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Stress relaxation of concrete beams caused by creep and shrinkage effects

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Abstract. Shrinkage and creep are two important physical properties of concrete material which cause increase of the deformation of the constantly loaded structure over long period of time, the feature known as rheology. Additional deformation of concrete structures at the end of the design working period (most commonly 50 years) caused by these phenomena is circa three times larger than the value of the immediate elastic deformation (or even larger in some cases). Hence, in accordance with the corresponding European standard, these effects should be taken in account while evaluating the serviceability limit state of concrete structures, and if significant, consideration of these phenomena is also needed for the verification of the ultimate limit state. In concrete material, creep occurs at all stress levels, and is dependent on many parameters, as cement class, concrete grade, relative humidity of the environment, surface of the structure in contact with the ambient air (drying surface), and the age of concrete (after casting) at the loading moment. Shrinkage of the concrete is independent on loading. It is caused by decrease of the pore water content in the hardened concrete, and is predominantly dependent on the ambient relative humidity. Relaxation describes stress reduction at a constant material strain, usually in prestressing steel tendons. In this study, the physical experiments of multiple concrete beams over time with respect to rheological processes are described. Each experimental system consists of two C35/45 beams horizontally bounded by a prestressed steel cylinder. Decrease of these pretension forces in cylinders over time have been monitored (stress relaxation). All together time histories of two forces are documented, based on measurements conducted in interior environment of an agricultural building. The experimental time histories of the pretension forces are then compared with the results of the finite element numerical analyses conducted in ANSYS software. Creep and shrinkage effects of the concrete material have been considered based on the corresponding European standard for design of the concrete structures. The time-histories of the forces in prestressed cylinders obtained from the numerical simulations are then compared with the experimental data, and discussed. It is concluded, that the estimation of the force decrease over analysed time with the creep and shrinkage effects considered according to the corresponding European

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standard appears to be slightly larger than the experimentally measured decrease of the force value, hence the assumption is more conservative.

1 Introduction

Concrete is a material, which undergoes slowly increasing deformation over long period of time while exposed to persistent mechanical stresses. This phenomenon is known as creep, and it occurs as all stress levels. Another important effect is shrinkage, which is independent on loading, and is caused by decrease of pore water content in the hardened concrete material. These rheological ("rheology" originates from Greek "study of flow") properties are described e.g. by Bažant and Jirásek [1]. Creep and shrinkage effects should be considered during the design process of the structures. The insufficient prediction or neglecting of creep might result into structural failures, as for example in case of Roissy Airport Terminal 2E collapse in May 2004 [2] [3].

Both rheological effects (creep and shrinkage) might be estimated according to European standard for concrete structures EN 1992-1-1 [4]. In EC2 [4], creep is handled by a creep coefficient (function of time), which is dependent on many parameters, e.g. relative humidity (RH), area of concrete member in contact with air (drying surface), concrete grade, age of concrete at the loading time. Shrinkage effects are defined by a shrinkage strain [4], also function of time dependent on the same parameters as creep. For purpose of numerical finite element analysis, the creep effects might be considered e.g. in software ANSYS [5] by utilization of some creep model from its library. Creep effects of concrete structures are analyzed e.g. by Daou et al. [6]. Additional shrinkage strain might be introduced for example by calibrated temperature loading of the concrete finite elements, or by introduction of deformation loads.

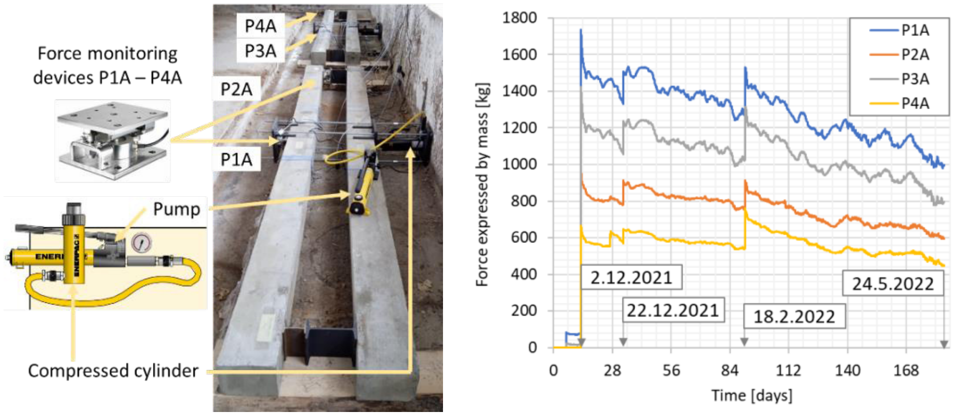
In this study, physical experiments of two sets of concrete beam couples are presented. The concrete beams are exposed to persistent forces (from pressurized cylinders), each pushing one set of the two beams towards each other. The decreases of these forces (relaxation) have been monitored over long period of time, and the data are well documented. Over time, the forces in the cylinders have been slightly increased. The time histories of relative humidity and ambient temperature are also provided (beams were located in interior of agricultural building in Czech Republic).

Additionally, numerical finite element analyses predicting the decrease of the monitored forces using ANSYS [5] and assumptions of creep and shrinkage by EC2 [4] have been conducted. The numerical models and workflows of these analyses are described, and the results are compared with the experimental data and discussed.

2 Physical model and experiments

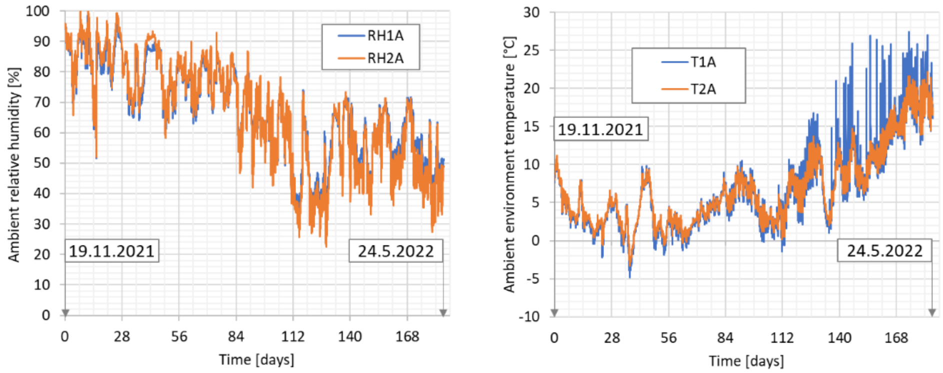
The experimental system consists of pair of concrete C35/45 beams, each of the same cross-section 150×200 mm, which are horizontally bounded by a system of steel plates, steel bars and a compressed steel cylinder (Fig. 1 a). Together experimental results from two sets of such beams are documented. The cylinders have been compressed on 2.12.2021, what is approximately 28 days after casting of the concrete beams. By pressuring of the cylinder, the force which acts in lateral direction pushing the two beams together has been introduced through the system of steel plates and bars. Force monitoring devices are placed directly into this system (Fig. 1 a – P1A, P3A), and the other monitoring devices are placed above the supports (Fig. 1 a – P2A, P4A). Time histories of these forces expressed by mass in kilograms are depicted in the Fig. 1 b. Force value is being monitored in the interval of 20 minutes over long period of time in order to investigate the rheological effects. In this study, the time interval up to 24.5.2022 is documented (Fig. 1 b). During this period, the decreasing force in the pressurized cylinder has been increased two times, 22.12.2021, and 18.2.2022. The first one has been neglected during the analysis (as it is rather small), the second one, which is larger has been taken into account also during the numerical analyses.

Rheological effects as creep and shrinkage are dependent on the temperature and relative humidity of the ambient environment. These factors were measured by two sensors each, and the time histories are graphically depicted in the Fig. 2.



(a) (b)

Fig. 1. a) Experiment set-up; b) Monitored forces expressed in mass.



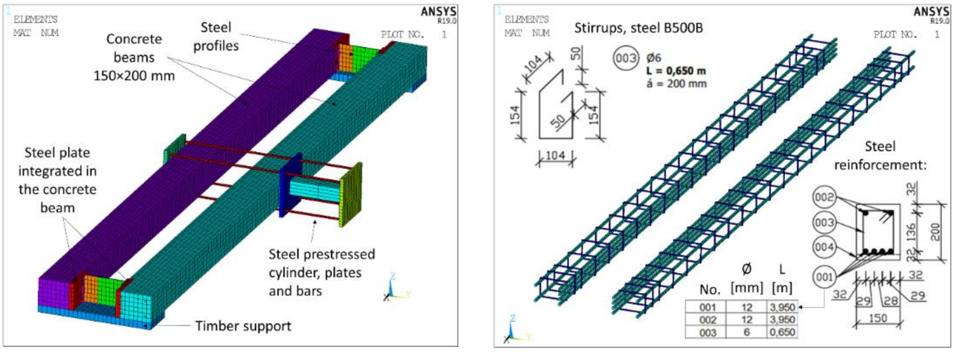
(a) (b)

Fig. 2. a) Relative humidity time history b) Ambient temperature time history.

3 Numerical analyses

3.1 Numerical finite element models

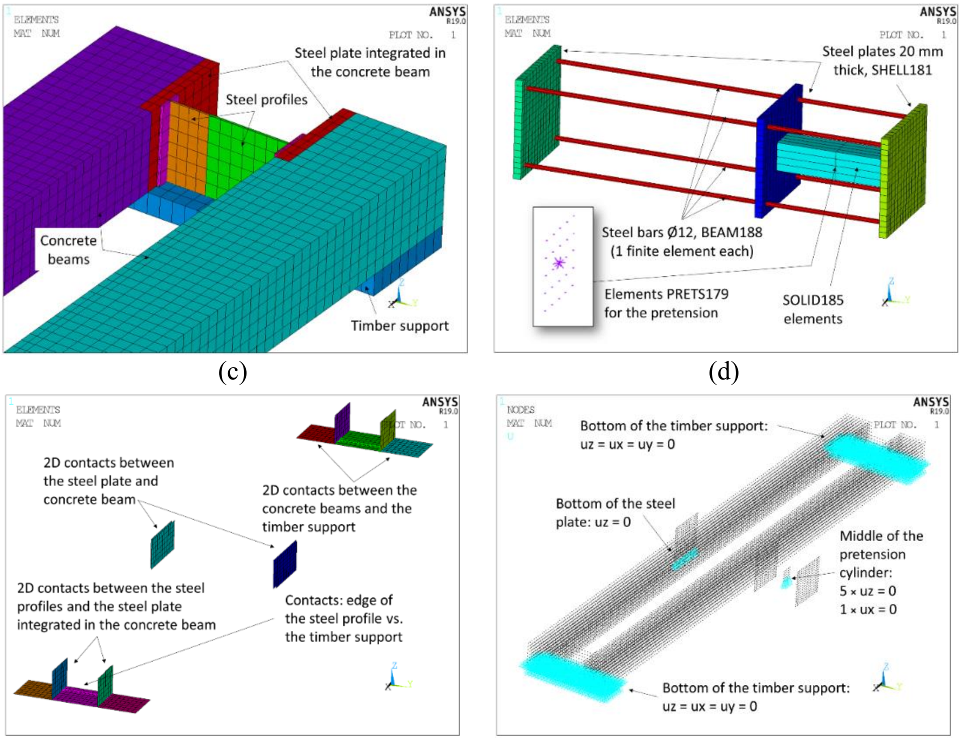
The numerical finite element model has been created in the software ANSYS [5]. Concrete beams are modelled by 3D structural SOLID185 elements, as well as the steel plates (integrated into the concrete beams) above the supports, timber supports, and the cylinder for pretension. The reinforcement has been modelled by specialized REINF184 elements. The geometry of the numerical model along with details, areas of defined contacts and boundary conditions (deformation constrains) is well documented in the Fig. 3 a – Fig. 3 f.



No.	Ø [mm]	L [m]
001	12	3,950
002	12	3,950
003	6	0,650

(a) (b)

Fig. 3. Numerical finite element model: a) Global geometry; b) Reinforcement detail.



(c) (d)

(e) (f)

Fig. 3. Numerical finite element model: c) Detail above the timber support; d) Detail of the system of loading; e) Detail of contacts; f) Nodes of numerical model and boundary conditions.

For concrete elements, linear elastic material model is used (E-modulus of 34 GPa, Poisson’s ratio 0.2), as well as for the reinforcing steel bars and all the other steel components (E = 210 GPa, Poisson’s ratio 0.3).

3.2 Rheological effects – creep

The creep effects are incorporated by the introduction of one of the implicit creep equations from ANSYS library [5] which determines the creep strain dependence. The equation dependent on time t, and four parameters noted C_1 to C_4 has been considered. The dependences on thermodynamic temperature T has been neglected by considering $C_4 = 0$, hence the equation of creep strain might be simplified into:

$$\varepsilon_{creep} = \frac{C_1 \cdot \sigma^{C_2} \cdot t^{(C_3+1)}}{C_3+1} \tag{1}$$

The creep parameters C_1 and C_3 were calibrated in order to obtain the same time history of the creep strain as is defined in accordance with EC2 [4], which defines the creep strain as:

$$\varepsilon_{creep}(t) = \varphi(t,t_0) \cdot \varepsilon = \varphi(t,t_0) \cdot \frac{\sigma}{E} \tag{2}$$

where $\varphi(t,t_0)$ is the creep coefficient determined in accordance with Annex B of EN 1992-1-1 [4], E is elastic modulus, σ is compressive stress applied at time t_0, which should not exceed the value of 0.45 f_{ck} (characteristic value of concrete compressive strength) otherwise creep non-linearity should be considered.

In order to avoid the dependence of creep coefficient $\varphi(t,t_0)$ on different stress levels σ (while combining the equations 1 and 2), the parameter C_2 has been considered as 1.0 during the parameter calibration process. Parameter values have been calibrated to C_1 = 1.94E–13 [$\mathrm{s}^{-(C3+1)} \cdot \mathrm{Pa}^{-1}$] and C_3 = –0.77497 [–] for the beam loaded in 28 days after concrete casting at relative humidity level 78.8%. For concrete beam loaded in 105 days after casting, C_1 = 1.47E–13 [$\mathrm{s}^{-(C3+1)} \cdot \mathrm{Pa}^{-1}$] and C_3 = –0.77486 [–]. The time dependence of E-modulus has been considered in accordance with CEB-FIP assumptions [7], which were utilized also in study by Singh et al. [8]:

$$E_c(t) = E_c \cdot \sqrt{exp\left\{ s \cdot \left[1 - \sqrt{\frac{28}{t/t_1}} \right] \right\}} \tag{3}$$

where E_c is the elastic modulus at the age of 28 days, t is the age of concrete after casting in days, t_1 is 1 day, s is the coefficient which depends on the type of cement aggregate, for normal hardening cement 0.25.

For each set of beams (measurements P1A+P2A and P3A+P4A in Fig. 1 b), two numerical analyses have been conducted to evaluate the creep effects. For P1A+P2A, first analysis is loading the beams by the deformation load which results in the initial force of 17.05 kN considering the parameters of 28 days old concrete. The second analysis is loading the 105 days old beams by the deformation to cause initial force of 2.43 kN. Analogically, for the P3A+P4A set of beams, the first analysis is loading of 28 days old beams by a deformation to imply the initial force of 13.96 kN, and the second analysis of 105 days old beams and 2.78 kN. In each case, the initial loading is introduced via the pretension elements PRETS179. Analogically, it is possible to introduce the initial force by defined thermal expansion coefficient and correspondingly applied thermal load. However, usually it takes 3 – 4 manual iterations in order to get the value of the initial force with desired precision, hence the PRETS179 elements are easier to use.

3.3 Rheological effects – shrinkage

In accordance with the chapter 3.1.4 (6) from EC2 [4], The shrinkage strain consists of two parts, drying shrinkage strain and autogenous shrinkage strain. The sum of these two values is the total shrinkage strain. Both parts were considered by the 3.1.4 (6) and Annex B of EN

1992-1-1 [4]. Cement class N is considered for the evaluation, mean compressive strength of the concrete as $f_{cm} = 43$ MPa, ambient relative humidity of 78.8% and the age of concrete at the beginning of drying (end of curing) as 28 days. The time history of the shrinkage strain for the cross-section of 150×200 mm is depicted in the Fig. 4. For each set of beams, one numerical analysis has been conducted considering the 28 days old concrete. For P1A+P2A, deformation load which implies the initial force of 17.05 kN, and for P3A+P4A 13.96 kN, analogically to previous cases. However, this time, there is no creep equation defined for the concrete material. In order to incorporate the total shrinkage strain (Fig. 4) into the numerical analysis, this time history of strain is considered as a thermal load (with negative sign) applied on all the concrete finite elements, while the thermal expansion coefficient is considered with the value of 1.0.

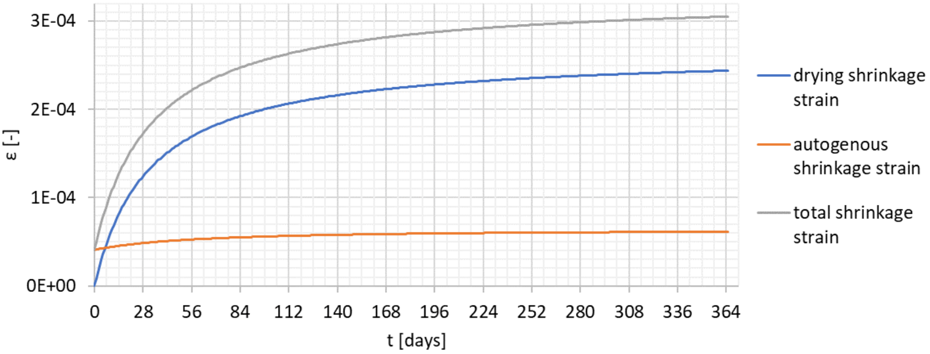


Fig. 4. Shrinkage strain.

3.4 Summary of analyses

Three independent structural analyses have been conducted for each of two sets of beams (measurements P1A+P2A and P3A+P4A in Fig. 1 b): two to evaluate the creep based on applied load in specific times, and one to evaluate the shrinkage effects, as described by the previous chapters. Summary of all 6 analyses is in the Table. 1.

Table 1. Summary of analyses.

Analysis number	Reference curve	E-modulus of concrete [GPa]	C_1 parameter [$s^{-(C3+1)}$ Pa^{-1}]	C_3 parameter [-]	Initial force [N]	Temperature load
#1	P1A	34.00	1.94E–13	–0.77497	17 049.2	No
#2	P1A	35.93	1.47E–13	–0.77486	2 425.0	No
#3	P1A	34.00	Not used	Not used	17 049.2	To fit shrinkage
#4	P3A	34.00	1.94E–13	–0.77497	13 959.6	No
#5	P3A	35.93	1.47E–13	–0.77486	2 776.2	No
#6	P3A	34.00	Not used	Not used	13 959.6	To fit shrinkage

4 Analyses results

During all of the 6 analyses, the decrease of force in the cylinder has been monitored (Fig. 5 for the reference measurement P1A and Fig. 6 for P3A). Analyses 1 and 2 (Fig. 5) are combined in the post processing in order to obtain the total time history of the decreasing force due to creep effects, based on the EC2 [4] assumptions. Analyses 4 and 5 analogically (Fig. 6). Force relaxation due to the shrinkage effects of concrete beams is evaluated in analyses 3 and 6. These results are then combined with the corresponding relaxation of the force caused by creep effects, and the final time history of force due to both rheological effects is determined (yellow curves in Fig. 5 and Fig. 6).

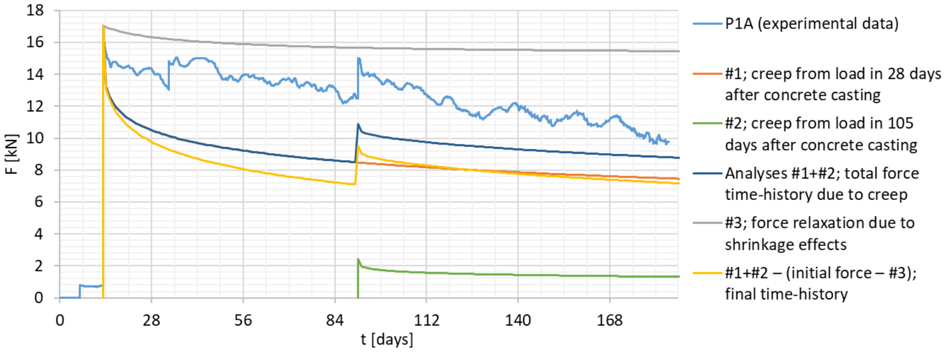


Fig. 5. Results of the first batch of analyses.

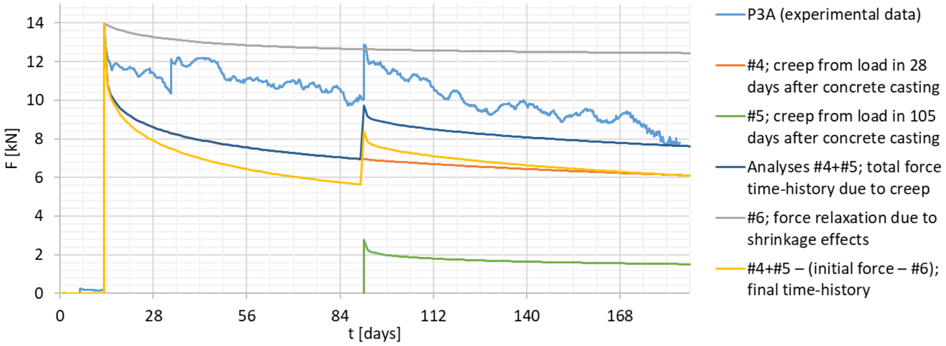


Fig. 6. Results of the second batch of analyses.

5 Discussion

For both of two sets of concrete beams, the relaxation of the force in the pressurized cylinder predicted by numerical analyses based on assumptions by EC2 [4] results in larger decrease of the force, then experimentally measured.

Although, it may be pointed out that the neglecting of the first increase of the force in the pressurized cylinder (Fig. 1 b, on time axis on 22.12.2021) would increase the force a little, so the final value depicted on the time axis (circa on 185th day) would be closer to the experiment value at that time. On the other hand, the creep effects in accordance with EC2 [4] has been determined considering the relative humidity of 78.8%, what is the average value of data up to circa 91st day (Fig. 2 a). Afterwards, the relative humidity is smaller, what would cause slightly larger creep. Analogically with the shrinkage effects. The largest portion of creep effects occurs at the beginning of the time interval, hence this simplification of considering the relative humidity at level of 78.8% might be applicable within the desired precision of the numerical analyses. The influence of temperature changes (Fig. 2 b) on creep and shrinkage have not been considered yet, which might slightly change the assumptions. These temperature effects might be introduced by incorporating the equations B.9 and B.10 from the Annex B of EN 1992-1-1 [4]. The influence of the different temperatures would be based on the averaged values from the whole analyzed time period, hence the local increases and decreases of force will not be captured. Hence, the most important is to compare the force from experiment with the force based on numerical analysis at the end of the analyzed time period.

Considering the values at the end of the analyzed period, the relative difference between numerical analysis and the experiment is approximately 20% (8 kN / 10 kN – Fig. 5) to 25% (6 kN / 8 kN – Fig. 6). This might be considered as a rather nice match, as the results are dependent on many factors (relative humidity, temperature, concrete material parameters) with noticeable variation. During the subsequent ongoing research, another analysis might be conducted with more precise consideration of the relative humidity, not neglecting the little increase of the force, taking into account the temperature influence, which is expected to be more precise in general (within the assumptions of the EC2). The comparison with experimental data might be done over larger period of time as well.

6 Conclusion

In this study, experimental data and numerical finite element analyses of two sets of concrete beam couple with horizontally bounded beams by a system of steel plates and bars are presented. The beams have been being pushed together in horizontal direction by a force in a pressurized cylinder. Decrease of the pretension force in the cylinder (Stress relaxation) has been monitored over longer period of time in order to evaluate the influences of rheological effects or concrete (creep and shrinkage). During the monitored time period, the decreasing force in cylinder was increased by additional pressurizing. The experimental data are well documented.

The finite element numerical analyses have been conducted in software ANSYS. Linear elastic material model for concrete has been used in combination with creep model (in ANSYS library noted as “modified time hardening”). The parameters of this creep model have been previously calibrated, so the response is in alignment with the assumptions of creep by EN 1992-1-1, over the considered time of the analysis.

For each set of concrete beam couple, three numerical analyses have been conducted, two to evaluate creep based on introduced force at certain point in time, and one to evaluate the effects of shrinkage. Together 6 numerical analyses have been conducted, and the results are processed in order to obtain the time history of the decreasing force. Results of analyses are compared with the experimental data, and are discussed. Overall, taking into account simplified assumptions, as neglecting the first little increase of force during the measurement, neglecting temperature dependence of creep as proposed by EC2, the results based on numerical analyses and experiments are in rather nice match (20 – 25% difference), while the assumptions by EC2 appears to be more conservative.

The paper presented workflow of the analysis process, which might be improved during the subsequent ongoing research, e.g. introduction of the temperature dependence, different approach in averaging the relative humidity, considering also the little so far neglected increase of the force.

Acknowledgments

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