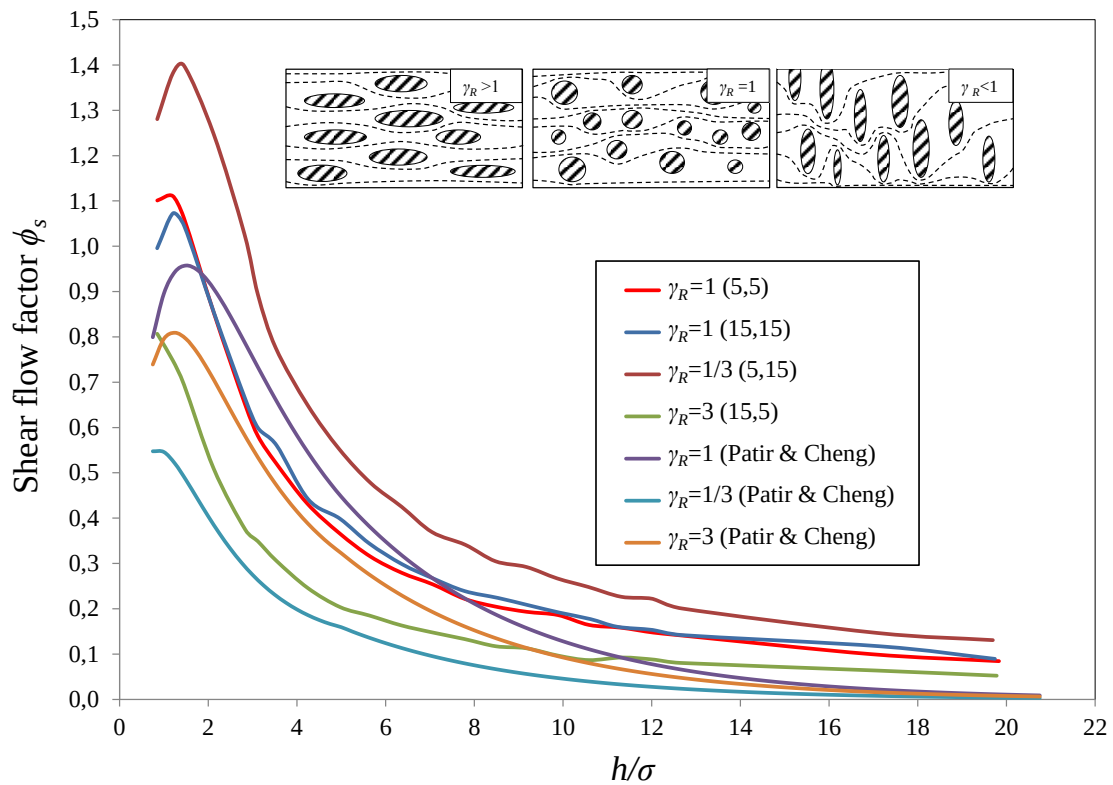


Fig. 66: Comparison of shear flow factors ϕ_s results

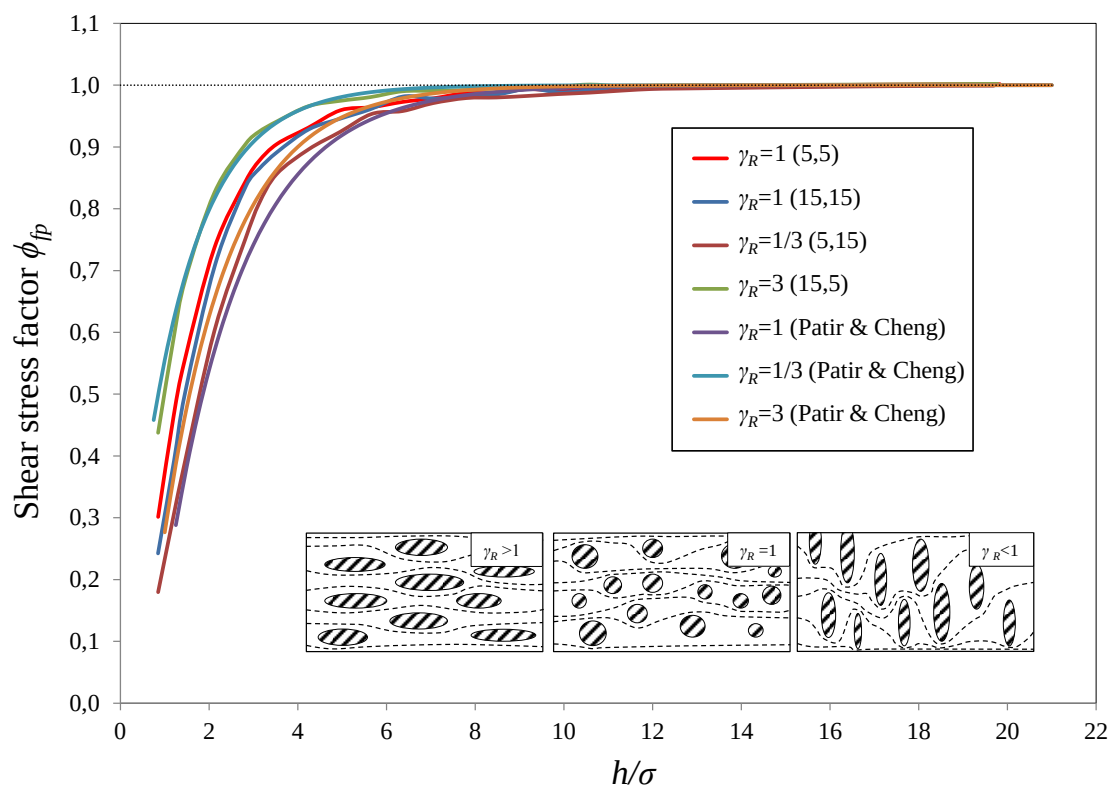


Fig. 67: Comparison of shear stress factors ϕ_{fp} results

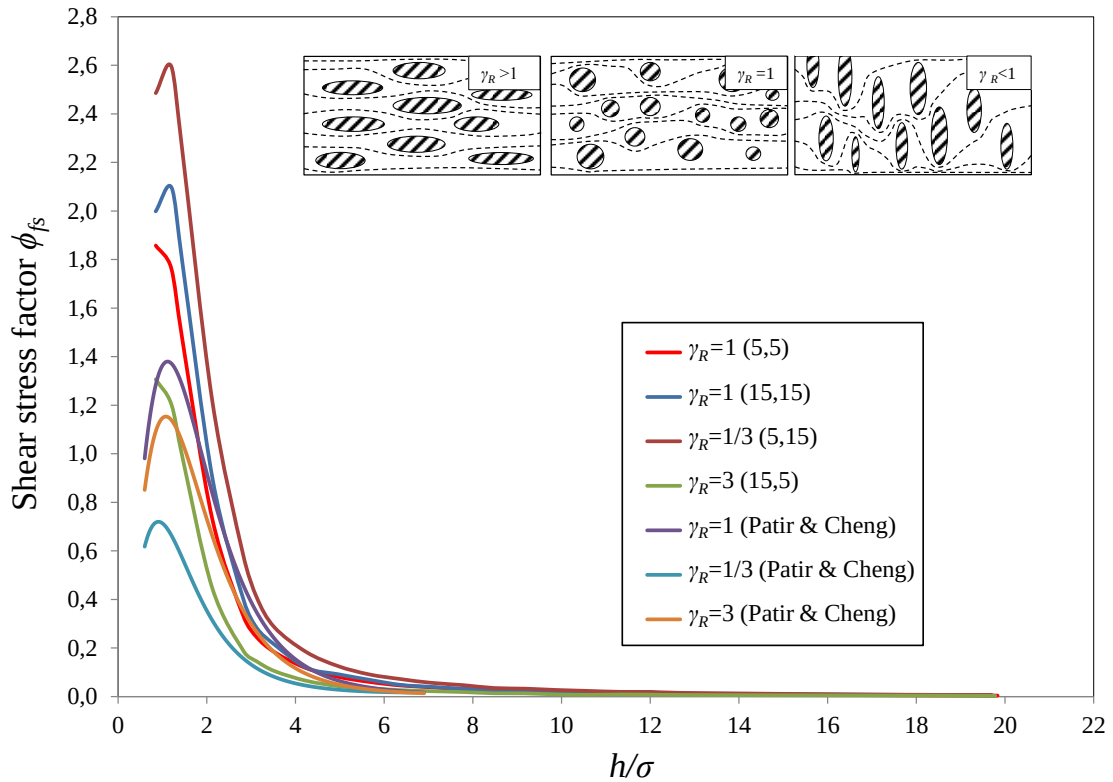


Fig. 68: Comparison of shear stress factors ϕ_{fs} results

To get smoother curves of each factor a finer FDM mesh would be needed, the error for iterative solution would have to be decreased, or the analysed surface area would have to be increased. These adjustments would negatively affect the solution time of all simulations. Considered could also be the code optimisation, possibly the implementation of the code in the programming language FORTRAN.

The optimal settings of numerical solution were studied by collective of authors M. Leighton et al. from Loughborough University in UK [20]. Authors, in the presentation connected to this article, stated that the resolution of FDM mesh of scanned (numerically generated) should be minimally 20 nodes per μm^2 . The minimal size of surface sample for pressure flow factors generation should be $17301 \mu\text{m}^2$ and for numerical solution of shear flow factors it has to have $30758 \mu\text{m}^2$. The averaging should be applied on minimally 10 results of statistically same surfaces.

When these recommendations are taken into consideration the applied settings seem unsuitable. The size of the analysed sample has to be increased several times together with at least two-times more dense mesh. During discussion with the collective of authors of these recommendations was learnt that with these settings and by using a highly optimised FORTRAN code running on a powerful LINUX machine, this simulation requires computational times of tens of hours. These settings are convenient to apply only with high demands on results' precision. However, after application of the recommended settings, further refinement of factors database can be presumed.



10 SLIDE BEARING DYNAMICS SOLUTION

10.1 INPUT DATA FOR SIMULATIONS

Input values for detailed solution of slide bearing are taken from commercially available software tool FEV Virtual Engine. It is a combination of MBS (Multi-Body System) principles and FEM (Finite Element Method). Based on the given data from an actual combustion engine and detailed knowledge of all machinery parts dimensions are from Virtual Engine gained reaction moments and forces of analysed bearings.

The current version of the code works with data taken from the pre-calculated hydrodynamic databases. Currently, it is possible to generate this hydrodynamic database for FEV Virtual Engine by using three different approaches. In all these approaches the core of the solver is based on the numerical solution of the Reynolds equation.

10.1.1 APPROACH 1 – MARTIN REBBERT

The first from these approaches is used for hydrodynamic databases generation directly inside the software FEV Virtual Engine. It is described in the reference [42]. Here is taken into consideration only one tilting angle (tilting angle in the minimal oil film thickness plane) of the bearing pin. The tilted state is derived from the untilted state. The oil film gap is given by the following formula, presented by Butenschön [3]:

$$H = 1 + \varepsilon \cos \varphi. \quad (88)$$

Where φ denotes dimensionless angle of rotation around the pins axis [rad], ε is the relative eccentricity [-] and H is the dimensionless oil film gap [-].

Dimensionless hydrodynamic pressure profile in the oil film gap of slide bearing is derived from the untilted state and has a form [42]:

$$P_{\text{tilted}} = P_{\text{untilted}} / (1 + \gamma Z)^3. \quad (89)$$

Here γ is the dimensionless tilting angle of the bearing pin in the plane of the narrowest oil film gap [-] in range between 0 (untilted pin – the axis of the pin is parallel with the axis of the shell) and 1 (contact between pin and shell), P_{tilted} is the dimensionless hydrodynamic pressure in tilted state [-], P_{untilted} is the dimensionless hydrodynamic pressure in untilted state [-], and Z is the dimensionless coordinate in bearing axis direction [-].

It is the simplest approach which causes some inaccuracies of results (due to consideration of only one tilting angle of bearing pin). For that reason the second, more realistic approach, was developed, it is described in reference [29].



10.1.2 APPROACH 2 – PAVEL NOVOTNÝ

Here two tilting planes are considered, the plane of the narrowest oil film gap (angle γ) and the plane perpendicular to the previously mentioned plane (angle δ). This approach then allows to describe the bearing oil film gap more realistically.

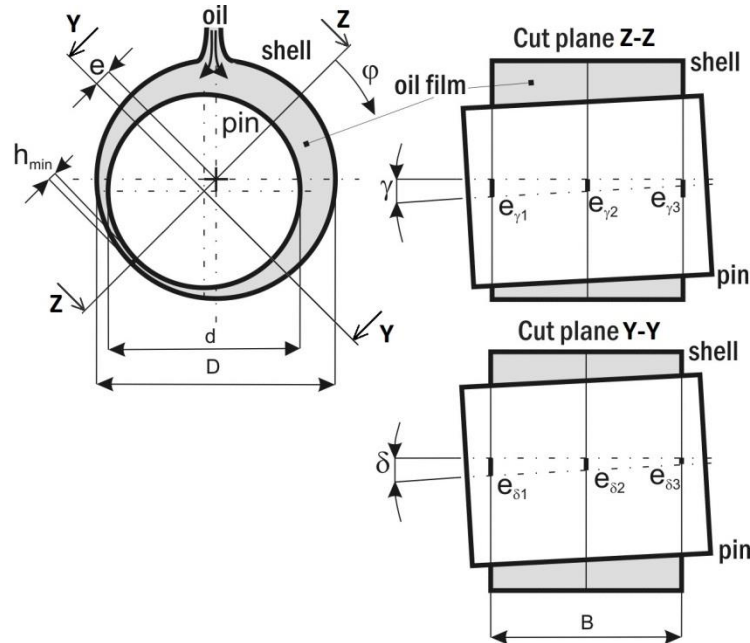


Fig. 69: Bearing pin tilting angles determination [29]

Dimensionless oil film gap, depending on dimensionless tilting angles, dimensionless rotating angle and dimensionless relative eccentricity, is given by the following equation:

$$H = H(\varphi, \varepsilon, \gamma, \delta) = 1 + \varepsilon \cos \varphi + Z(-\gamma \cos \varphi + \varepsilon \gamma \cos \varphi - \delta \sin \varphi). \quad (90)$$

Here φ denotes, again, the dimensionless angle of rotation around the bearing axis [rad], ε is relative eccentricity [-], γ is the dimensionless tilting angle of the bearing pin in the plane of the narrowest oil film gap [-] in range between 0 (untilted pin – the axis of the pin is parallel with the axis of the shell) and 1 (contact between pin and shell), δ is the dimensionless tilting angle of the bearing pin in the plane perpendicular to the first one [-], Z is the dimensionless coordinate in the bearing axis [-], and H is the dimensionless oil film gap [-].

This approach takes into consideration all states of pin tilting (including unrealistic states). An example of this state is when the bearing pin is misaligned for relative eccentricity of almost 1. Then there is almost no space for pin tilting (contact between pin and shell is going to occur). However, these cases are not detected by this approach and thus also the unrealistic cases are calculated. Therefore, the third approach for oil film gap determination for generator of hydrodynamic databases for FEV Virtual Engine was developed.



10.1.3 APPROACH 3 – VONCKEN - MARŠÁLEK

This approach was suggested, during a study visit of the author of this doctoral thesis in company FEV GmbH, by Dr. Ir. A.P.J. Voncken. Author of this thesis then elaborated on this approach and implemented this approach into FORTRAN subroutine. This approach takes into account only realistic states of bearing pin tilting.

In the following passage the evidence and complete derivation of this theory of general bearing pin tilting is written down.

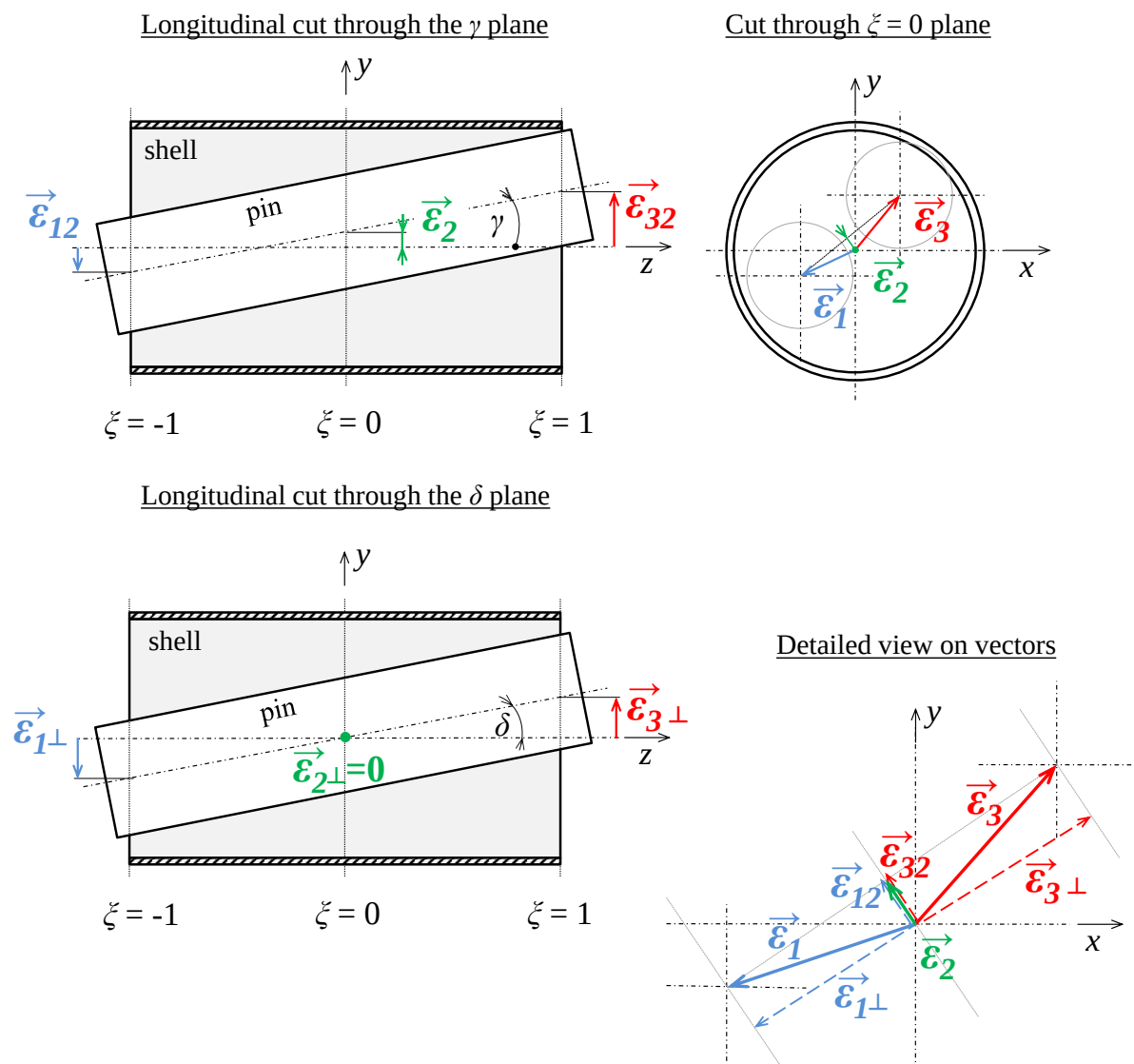


Fig. 70: Bearing pin tilting

The dimensionless oil film gap is given by the following formula:

$$H = H_0 \cdot \left(1 - \overline{\varepsilon}_{(\xi)} \cdot \begin{pmatrix} \cos \psi \\ \sin \psi \end{pmatrix} \right). \quad (91)$$



Here φ is an angle pointed to the minimal oil film thickness (in the middle of the width of the shell – in cut plane $\xi = 0$) and ψ is general angle around circumference of the bearing.

$$\vec{\varepsilon}_2 = \frac{\vec{\varepsilon}_1 + \vec{\varepsilon}_3}{2} \quad (92)$$

$$\vec{\varepsilon}_{(\xi)} = \vec{\varepsilon}_1 \cdot \left(\frac{1-\xi}{2} \right) + \vec{\varepsilon}_3 \cdot \left(\frac{1+\xi}{2} \right) \quad (93)$$

$$\vec{\varepsilon}_2 \equiv \vec{\varepsilon}_{(0)} \Rightarrow \vec{\varepsilon}_{(0)} = \vec{\varepsilon}_1 \cdot \left(\frac{1-0}{2} \right) + \vec{\varepsilon}_3 \cdot \left(\frac{1+0}{2} \right) = \frac{1}{2} \cdot \vec{\varepsilon}_1 + \frac{1}{2} \cdot \vec{\varepsilon}_3 = \frac{\vec{\varepsilon}_1 + \vec{\varepsilon}_3}{2} \quad (94)$$

Relative eccentricity ε in x and y directions are given by:

$$\vec{\varepsilon}_2 = \vec{\varepsilon} = \vec{\varepsilon} \cdot \begin{pmatrix} \sin \varphi \\ \cos \varphi \end{pmatrix} \Rightarrow \varepsilon = \sqrt{\varepsilon_{2y}^2 + \varepsilon_{2x}^2}, \quad (95)$$

$$\varepsilon_2 = \sqrt{(\varepsilon_2 \cdot \sin \varphi)^2 + (\varepsilon_2 \cdot \cos \varphi)^2}, \quad (96)$$

$$\varepsilon = \sqrt{(\varepsilon \cdot \sin \varphi)^2 + (\varepsilon \cdot \cos \varphi)^2}, \quad (97)$$

$$\varepsilon = \varepsilon \cdot \begin{pmatrix} \sin \varphi \\ \cos \varphi \end{pmatrix}. \quad (98)$$

TILTING ANGLE IN THE PLANE OF NARROWEST (MINIMAL) OIL FILM LAYER – ANGLE γ

Projection of the vector $\vec{\varepsilon}_3$ onto vector $\vec{\varepsilon}_2$ is measure for tilting in $\vec{\varepsilon}_2$ direction $\Rightarrow$ tilting angle γ . γ is dimensionless measure for possible tilting angle from $\gamma = 0$ to $\gamma = 1$ (contact).

$$\gamma = \left(\frac{\varepsilon_{32} - \varepsilon}{1 - \varepsilon} \right) \quad (99)$$

In parentheses is value “1” because ε can have maximal value “1”. In parentheses is ε because this vector γ is related to $\varepsilon (= \varepsilon_2)$.



The evidence (proof):

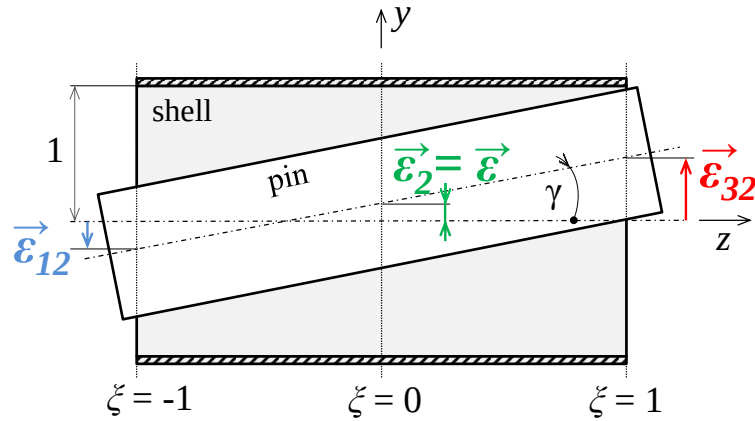


Fig. 71: Longitudinal cut through the γ plane

$\gamma_{\max}$ occurs when $\vec{\varepsilon}_3$ is in contact ($\vec{\varepsilon}_3=1$) and $\vec{\varepsilon}_2=0$.

$$\gamma_{\max} = \frac{\vec{\varepsilon}_3 - \vec{\varepsilon}_2}{1 - \vec{\varepsilon}_2} = \frac{1 - 0}{1 - 0} = 1 \quad (100)$$

$\gamma = \frac{1}{2}$ if $\vec{\varepsilon}_3 = \frac{1}{2}$ and $\vec{\varepsilon}_2 = 0$ (for example):

$$\gamma_{\frac{1}{2}} = \frac{\vec{\varepsilon}_3 - \vec{\varepsilon}_2}{1 - \vec{\varepsilon}_2} = \frac{\frac{1}{2} - 0}{1 - 0} = \frac{1}{2}. \quad (101)$$

Or if $\vec{\varepsilon}_3 = \frac{2}{3}$ and $\vec{\varepsilon}_2 = \frac{1}{2}$ then:

$$\gamma = \frac{\vec{\varepsilon}_3 - \vec{\varepsilon}_2}{1 - \vec{\varepsilon}_2} = \frac{\frac{2}{3} - \frac{1}{3}}{1 - \frac{1}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{3}{6} = \frac{1}{2}. \quad (102)$$

Derivation to be continued using the above presented procedure.

Furthermore:

$$\gamma = \frac{\vec{\varepsilon}_{32} - \vec{\varepsilon}}{1 - \vec{\varepsilon}} \Rightarrow \gamma \cdot (1 - \vec{\varepsilon}) = \vec{\varepsilon}_{32} - \vec{\varepsilon} \Rightarrow \boxed{\vec{\varepsilon}_{32}} = \vec{\varepsilon} + \gamma \cdot (1 - \vec{\varepsilon}), \quad (103)$$

$$\gamma = \frac{\vec{\varepsilon} - \vec{\varepsilon}_{12}}{1 - \vec{\varepsilon}} \Rightarrow \gamma \cdot (1 - \vec{\varepsilon}) = \vec{\varepsilon} - \vec{\varepsilon}_{12} \Rightarrow \boxed{\vec{\varepsilon}_{12}} = \vec{\varepsilon} - \gamma \cdot (1 - \vec{\varepsilon}). \quad (104)$$



TILTING ANGLE IN PLANE WHICH IS PERPENDICULAR TO THE PLANE OF THE NARROWEST (MINIMAL) OIL FILM LAYER - ANGLE δ

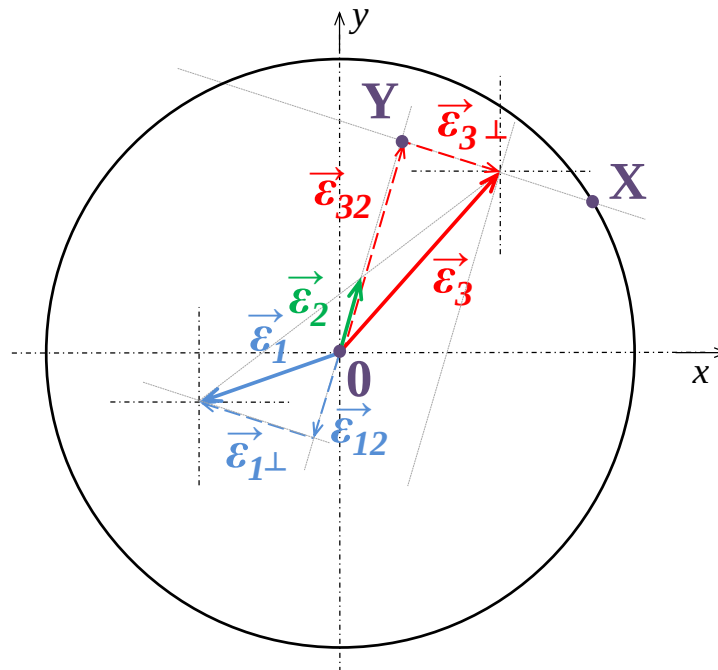


Fig. 72: Cut through the $\xi=0$ plane

Vectors are related to the vector $\vec{\varepsilon}_2$ because vector $\vec{\varepsilon}_2$ is measured in the middle of the shell width.

The [YX] line shows a maximal possible distance which the pin can (theoretically) reach. (In cut plane 3 and in plane of the narrowest (minimal) oil film layer.)

The redrawn figure then looks as follows:

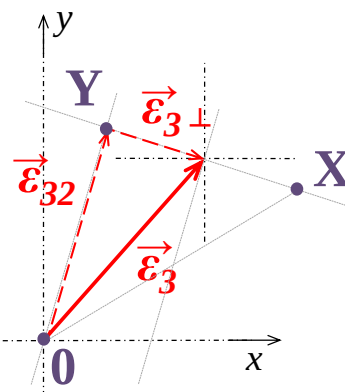


Fig. 73: Detailed view on vectors

Where $\vec{\varepsilon}_{32}$ vector is vector $\vec{\varepsilon}_3$ projected onto the $\vec{\varepsilon}_2$ vector direction.



$\vec{\varepsilon}_3$ is the vector of relative eccentricity in cut 3 (on the right side of the shell width).

$\vec{\varepsilon}_{3\perp}$ is the perpendicular part of the $\vec{\varepsilon}_3$ vector, projected onto the $\vec{\varepsilon}_2$ vector (onto its direction).

$$\overline{OX} = 1 \quad (105)$$

If a situation that vector $\vec{\varepsilon}_{3\perp}$ will run from point Y to the point X will occur then the maximal available angle δ will be 0. (There will be no space for tilting.)

Then the maximal available dimension (relative) for tilting is the $\overline{XY}$ dimension.

$$\text{Hypotenuse}^2 = \overline{OY}^2 + \overline{YX}^2 \Rightarrow \overline{YX}^2 = \text{Hypotenuse}^2 - \overline{OY}^2 \quad (106)$$

In our case the hypotenuse = 1.

$$\overline{YX}^2 = 1^2 - \overline{OY}^2 \Rightarrow \overline{YX} = \sqrt{1^2 - |\vec{\varepsilon}_{32}|^2} \Rightarrow \overline{YX} = \sqrt{1 - (\varepsilon + \gamma \cdot (1 - \varepsilon))^2} \quad (107)$$

Tilting angle δ is then expressed as a ratio of $\vec{\varepsilon}_{3\perp}$ to the maximal possible dimension of the $\vec{\varepsilon}_{3\perp}$ vector ($\overline{YX}$).

The evidence (proof):

If the $\vec{\varepsilon}_{32} = 0.5$ then:

$$\overline{YX} = \sqrt{1^2 - |\vec{\varepsilon}_{32}|^2} = \sqrt{1 - 0.5^2} = 0.866. \quad (108)$$

If the δ angle will be 0.5 then $\vec{\varepsilon}_{3\perp}$ will be $0.5 \cdot 0.866 = 0.433$ - (half of the possible value).

If the δ angle will be 1/3 then $\vec{\varepsilon}_{3\perp}$ will be one third of the possible value.

$$\boxed{\delta} = \frac{\vec{\varepsilon}_{3\perp}}{\overline{YX}} = \frac{\vec{\varepsilon}_{3\perp}}{\sqrt{1 - (\varepsilon + \gamma \cdot (1 - \varepsilon))^2}} \quad (109)$$

And then:

$$\boxed{\vec{\varepsilon}_{3\perp}} = \delta \cdot \sqrt{1 - (\varepsilon + \gamma \cdot (1 - \varepsilon))^2}. \quad (110)$$

If we substitute equation (91) into equation (93) then:

$$H = H_0 \cdot \left\{ 1 - \left(\frac{\cos \psi}{\sin \psi} \right) \cdot \left(\frac{\varepsilon_1 + \varepsilon_3}{2} \right) - \xi \cdot \left(\frac{\cos \psi}{\sin \psi} \right) \cdot \left(\frac{\varepsilon_3 - \varepsilon_1}{2} \right) \right\}. \quad (111)$$



Expression $\varepsilon_3 - \varepsilon_1$ will be extended by $+\varepsilon_2 - \varepsilon_2$. Then its value will be without any change but we will obtain relation between epsilons in the middle of the shell width and at the edges.

$$(\varepsilon_3 - \varepsilon_1 + \varepsilon_2 - \varepsilon_2) = (\varepsilon_3 - \varepsilon_2) - (\varepsilon_1 - \varepsilon_2) \quad (112)$$

$$\frac{\vec{\varepsilon}_1 + \vec{\varepsilon}_3}{2} \equiv \vec{\varepsilon}_2 = \varepsilon \cdot \begin{pmatrix} \cos \varphi \\ \sin \varphi \end{pmatrix} \quad (113)$$

Into equation (111) are constituted expressions (112) and (113), and the resulting equation has a form:

$$H = H_0 \cdot \left\{ 1 - \varepsilon \cdot \begin{pmatrix} \cos \psi \\ \sin \psi \end{pmatrix} \cdot \begin{pmatrix} \cos \varphi \\ \sin \varphi \end{pmatrix} - \frac{\xi}{2} \cdot \begin{pmatrix} \cos \psi \\ \sin \psi \end{pmatrix} \cdot [(\varepsilon_3 - \varepsilon_2) - (\varepsilon_1 - \varepsilon_2)] \right\}. \quad (114)$$

Furthermore:

$$\vec{a} = \vec{a}_x \cdot \vec{e}_x + \vec{a}_y \cdot \vec{e}_y, \quad (115)$$

$$\vec{a}_x = \vec{a} \cdot \vec{e}_x, \quad (116)$$

$$\vec{a}_y = \vec{a} \cdot \vec{e}_y, \quad (117)$$

$$\vec{a} = (\vec{a} \cdot \vec{e}_x) \cdot \vec{e}_x + (\vec{a} \cdot \vec{e}_y) \cdot \vec{e}_y. \quad (118)$$

Where: $\vec{e}_y$ is the unit vector in y direction, $\vec{e}_x$ is the unit vector in x direction, $\vec{a}_y$ is the y component of vector $\vec{a}$ and $\vec{a}_x$ is the x component of vector $\vec{a}$.

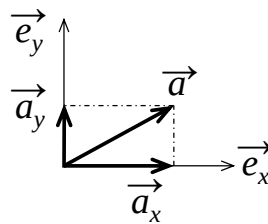


Fig. 74: Unit vectors using

PARENTHESES $[(\varepsilon_3 - \varepsilon_2) - (\varepsilon_1 - \varepsilon_2)]$ **EXPLANATION**

$$\varepsilon_3 - \varepsilon_2 = (\varepsilon_3 - \varepsilon) \cdot \begin{pmatrix} \cos \varphi \\ \sin \varphi \end{pmatrix} + [(\varepsilon_3 - \varepsilon_2) \cdot \begin{pmatrix} -\sin \varphi \\ \cos \varphi \end{pmatrix}] \cdot \begin{pmatrix} -\sin \varphi \\ \cos \varphi \end{pmatrix} \quad (119)$$



Here $(\varepsilon_{32} - \varepsilon) \cdot \begin{pmatrix} \cos \varphi \\ \sin \varphi \end{pmatrix}$ represents the direction of projection onto $\vec{\varepsilon}_2$ vector and $\left[(\varepsilon_3 - \varepsilon_2) \cdot \begin{pmatrix} -\sin \varphi \\ \cos \varphi \end{pmatrix} \right] \cdot \begin{pmatrix} -\sin \varphi \\ \cos \varphi \end{pmatrix}$ represents the direction perpendicular to the $\vec{\varepsilon}_2$ vector.

By substitution into $\vec{\varepsilon}_{32}$ vector the following formula is obtained:

$$\varepsilon_3 - \varepsilon_2 = [\gamma \cdot (1 - \varepsilon)] \cdot \begin{pmatrix} \cos \varphi \\ \sin \varphi \end{pmatrix} + \left[(\varepsilon_3 - \varepsilon_2) \cdot \begin{pmatrix} -\sin \varphi \\ \cos \varphi \end{pmatrix} \right] \cdot \begin{pmatrix} -\sin \varphi \\ \cos \varphi \end{pmatrix}, \quad (120)$$

$$\varepsilon_1 - \varepsilon_2 = (\varepsilon_{12} - \varepsilon) \cdot \begin{pmatrix} \cos \varphi \\ \sin \varphi \end{pmatrix} + \left[(\varepsilon_1 - \varepsilon_2) \cdot \begin{pmatrix} -\sin \varphi \\ \cos \varphi \end{pmatrix} \right] \cdot \begin{pmatrix} -\sin \varphi \\ \cos \varphi \end{pmatrix}. \quad (121)$$

Here $(\varepsilon_{12} - \varepsilon) \cdot \begin{pmatrix} \cos \varphi \\ \sin \varphi \end{pmatrix}$ represents the direction of projection onto $\vec{\varepsilon}_2$ vector and $\left[(\varepsilon_1 - \varepsilon_2) \cdot \begin{pmatrix} -\sin \varphi \\ \cos \varphi \end{pmatrix} \right] \cdot \begin{pmatrix} -\sin \varphi \\ \cos \varphi \end{pmatrix}$ represents the direction perpendicular to the $\vec{\varepsilon}_2$ vector.

By substitution into $\vec{\varepsilon}_{12}$ vector the following formula is obtained:

$$\varepsilon_1 - \varepsilon_2 = [-\gamma \cdot (1 - \varepsilon)] \cdot \begin{pmatrix} \cos \varphi \\ \sin \varphi \end{pmatrix} + \left[(\varepsilon_1 - \varepsilon_2) \cdot \begin{pmatrix} -\sin \varphi \\ \cos \varphi \end{pmatrix} \right] \cdot \begin{pmatrix} -\sin \varphi \\ \cos \varphi \end{pmatrix}. \quad (122)$$

By substitution of equation (120) and (122) into equation (114) the next equation is gained:

$$H = H_0 \cdot \left\{ 1 - (\varepsilon + \xi \cdot \gamma \cdot (1 - \varepsilon)) \cdot \begin{pmatrix} \cos \varphi \\ \sin \varphi \end{pmatrix} \cdot \begin{pmatrix} \cos \psi \\ \sin \psi \end{pmatrix} - \frac{\xi}{2} \cdot \begin{pmatrix} \cos \psi \\ \sin \psi \end{pmatrix} \cdot \begin{pmatrix} -\sin \varphi \\ \cos \varphi \end{pmatrix} \cdot (\varepsilon_{3\perp} - \varepsilon_{1\perp}) \right\}. \quad (123)$$

Furthermore, if the distance between point V and W has to be known, the sum of both vectors is simply done by formula:

$$\varepsilon_{3\perp} + \varepsilon_{1\perp} = \boxed{2 \cdot \varepsilon_{3\perp}} = 2 \cdot \varepsilon_{1\perp}. \quad (124)$$

Despite the fact that the vectors have opposite signs this operation is possible. (It is about the distance, not about the sum.)

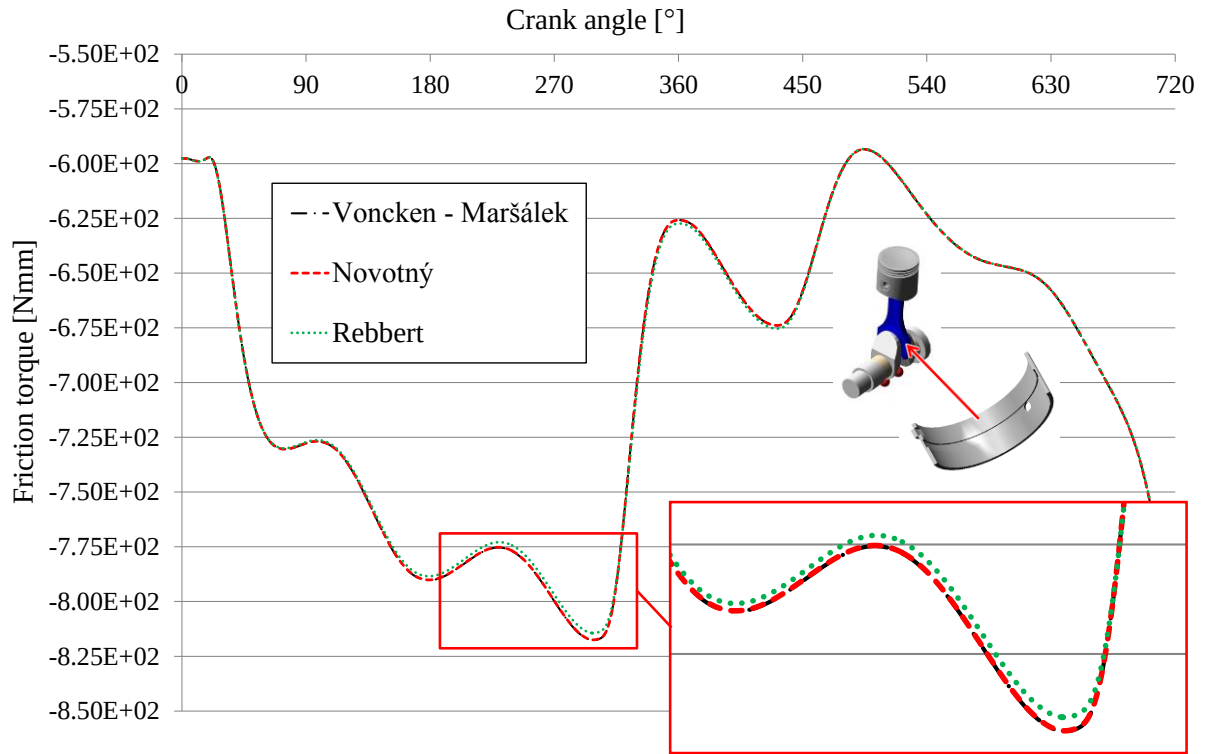


Fig. 76: Comparison of implemented approaches for hydrodynamic databases generation

10.2 DESCRIPTION OF DETAILED SOLVER OF SLIDE BEARING DYNAMICS

The fundamental equation describing the behaviour of hydrodynamic lubricating layer is still the Reynolds equation and it has a form:

$$\frac{\partial}{\partial x} \left(\frac{\rho h^3}{12\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{\rho h^3}{12\eta} \frac{\partial p}{\partial z} \right) - \frac{U}{2} \frac{\partial(\rho h)}{\partial x} - \frac{\partial(\rho h)}{\partial t} = 0. \quad (130)$$

The modified Reynolds equation with flow factors has a form:

$$\frac{\partial}{\partial x} \left(\phi_x \frac{\rho h^3}{12\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(\phi_z \frac{\rho h^3}{12\eta} \frac{\partial p}{\partial z} \right) - \frac{U}{2} \frac{\partial(\rho \bar{h}_T)}{\partial x} - \frac{\partial(\rho \bar{h}_T)}{\partial t} - \frac{U}{2} \sigma \frac{\partial \phi_s}{\partial x} = 0. \quad (131)$$

Here p is the hydrodynamic pressure [Pa], h is the nominal oil film gap height [m], η is the dynamic oil viscosity [Pa·s], U is the circumferential velocity of the bearing pin [$\text{m}\cdot\text{s}^{-1}$], x and z are coordinates and ϕ_x , ϕ_z and ϕ_s pressure and shear flow factors, taken from the pre-calculated database and $\bar{h}_T$ is the average value calculated from the local oil film height h_T [m].

The oil film thickness is defined as:

$$h = R - r + e \cos(\varphi), \quad (132)$$



where R is the radius of bearing shell [m], r is the radius of bearing pin [m], e is the eccentricity between bearing pin and shell [m] and φ is the rotational angle around the bearing pin axis [rad]. Thickness of oil film pressure in this case is determined similarly as in chapter 10.1.2 Approach 2 – Pavel Novotný.

Established is parameter c – the half of bearing clearance ($c = R - r = \text{clearance} / 2$) and dimensionless variables according to the following equations:

$$\begin{aligned}
 H &= \frac{h}{R-r} = \frac{h}{c}, \quad \bar{\rho} = \frac{\rho}{\rho_0}, \quad \bar{\eta} = \frac{\eta}{\eta_0}, \quad \varphi = \frac{x}{R}, \quad Z = \frac{z}{B}, \\
 v &= \frac{B}{R} = \frac{2B}{D}, \quad P = \frac{pc^2}{12R^2\eta_0}, \quad \bar{U}_n = \frac{U_n}{c} = \dot{H}, \quad \bar{U} = \frac{U}{2R}, \\
 \bar{\sigma} &= \frac{\sigma}{c\rho_0}, \quad \xi_x = \phi_x \frac{\bar{\rho}H^3}{\bar{\eta}}, \quad \xi_z = \phi_z \frac{\bar{\rho}H^3}{\bar{\eta}}.
 \end{aligned} \tag{133}$$

The form of the Reynolds equation is given by the following formulae (terms).

Poisseuille terms:

$$\frac{\partial}{\partial x} \left(\phi_x \frac{\rho h^3}{12\eta} \frac{\partial p}{\partial x} \right) = c\rho_0 \frac{\partial}{\partial \varphi} \left(\xi_x \frac{\partial P}{\partial \varphi} \right), \tag{134}$$

$$\frac{\partial}{\partial z} \left(\phi_z \frac{\rho h^3}{12\eta} \frac{\partial p}{\partial z} \right) = c\rho_0 \frac{R^2}{B^2} \frac{\partial}{\partial Z} \left(\xi_z \frac{\partial P}{\partial Z} \right) = c\rho_0 \frac{1}{v^2} \frac{\partial}{\partial Z} \left(\xi_z \frac{\partial P}{\partial Z} \right). \tag{135}$$

Couette term:

$$\frac{U}{2} \frac{\partial(\rho h)}{\partial x} = c\rho_0 \frac{U}{2R} \frac{\partial(\bar{\rho}H)}{\partial \varphi} = c\rho_0 \bar{U} \frac{\partial(\bar{\rho}H)}{\partial \varphi}. \tag{136}$$

Transient term:

$$\frac{\partial(\rho h)}{\partial t} = \dot{\rho}h + \rho\dot{h} = \dot{\rho}h + \rho U_n = \dot{\rho}h + \rho c \bar{U}_n = c\rho_0 (\dot{\bar{\rho}}H + \bar{\rho}\dot{H}). \tag{137}$$

Shear flow factor term:

$$\frac{U}{2} \sigma \frac{\partial \phi_s}{\partial x} = c\rho_0 \frac{U}{c\rho_0 2R} \sigma \frac{\partial \phi_s}{\partial \varphi} = c\rho_0 \bar{U} \bar{\sigma} \frac{\partial \phi_s}{\partial \varphi}. \tag{138}$$

Reynolds equation in dimensionless form:



$$\frac{\partial}{\partial \phi} \left(\xi_x \frac{\partial P}{\partial \phi} \right) + \frac{1}{\nu^2} \frac{\partial}{\partial Z} \left(\xi_z \frac{\partial P}{\partial Z} \right) - \bar{U} \frac{\partial(\bar{\rho}H)}{\partial \phi} - (\dot{\bar{\rho}}H + \bar{\rho}\dot{H}) - \bar{U}\bar{\sigma} \frac{\partial \phi_s}{\partial \phi} = 0. \quad (139)$$

Reynolds equation in dimensionless form with constant oil viscosity and density:

$$\frac{\partial}{\partial \phi} \left(\xi_x \frac{\partial P}{\partial \phi} \right) + \frac{1}{\nu^2} \frac{\partial}{\partial Z} \left(\xi_z \frac{\partial P}{\partial Z} \right) - \bar{U} \frac{\partial H}{\partial \phi} - \dot{H} - \bar{U}\bar{\sigma} \frac{\partial \phi_s}{\partial \phi} = 0. \quad (140)$$

Discretisation of each term of the Reynolds equation is done by using the Finite Difference Method.

Discretised Poisseuille term:

$$\left(\frac{\partial}{\partial \phi} \left(\xi_x \frac{\partial P}{\partial \phi} \right) \right)_{i,j} \cong \frac{\left(\xi_x \frac{\partial P}{\partial \phi} \right)_{i+1/2,j} - \left(\xi_x \frac{\partial P}{\partial \phi} \right)_{i-1/2,j}}{h_\phi}. \quad (141)$$

Inner derivation:

$$\left(\xi_x \frac{\partial P}{\partial \phi} \right)_{i+1/2,j} \cong \xi_{xi+1/2,j} \frac{P_{i+1,j} - P_{i-1,j}}{2h_\phi}. \quad (142)$$

Combination:

$$\left(\frac{\partial}{\partial \phi} \left(\xi_x \frac{\partial P}{\partial \phi} \right) \right)_{i,j} \cong \frac{\xi_{xi+1/2,j} P_{i+1,j} - (\xi_{xi+1/2,j} + \xi_{xi-1/2,j}) P_{i,j} + \xi_{xi-1/2,j} P_{i-1,j}}{h_\phi^2} = a_1 (\xi_{xi+1/2,j} P_{i+1,j} - (\xi_{xi+1/2,j} + \xi_{xi-1/2,j}) P_{i,j} + \xi_{xi-1/2,j} P_{i-1,j}) \quad (143)$$

Where: $a_1 = \frac{1}{h_\phi^2}$ and $a_2 = \frac{1}{(\nu h_z)^2}$.

Explanation of ξ term:

$$\xi_{xi\pm 1/2,j} = \frac{\xi_{xi+1,j} \pm \xi_{xi,j}}{2}, \quad \xi_{zi\pm 1/2,j} = \frac{\xi_{zi+1,j} \pm \xi_{zi,j}}{2}. \quad (144)$$

Explanation of Couette term:

$$\begin{aligned} \bar{U} \frac{\partial \bar{\rho}H}{\partial \phi} \Big|_{i,j} &\cong \bar{U}_{i,j} \frac{1.5\bar{\rho}_{i,j}H_{i,j} - 2.0\bar{\rho}_{i-1,j}H_{i-1,j} + 0.5\bar{\rho}_{i-2,j}H_{i-2,j}}{h_\phi} \\ &= a_3 U_{i,j} (1.5\bar{\rho}_{i,j}H_{i,j} - 2.0\bar{\rho}_{i-1,j}H_{i-1,j} + 0.5\bar{\rho}_{i-2,j}H_{i-2,j}) \end{aligned} \quad (145)$$



Here: $a_3 = \frac{1}{2Rh_\varphi}$.

Explanation of shear flow factor term:

$$\bar{U}\bar{\sigma}\left.\frac{\partial\phi_s}{\partial\varphi}\right|_{i,j} \cong \bar{U}_{i,j}\bar{\sigma}_{i,j}\frac{\phi_{si+1,j}-\phi_{si-1,j}}{2h_\varphi} = a_4U_{i,j}\bar{\sigma}_{i,j}(\phi_{si+1,j}-\phi_{si-1,j}). \quad (146)$$

Where: $a_4 = \frac{1}{4Rh_\varphi}$.

Final shape of the discretised Reynolds equation:

$$\begin{aligned} & +a_1(\xi_{xi+1/2,j}P_{i+1,j} - (\xi_{xi+1/2,j} + \xi_{xi-1/2,j})P_{i,j} + \xi_{xi-1/2,j}P_{i-1,j}) \\ & +a_2(\xi_{zi,j+1/2}P_{i,j+1} - (\xi_{zi,j+1/2} + \xi_{zi,j-1/2})P_{i,j} + \xi_{zi,j-1/2}P_{i,j-1}) \\ & -a_3U_{i,j}(1.5H_{i,j} - 2H_{i-1,j} + 0.5H_{i-2,j}) \\ & -\dot{H}_{i,j} - a_4U_{i,j}\bar{\sigma}_{i,j}(\phi_{si+1,j} - \phi_{si-1,j}) \\ P_{i,j} = \tilde{P}_{i,j} - \omega_{SOR}\frac{r_{i,j}}{J_{i,j}} = \tilde{P}_{i,j} - \omega_{SOR}\frac{-\dot{H}_{i,j} - a_4U_{i,j}\bar{\sigma}_{i,j}(\phi_{si+1,j} - \phi_{si-1,j})}{-a_1(\xi_{xi+1/2,j} + \xi_{xi-1/2,j}) - a_2(\xi_{zi,j+1/2} + \xi_{zi,j-1/2})}. \end{aligned} \quad (147)$$

ω_{SOR} is the overrelaxation coefficient used for SOR method.

The computational approach takes also into consideration a simple cavitation model.

Residuum in point i, j :

$$\begin{aligned} r_{i,j} = & +a_1(\xi_{xi+1/2,j}P_{i+1,j} - (\xi_{xi+1/2,j} + \xi_{xi-1/2,j})P_{i,j} + \xi_{xi-1/2,j}P_{i-1,j}) \\ & +a_2(\xi_{zi,j+1/2}P_{i,j+1} - (\xi_{zi,j+1/2} + \xi_{zi,j-1/2})P_{i,j} + \xi_{zi,j-1/2}P_{i,j-1}) \\ & -a_3U_{i,j}(1.5H_{i,j} - H_{i-1,j} + 0.5H_{i-2,j}) \\ & -\dot{H}_{i,j} - a_4U_{i,j}\bar{\sigma}_{i,j}(\phi_{si+1,j} - \phi_{si-1,j}) \end{aligned} \quad (148)$$

Jacobian in point i, j :

$$J_{i,j} = -a_1(\xi_{xi+1/2,j} + \xi_{xi-1/2,j}) - a_2(\xi_{zi,j+1/2} + \xi_{zi,j-1/2}). \quad (149)$$

During the numerical solution equilibrium state is considered between inner and outer forces acting on the bearing pin relatively to the bearing shell. Velocity U_2 [m·s⁻¹] then equals to 0. As inner forces are forces in the oil film layer. Inertia forces, elastic forces, and damping forces are included in outer forces and are determined separately – see previous chapter 10.1 Input Data for Simulations.

$$\mathbf{F}_{\text{Inner}} + \mathbf{F}_{\text{Outer}} = \mathbf{0} \quad (150)$$

This equilibrium of inner and outer forces is solved numerically. It is a nonlinear system of equations. The initial nonlinear function is given by equation:



$$\mathbf{f}(x, y, z, \vartheta, \zeta, \alpha, t) = \begin{bmatrix} f_x \\ f_y \\ f_z \\ f_\vartheta \\ f_\zeta \\ f_\alpha \end{bmatrix} = \mathbf{F}_{\text{Inner}} + \mathbf{F}_{\text{Outer}} = \mathbf{0}. \quad (151)$$

This system of nonlinear functions is solved iteratively using Newton-Raphson algorithm. Regarding time-efficiency it is better to use simpler differential methods instead of more complicated methods of numerical derivations. The used Newton-Raphson algorithm has a form:

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \\ z_{k+1} \\ \vartheta_{k+1} \\ \zeta_{k+1} \\ \alpha_{k+1} \end{bmatrix} = \mathbf{X}_{k+1} = \mathbf{X}_k - \omega_{EQ} \mathbf{J}_k^{-1} \mathbf{f}_k. \quad (152)$$

Jacobian has a form:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_y}{\partial x} & \frac{\partial f_z}{\partial x} & \frac{\partial f_\vartheta}{\partial x} & \frac{\partial f_\zeta}{\partial x} & \frac{\partial f_\alpha}{\partial x} \\ \frac{\partial f_x}{\partial y} & \frac{\partial f_y}{\partial y} & & & & \\ \frac{\partial f_x}{\partial z} & & & & & \\ \frac{\partial f_x}{\partial \vartheta} & & & & & \\ \frac{\partial f_x}{\partial \zeta} & & & & & \\ \frac{\partial f_x}{\partial \alpha} & & & & & \frac{\partial f_\alpha}{\partial \alpha} \end{bmatrix}. \quad (153)$$

Integration of inner forces in the coordinate system of bearing pin is given by the following formulae:

$$F_x = \sum_{i=1}^m \sum_{j=1}^n p_{Ci,j} dA \cos(\varphi_{Ci}) - \sum_{i=1}^m \sum_{j=1}^n \tau_{Ci,j} dA \sin(\varphi_{Ci}), \quad (154)$$



$$F_y = \sum_{i=1}^m \sum_{j=1}^n p_{Ci,j} dA \sin(\varphi_{Ci}) + \sum_{i=1}^m \sum_{j=1}^n \tau_{Ci,j} dA \cos(\varphi_{Ci}), \quad (155)$$

$$F_z = 0, \quad (156)$$

$$M_x = \sum_{i=1}^m \sum_{j=1}^n p_{Ci,j} \cdot dA \cdot z_{Ci,j} \cdot \sin(\varphi_{Ci}) + \sum_{i=1}^m \sum_{j=1}^n \tau_{Ci,j} \cdot dA \cdot z_{Ci,j} \cdot \cos(\varphi_{Ci}), \quad (157)$$

$$M_y = \sum_{i=1}^m \sum_{j=1}^n p_{Ci,j} \cdot dA \cdot z_{Ci,j} \cdot \cos(\varphi_{Ci}) - \sum_{i=1}^m \sum_{j=1}^n \tau_{Ci,j} \cdot dA \cdot z_{Ci,j} \cdot \sin(\varphi_{Ci}), \quad (158)$$

$$M_z = \sum_{i=1}^m \sum_{j=1}^n \tau_{Ci,j} R dA. \quad (159)$$

Explanation of individual variables is presented in the following figure:

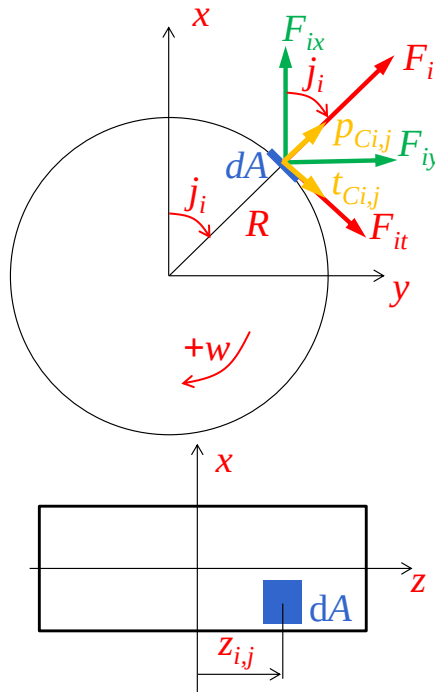


Fig. 77: Inner forces in the coordinate system of bearing shell

To integrate normal pressure and stress trapezoidal rule is used:

$$p_N = \frac{dA}{4} (p_{i,j} + p_{i+1,j} + p_{i,j+1} + p_{i+1,j+1}), \quad (160)$$

$$\tau_N = \frac{dA}{4} (\tau_{i,j} + \tau_{i+1,j} + \tau_{i,j+1} + \tau_{i+1,j+1}). \quad (161)$$

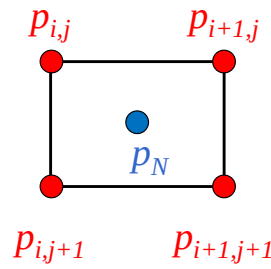


Fig. 78: Illustrative figure of the trapezoidal rule

For completeness, a list of kinematic variables of the slide bearing calculation is listed here.

Pin displacement $\mathbf{X}$, pin velocity $\mathbf{V}$ and pin acceleration $\mathbf{A}$ are given by the following formulae:

$$\begin{bmatrix} x \\ y \\ z \\ g \\ \zeta \\ \alpha \end{bmatrix} = \mathbf{X}, \quad \begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_g \\ \omega_\zeta \\ \omega \end{bmatrix} = \mathbf{V}, \quad \begin{bmatrix} a_x \\ a_y \\ a_z \\ \varepsilon_g \\ \varepsilon_\zeta \\ \varepsilon_\alpha \end{bmatrix} = \mathbf{A}. \quad (162)$$

Where α is the angle of pin rotation related to the bearing shell, ω is the angular velocity of the bearing pin related to the bearing shell, and ε_α is the angular acceleration related to the bearing shell.

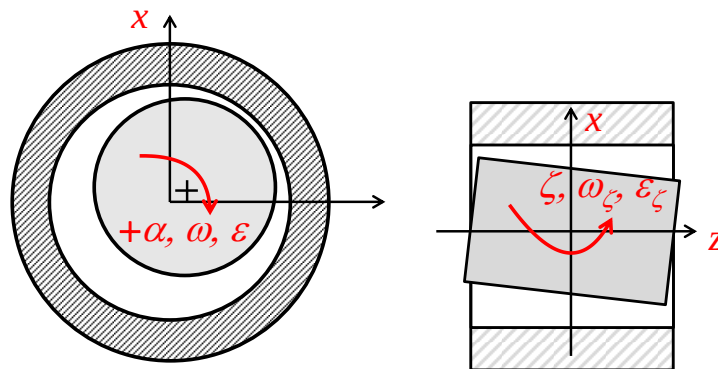


Fig. 79: Illustrative figure of kinematic values

The calculation of kinematic values is given by the following equations.

Pin displacement in a time step $t+\Delta t$:



$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \\ z_{k+1} \\ g_{k+1} \\ \zeta_{k+1} \\ \alpha_{k+1} \end{bmatrix} = \mathbf{X}_{k+1}. \quad (163)$$

Pin velocity and acceleration are determined by using Newmark method:

$$\mathbf{V}_{k+1} = \dot{\mathbf{X}}_{k+1} = \frac{\gamma_N}{\beta_N \Delta t} (\mathbf{X}_{k+1} - \mathbf{X}_k) - \left(\frac{\gamma_N}{\beta_N} - 1 \right) \dot{\mathbf{X}}_k - \left(\frac{\gamma_N}{2\beta_N} - 1 \right) \Delta t \ddot{\mathbf{X}}_k, \quad (164)$$

$$\mathbf{A}_{k+1} = \ddot{\mathbf{X}}_{k+1} = \frac{1}{\beta_N \Delta t^2} (\mathbf{X}_{k+1} - \mathbf{X}_k) - \frac{1}{\beta_N \Delta t} \dot{\mathbf{X}}_k - \left(\frac{1}{2\beta_N} - 1 \right) \ddot{\mathbf{X}}_k. \quad (165)$$

Where $\beta_N = 0.25$ and $\gamma_N = 0.5$.

Tangential velocity of oil film gap (change of oil film gap in tangential direction) (stated in the so called Couette term of the Reynolds equation $\left(\frac{U}{2} \frac{\partial(\rho h)}{\partial x} \right)$) is given by equation:

$$U = \omega R - v_x \sin \varphi + v_y \cos \varphi. \quad (166)$$

Normal velocity of oil film gap (squeeze) (so called transient term of the Reynolds equation $\left(\frac{\partial(\rho h)}{\partial t} \right)$) is given by equation:

$$\frac{\partial(\rho h)}{\partial t} = \dot{\rho} h + \rho \dot{h} = \dot{\rho} h + \rho U_n. \quad (167)$$

Normal velocity of bearing pin (U_n) is given by equation:

$$U_n = \dot{h} = -v_x \cos \varphi - v_y \sin \varphi. \quad (168)$$

10.3 RESULTS OF NUMERICAL SIMULATIONS OF SLIDE BEARINGS BEHAVIOUR

Differences between results after the application of four different approaches are going to be demonstrated on the connecting rod bearing of two different combustion engines.

The first approach is for the case when the surface roughness pattern with the Peklenik number $\gamma_R = 3$ is incorporated in the solution of slide bearing dynamics. (Surface pattern supports the oil flow in bearing.)



The second approach is for surface pattern with Peklenik number $\gamma_R = 1/3$ (surface pattern restricts the oil flow in the bearing).

The third approach is for the case when the surfaces pattern is directionally independent – Peklenik number $\gamma_R = 1$.

The last approach is without the consideration of flow and shear stress factors.

Parameters of analysed slide bearings and engines are listed in the following table.

Table 27: Engines/bearings description

		Škoda EA111 1.2 HTP		Mercedes-Benz OM 651.961	
Engine	Bore	76.5	[mm]	83	[mm]
	Stroke	86.9	[mm]	99	[mm]
	Volume	1198	[cm ³]	2143	[cm ³]
	Compression	10.5	[-]	16.2	[-]
	Power output	51/5400	[kW/min ⁻¹]	150/4200	[kW/min ⁻¹]
	Torque	112/3000	[Nm/min ⁻¹]	500/1600-1800	[Nm/min ⁻¹]
Bearing	Diameter	42	[mm]	52	[mm]
	Width	16	[mm]	20.5	[mm]

To see the behaviour of given bearing/engine better, simulations for a set of bearing clearances and for two variants of engine speed were done.

The set of graphs showing results of simulations follows. Furthermore, the results of simulation of internal combustion engine Škoda EA 111, 1.2 HTP bearing are labelled just as Škoda and results of internal combustion engine Mercedes-Benz OM 651.961 bearing just as Mercedes-Benz.



10.3.1 MERCEDES-BENZ

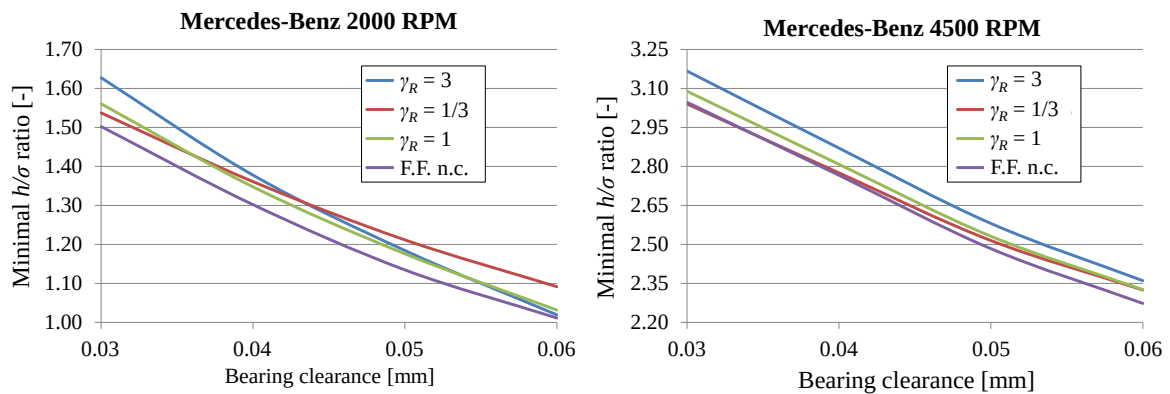


Fig. 80: Minimal h/σ ratio Mercedes-Benz

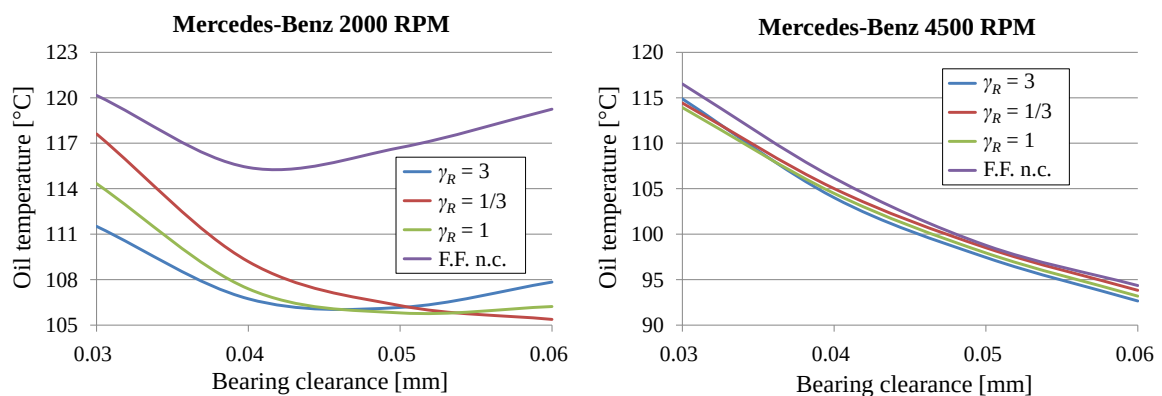


Fig. 81: Oil temperature Mercedes-Benz

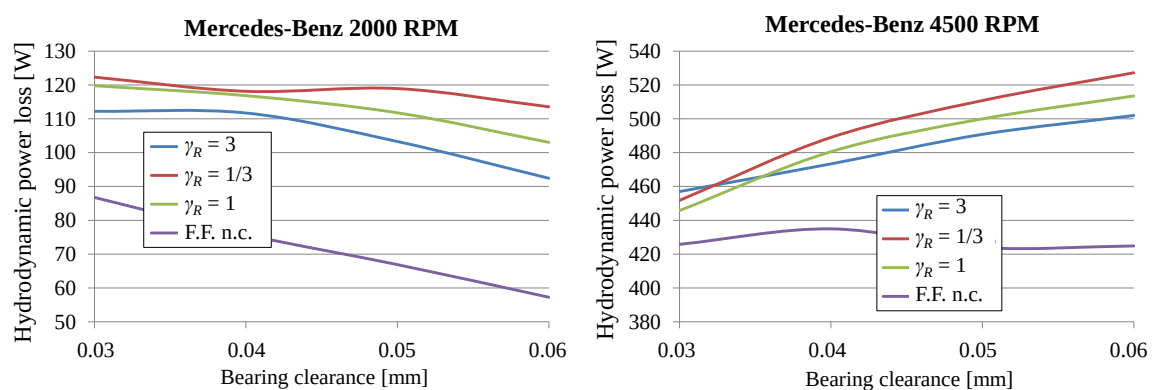


Fig. 82: Hydrodynamic power loss Mercedes-Benz

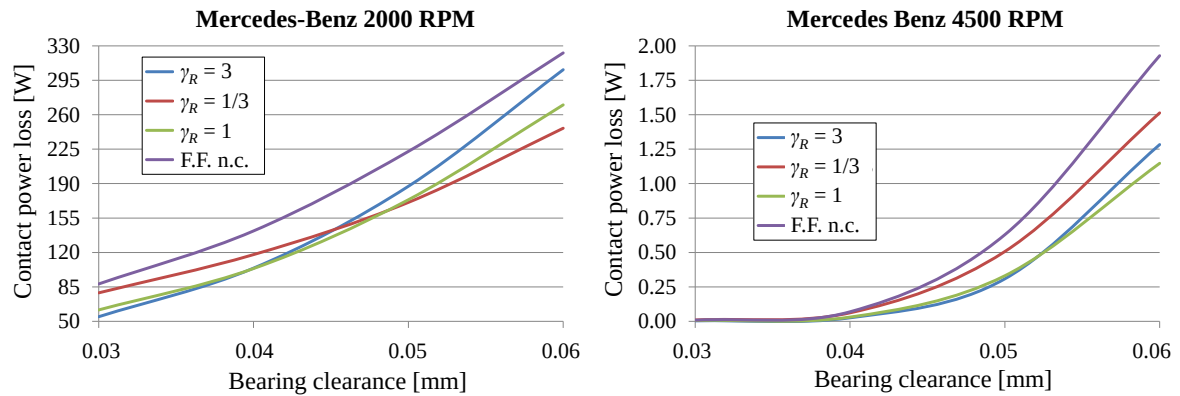


Fig. 83: Contact power loss Mercedes-Benz

10.3.2 ŠKODA

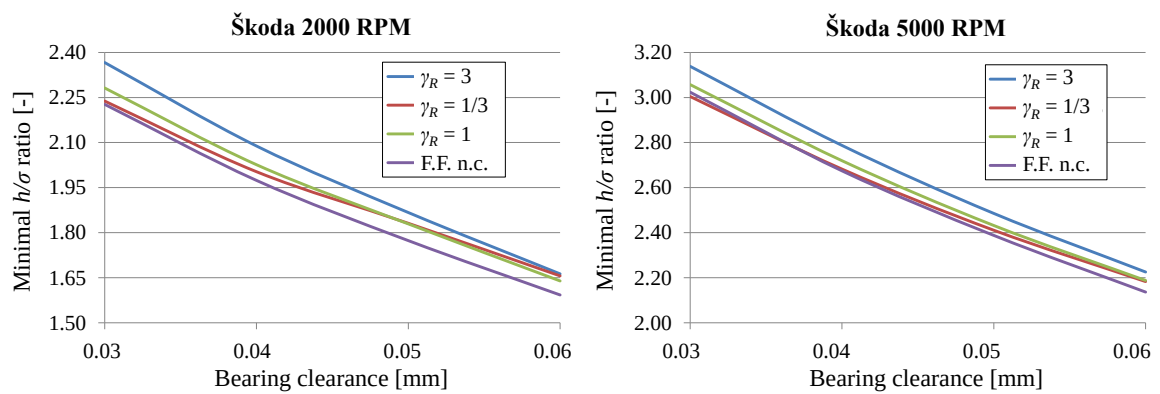
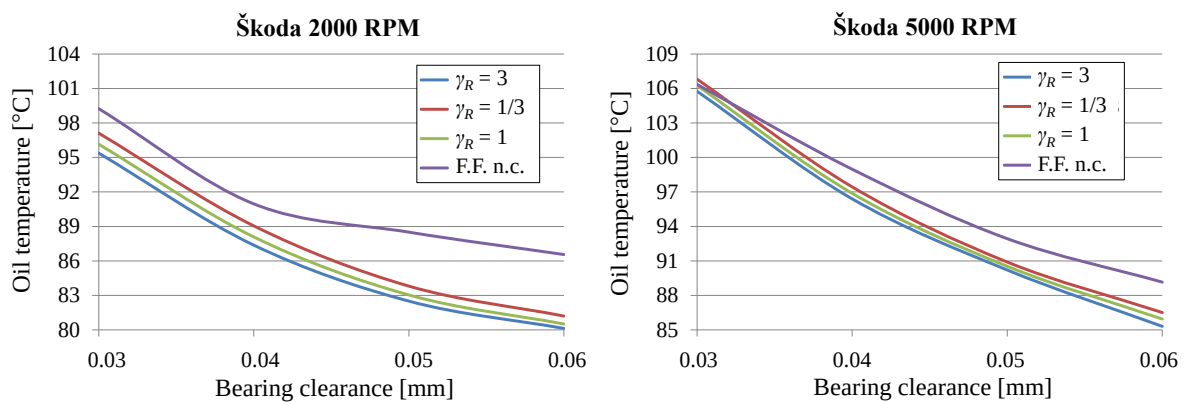
Fig. 84: Minimal h/σ ratio Škoda

Fig. 85: Oil temperature Škoda

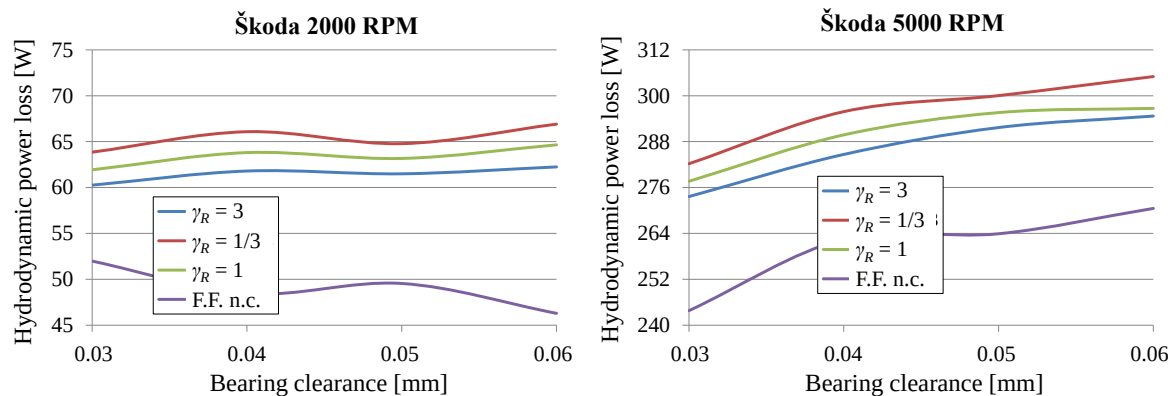


Fig. 86: Hydrodynamic power loss Škoda

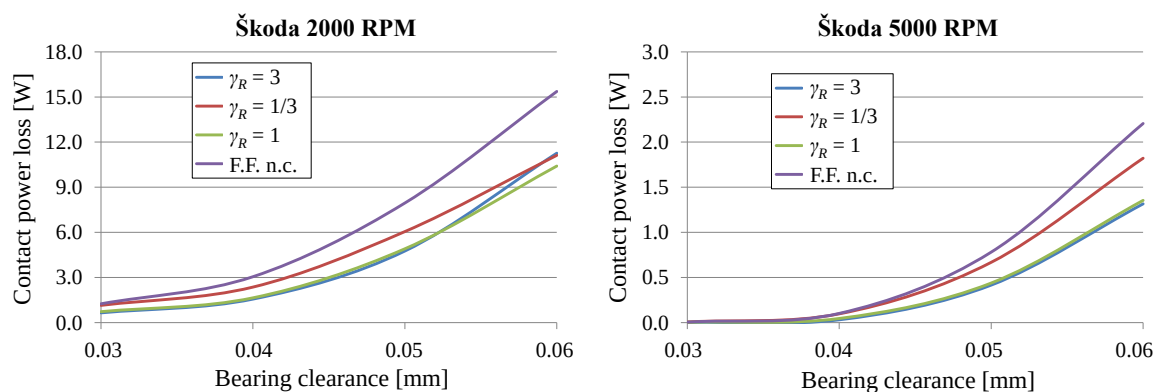


Fig. 87: Contact power loss Škoda

10.4 DISCUSSION ON THE PRESENTED RESULTS

From individual results a big difference is evident between variants where all determined factors were considered and where all determined factors were neglected.

In the case of the analysed Mercedes-Benz connection rod slide bearing and also in the case of Škoda slide bearing an expected effect on the minimal value of h/σ ratio in dependence on changed slide bearing clearance can be seen. In both cases the variant when $\gamma_R=3$ (directional dependence of surface pattern helps oil flow in bearing), shows increased values of this h/σ ratio in comparison with all other cases.

From these results also values of dry contact power loss are logically calculated. Approaches with the lowest values of h/σ ratio have logically the highest values of dry contact power loss.

Another expected result is also that the consideration of shear stress factors ϕ_{fp} and ϕ_{fs} has a relatively great influence on the hydrodynamic power loss. Calculation of hydrodynamic shear stress is given by equation (73) and not by the commonly used equation (72). In this place it is again possible to see the positive impact of that surface pattern helping the oil flow.



The shape of individual curves of oil heat up dependence is based on the information stated in the first paragraph of this chapter. The case with the lowest values of h/σ ratio (surfaces are close to each other) does not allow as good oil flow as, for example, the case when $\gamma_R=3$. Therefore there are higher oil film temperatures.

If we focus on the comparison of results of the two different engine analyses (especially from the bearing load point of view), in case of the highly loaded Mercedes-Benz combustion engine it is possible to see a significantly higher effect of all flow and shear stress factors on the results. The importance of considering all flow and shear stress factors increases together with the load of the analysed machinery parts.

From all the results presented, it is possible to immediately see the difference between cases when all factors were considered or neglected. Such a sophisticated and complex computational model can help develop not only design, but also surface topography of slide bearings and thereby develop more advanced machines, which the use of slide bearings.



11 INDUSTRIAL APPLICATION

Although it may seem unbelievable, it is still not common for combustion engine developers to use such complex computational codes for combustion engine behaviour simulation as is the one presented here. Therefore, an interest in the development of similar software tool from the side of our potential industrial partners was noticed even before the development process started.

Therefore, it can be stated that this doctoral thesis is definitely not solely academic. On the contrary – our Institute of Automotive Engineering, as an academic department, here has a role of applicator and developer of current scientific knowledge with the factual connection to the real problems from industrial sphere.

This doctoral thesis was developed in close cooperation with the company FEV GmbH, operating on the automotive market, which was interested in the concept of the impact of surface pattern on the hydrodynamic lubricating layer.

A similar computational approach may be, in fact, applied on all cases where the hydrodynamic (mixed) lubrication regime is presented. Taking into consideration trends of each factor (pressure and shear flow factors and shear stress factors), a great contribution of this complex approach can be predicted, especially in the development process of highly loaded machinery parts, working mostly under mixed lubrication regime (low values of (h/σ) ratio). Example of such machinery parts can be: compression piston rings, piston bearings, valve rocker levers, gear-wheels, etc. Further application of this approach to the other, above mentioned, machinery parts design process is planned.

The developed comprehensive approach has a potential to be applied in the industrial practice.



CONCLUSIONS

There is no doubt about the irreplaceability of slide bearings in numerous machinery applications. On the contrary – slide bearings find their application, thanks to modern materials and more sophisticated design, also in places where it was problematic or inappropriate in the past.

Perfection and credibility of the design of individual components nowadays play an important role in the development of any product as early as in the process of virtual prototyping. Therefore, a continuous improvement of computational models and their adaptation to the possibilities of contemporary computer technology are being used more and more often.

This statement is supported by the fact that automotive companies from the whole world are interested in modern trends in this area. One can say that only this strategy (the connection of the sophisticated computational models with the experimental testing) leads to the successful development in the field of modern combustion engines with higher efficiency and lower fuel consumption (as it is required from the side of ministry, environment, and consumers).

The comprehensive and generally applicable numerical model of the slide bearing has the potential to contribute to the development process of, not only, slide bearings, but also to the development process of all machineries using slide bearings.

The approach, described in this doctoral thesis, can be applied not only in the slide bearing design (optimisation) process. Its presumed application is also in the field of highly loaded machinery parts, working mainly under mixed lubrication regime. The contribution of this approach is going to be proved by these applications. This statement is supported by the results from performed analyses. Among these components may also be listed compression piston rings, piston bearings, valve rocker levers, gear-wheels, and others.

This doctoral thesis was written with the aim to describe all used numerical methods and principles in as much detail as possible and as much comprehensibly as possible. Readers of this thesis then should be able, based on the information provided, to create similar computational algorithms. This strategy goes hand in hand with the initial plan of this doctoral thesis – to inform general public about the developed approach.



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LIST OF USED SHORTCUTS AND SYMBOLS

a	[m]	radius of contact area – Hertz theory
$\mathbf{A}$	[-]	matrix of acceleration
A_1	[-]	parameter for analytical shear flow factor calculation
a_1-a_4	[-]	parameters for numerical solution of Reynolds equation
A_2	[-]	parameter for analytical shear flow factor calculation
A_3	[-]	parameter for analytical shear stress factor calculation
$AACF$	[-]	Areal Autocorrelation Function
A_e	[m ²]	area of surface roughness element
A_{el}	[m ²]	contact area for elastic behaviour of material
A_{ep}	[m ²]	contact area for elasto-plastic behaviour of material
A_p	[m ²]	contact area for plastic behaviour of material
A_R	[m ²]	sum of all micro-contact areas
B	[m]	bearing width
B_{MBS}	[N·s·m ⁻¹]	damping
C	[-]	parameter for flow factors analytical calculation
c	[m]	bearing clearance
d	[m]	diameter of bearing pin
D	[m]	diameter of bearing shell
D_f	[-]	fractal dimension
D_R	[-]	parameter for shear stress factors analytical calculation
e	[m]	bearing eccentricity
E	[Pa]	Young's modulus
e	[-]	Euler's number
E'	[Pa]	combined Young's modulus



$e_{\gamma 1}$	[m]	eccentricity of bearing pin, measured in the cut plane Z-Z and in the bearing width $B=0$
$e_{\gamma 2}$	[m]	eccentricity of bearing pin, measured in the cut plane Z-Z and in the bearing width $B=B/2$
$e_{\gamma 3}$	[m]	eccentricity of bearing pin, measured in the cut plane Z-Z and in the bearing width $B=B$
$e_{\delta 1}$	[m]	eccentricity of bearing pin, measured in the cut plane Y-Y and in the bearing width $B=0$
$e_{\delta 2}$	[m]	eccentricity of bearing pin, measured in the cut plane Y-Y and in the bearing width $B=B/2$
$e_{\delta 3}$	[m]	eccentricity of bearing pin, measured in the cut plane Y-Y and in the bearing width $B=B$
$F_{5/2}$	[-]	function of contact pressure increase
F_{c_MBS}	[N]	contact force
F_{el}	[N]	contact force for elastic behaviour of material
F_{ep}	[N]	contact force for elasto-plastic behaviour of material
F_p	[N]	contact force for plastic behaviour of material
G	[m]	fractal parameter of surface heights variations
h	[m]	nominal oil film height
H	[-]	dimensionless oil film height
H_0	[-]	nominal dimensionless oil film height
h_a	[m]	height of the surface roughness point in element key point a
h_b	[m]	height of the surface roughness point in element key point b
h_c	[m]	height of the surface roughness point in element key point c
h_d	[m]	height of the surface roughness point in element key point d
H_m	[Pa]	hardness of softer material
h_{min}	[m]	minimal oil film thickness
h_R	[m]	height of the contact



h_T	[m]	local oil film height
I	[A]	electric current
K	[-]	Greenwood & Tripp contact parameter
L	[m]	length of surface sample
l_a	[m]	length of surface element side a
l_{ac}	[m]	length of surface element diagonal
l_b	[m]	length of surface element side b
l_c	[m]	length of surface element side c
l_d	[m]	length of surface element side d
L_s	[m]	“cutt off” length
M	[-]	number of consecutive surface ridges
m_f	[-]	number of the first rough of the hydrodynamic pressure matrix
m_l	[-]	number of the last rough of the hydrodynamic pressure matrix
m_s	[-]	autocorrelation length in direction y
n	[-]	frequency index
n_f	[-]	number of the first column of the hydrodynamic pressure matrix
$n_{F5/2}$	[-]	number of values of the Greenwood & Tripp contact pressure function
n_l	[-]	number of the last column of the hydrodynamic pressure matrix
n_o	[-]	amount of tested data
n_{rs}	[-]	number of surface roughness peaks on the contact area
n_s	[-]	autocorrelation length in direction x
p	[Pa]	hydrodynamic oil film pressure
P	[-]	dimensionless pressure
P_{av}	[Pa]	average contact pressure – Hertz theory
P_c	[Pa]	contact pressure



$P_{Contact}$	[Pa]	contact pressure in slide bearing
$P_{Hydrodynamic}$	[Pa]	hydrodynamic pressure in slide bearing
P_{max}	[Pa]	maximal contact pressure – Hertz theory
p_N	[Pa]	normal hydrodynamic oil film pressure
P_{SOR}	[Pa]	hydrodynamic pressure calculated by using SOR method
P_{tilted}	[-]	dimensionless oil film pressure in the tilted state of the pin
P_{Total}	[Pa]	overall pressure in slide bearing
$P_{untilted}$	[-]	dimensionless oil film pressure in the untilted state of the pin
q_{rough_x}	[m ³ /s]	flow between rough surfaces in direction x
q_{rough_y}	[m ³ /s]	flow between rough surfaces in direction y
q_{smooth_x}	[m ³ /s]	flow between smooth surfaces in direction x
q_{smooth_y}	[m ³ /s]	flow between smooth surfaces in direction y
R	[m]	radius of the bearing shell
r	[m]	radius of the bearing pin
R'	[m]	combined radius of curvature – Hertz theory
r'	[-]	parameter for flow factors analytical calculation
$R_{1(2) \times (y)}$	[m]	radius of contact body 1(2) in direction x (y)
Ra	[m]	average roughness of 2D rough profile
Rm	[Mpa]	tensile strength of material
$Rp0,2$	[MPa]	yield point of material
R_{piston}	[Ω]	resistivity of the piston
R_{piston_liner}	[Ω]	resistivity of the piston liner
Rq	[m]	Root Mean Square of 2D rough profile
r_r	[-]	residuum for solution of hydrodynamic pressure
r_s	[m]	average radius of micro contact radius



R_T	[Ω]	resistivity between piston and piston liner
R_x	[m]	reduced radius of contact bodies in x direction
$r_{x,y,z}$	[m]	translation boundary condition for FEM analysis of rough surfaces
R_y	[m]	reduced radius of contact bodies in y direction
Sa	[m]	average roughness of 3D rough profile
S_{abc}	[m]	circumference of surface element triangle abc
S_{acd}	[m]	circumference of surface element triangle acd
Sdq	[$^\circ$]	RMS surface slope
Sdr	[-]	developed interfacial area ratio
Sds	[1/m ²]	density of surface summits
Sku	[-]	kurtosis of the 3D surface texture
Sp	[m]	maximal peak height of the surface texture
Spc	[mm ⁻²]	arithmetic mean peak curvature
Spd	[mm ⁻²]	density of peaks
Sq	[m]	Root Mean Square (RMS) of 3D rough profile
s_R	[-]	parameter for analytical shear stress factor calculation
Ssc	[1/m ²]	mean summit curvature
Ssk	[-]	skewness of the 3D surface texture
Sv	[m]	maximal valley depth of the surface texture
Svk	[m]	reduced valley depth
Sz	[m]	maximal height of surface profile
t	[s]	time
t_{ci}	[-]	observed frequency
$\tilde{t}_{ci}$	[-]	expected frequency
U	[m·s ⁻¹]	relative velocity



U_1	$[\text{m}\cdot\text{s}^{-1}]$	circumferential speed of the surface 1
U_2	$[\text{m}\cdot\text{s}^{-1}]$	circumferential speed of the surface 2
U_c	$[\text{V}]$	voltage in the contact between piston and piston liner
U_n	$[\text{m}\cdot\text{s}^{-1}]$	normal velocity of the bearing pin
U_R	$[\text{V}]$	voltage
$\bar{U}$	$[-]$	dimensionless relative velocity
$\bar{U}_n$	$[-]$	dimensionless normal velocity of the bearing pin
v_{MBS}	$[\text{m}\cdot\text{s}^{-1}]$	relative velocity of contact surfaces
W	$[\text{N}]$	contact force – Hertz theory
x	$[\text{m}]$	coordinate
X	$[-]$	dimensionless coordinate
$\mathbf{X}$	$[-]$	matrix of translations
Y	$[-]$	dimensionless coordinate
y	$[\text{m}]$	coordinate
Y-Y	$[-]$	cut plane Y-Y
z	$[\text{m}]$	coordinate
Z	$[-]$	dimensionless coordinate
Z_{Patir}	$[-]$	variable used for average local oil film thickness calculation
z_r	$[\text{m}]$	matrix of surface roughness height
Z-Z	$[-]$	cut plane Z-Z
α_1	$[-]$	parameter for analytical shear flow factor calculation
α_2	$[-]$	parameter for analytical shear flow factor calculation
α_3	$[-]$	parameter for analytical shear flow factor calculation
α_4	$[-]$	parameter for analytical shear stress factor calculation
α_5	$[-]$	parameter for analytical shear stress factor calculation



α_6	[-]	parameter for analytical shear stress factor calculation
α_f	[-]	offset parameter of line
β	[m]	average radius of curvature of rough surface asperities
β_f	[-]	angular coefficient of the line
β_N	[-]	coefficient for Newmark numerical algorithm
γ	[-]	bearing pin tilting angle measured in the plane of the minimal oil film thickness
γ_f	[-]	parameter of scale changes
γ_N	[-]	coefficient for Newmark numerical algorithm
γ_R	[-]	Peklenik number
δ	[-]	tilting angle in the cut plane Y-Y - in the plane perpendicular to the minimal oil film plane
δ_e	[m]	length of surface roughness element side
δ_R	[m]	surfaces penetration (deflection)
δ_{R1}	[m]	amplitude of surface 1 asperity, measured from the zero-level plane
δ_{R2}	[m]	amplitude of surface 2 asperity, measured from the zero-level plane
Δx	[m]	differential length of element size in x direction
Δy	[m]	differential length of element size in y direction
ε	[-]	relative eccentricity of the bearing pin
ζ	[Ω m]	resistance of the steel
η	[Pa·s]	dynamic oil viscosity
η_0	[Pa·s]	nominal dynamic oil viscosity
η_r	[m ⁻²]	surface density of asperities of rough surface – for Greenwood & Tripp theory
$\bar{\eta}$	[-]	dimensionless dynamic oil viscosity
κ_u	[-]	points lower than his eight neighbouring points
κ_v	[-]	points higher than his eight neighbouring points



$\lambda_{x0.5}$	[-]	number of differential lengths of element size in x direction to reach value of $AACF/2$
$\lambda_{y0.5}$	[-]	number of differential lengths of element size in y direction to reach value of $AACF/2$
ν	[-]	Poisson's constant
ν	[-]	Poisson's constant
ξ	[-]	cut planes specification
ξ_x	[-]	dimensionless variable for flow factor x
ξ_z	[-]	dimensionless variable for flow factor z
π	[-]	Ludolf's number
ρ	[kg·m ⁻³]	oil density
ρ_0	[kg·m ⁻³]	nominal oil density
ρ_{Jacobi}	[-]	Successive Over Relaxation parameter
$\bar{\rho}$	[-]	dimensionless oil density
σ	[m]	combined RMS of both surfaces
σ_s	[-]	standard deviation
$\bar{\sigma}$	[m]	dimensionless combined RMS of both surfaces
τ_N	[Pa]	normal oil film tension
ν	[-]	dimensionless bearing width
φ	[rad]	angle of the bearing pin rotation measured from the minimal oil film thickness plane
ϕ	[-]	random phase
ϕ_{fp}	[-]	shear stress factor p
ϕ_{fs}	[-]	shear stress factor s
ϕ_s	[-]	shear flow factor
ϕ_x	[-]	pressure flow factor in x direction



ϕ_y	[-]	pressure flow factor in y direction
χ^2	[-]	Chi-squared criterion
ψ	[-]	general angle around circumference of the bearing
ω_{c1}	[m]	criterion for fully elastic state of material
ω_{c2}	[m]	criterion for elasto-plastic state of material
ω_{c3}	[m]	criterion for fully plastic state of material
ω_{SOR}	[-]	Successive Over Relaxation parameter
DK1-DK3		shot-peened surface number 1-3
AACF		Areal Autocorrelation Function
FD		Fractal Dimension
FEM		Finite Element Method
G1-G7		grinded surface number 1-7
H1-H5		honed surface number 1-5
L1		lapped surface number 1
LS		loading state
M1-M6		measurement number 1-6
MBS		Multi Body System
SOR		Successive Over Relaxation
SP1		superfinished surface number 1
T1-T4		turned surface number 1-4
VD		surface burnished by diamond
VK		surface burnished by ceramics
ZP		Zero Plane



LIST OF ATTACHMENTS

- Attachment No. 1: Author list of publications