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## Focusing on Problem of Lateral Torsional Buckling of Beams with Web Holes

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### Abstract

This paper deals with the problems of analysis and the design of perforated thin-walled steel Sigma beams with respect to lateral torsional buckling (further LTB). The application of Sigma profiles is in the storage systems and expedient flooring structures as floor girders. In some cases, the beams are not laterally restraint and therefore the effect of lateral torsional buckling may occur. First the theoretical calculations according to the design procedure listed in European standard EN 1993-1-1 and Czech national annex NB.3 are being performed. These calculations are valid for the beams with the uniform cross-section only without any rules for the beams weakened by holes. Therefore the substitute cross-section is being established and the cross-section characteristics are defined as the weighted average of the characteristics for the full section and for the section weakened by holes. The calculated elastic critical moments for the substitute cross-section properties are verified by the FEM calculations on the beam with variable cross-section. This theoretical approach is verified by testing in laboratory conditions. The experiments are being carried out on the specimens of three different beam lengths. The test results are statistically evaluated according to EN 1090 and the characteristics bending resistances obtained from tests are compared with the values calculated according to EN 1993-1-1 and Czech national annex NB.3 using the cross-section characteristics of substitute cross-section.

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## 1. Introduction

The Sigma beams are mono-symmetric profiles loaded in the plane perpendicular to the axis of symmetry. The application of the Sigma beams is in the built-in floors in warehouses as the floor girders. In the structure the floor girders are bolted through the truss plate to the columns, which support the floor. In some cases, the floor structure is covered by the steel grids which are not connected with the floor girders (the beams are not restrained to LTB). The Sigma beams are weakened by the web holes of diameter 65 mm presented at the position of the neutral axis which are used as grommets for utility networks.

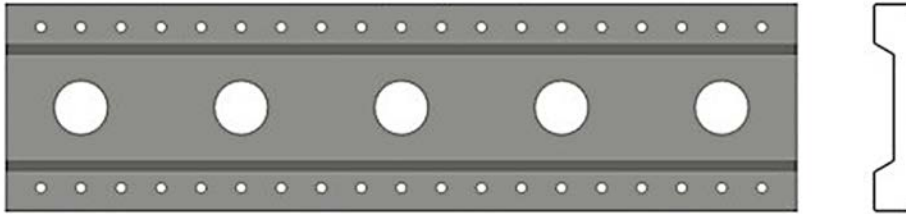


Fig. 1. Sigma beams with web holes.

## 2. Calculation according to Eurocode

The bending resistance calculation of the beams with respect to LTB is listed in the Czech national annex NB.3 of EN 1993-1-1 (further EC calculation or EC design procedure [1]). The procedure in this national annex is valid for at least mono-symmetrical cross-sections loaded in the plane perpendicular to the axis of symmetry (for the examples of cross-sections see Figure 2). It is assumed, that the transverse load through the shear center [2,3]. In the case of the beam of uniform cross-section, which is symmetrical about the major or the minor axis, the Czech national annex NB.3 provides general formulas for calculation of elastic critical moment  $M_{cr}$  for LTB in bending about axis  $y$ - $y$ .

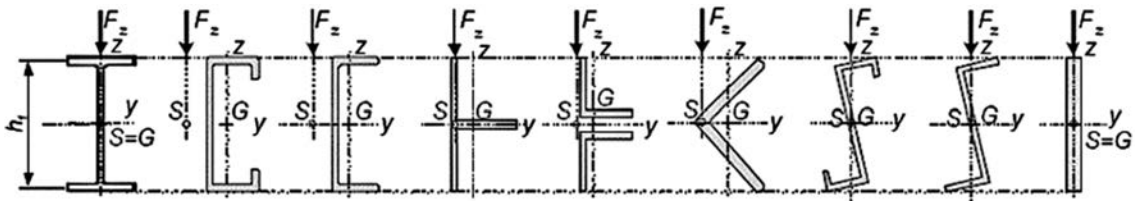


Fig. 2. Examples of cross-sections.

### 2.1. Cross-section properties

For the calculation of the bending resistances of Sigma beams according to EC design procedures the “substitute” cross-section of the beam is being established. The substitute cross-section properties are defined as the weighted average of the cross-section properties of the full section (section A in Figure 3) and the weakened section (section B in Figure 3).

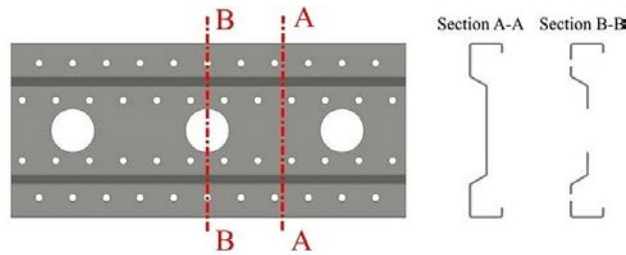


Fig. 3. Designated sections for calculation substitute cross-section properties.

2.2. Calculated resistances

Four different load cases are considered: continuous load, pointload in the middle of the span, two pointloads in the third of the span [4] and beam loaded with the end moments (see Figure 4).

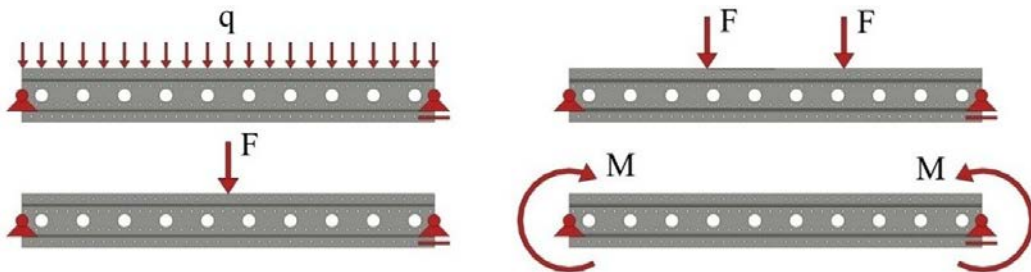


Fig. 4. Considered load schemes.

In the Table 1 are listed the characteristic bending resistances with respect to lateral torsional buckling determined for beam lengths 2, 3 and 4 meters according to EN 1993-1-1 and Czech National Annex NB.3 using the cross-section properties of substitute cross-section. The bending resistances are determined for buckling curve b, which is recommended by EN 1993-1-3 for thin-walled profiles.

Table 1. Calculated bending resistances.

Supports in torsion	Load	Beam length L	$L_{SUPP}$ [mm]	$k_z$	$k_w$	$C_1$	$C_2$	$\kappa_{wt}$	$\zeta_g$	$\zeta_j$	$\mu_{cr}$	$M_{cr}$ [kNm]	$M_{el,Rd}$ [kNm]	$\bar{\lambda}_{LT}$	$\chi_{LT}$	$M_{b,Rd}$ [kNm]
Freely supported in torsion (free warping at both ends)	Continuous load	2 m	1 960	1,00	1,00	1,13	0,46	3,65	3,42	0,00	2,86	11,35	22,61	1,41	0,38	<b>8,52</b>
		3 m	2 960	1,00	1,00	1,13	0,46	2,42	2,27	0,00	2,01	5,28	22,61	2,07	0,20	<b>4,45</b>
		4 m	3 960	1,00	1,00	1,13	0,46	1,81	1,69	0,00	1,61	3,18	22,61	2,67	0,12	<b>2,80</b>
	Pointload in L/2	2 m	1 960	1,00	1,00	1,36	0,55	3,65	3,42	0,00	3,19	12,68	22,61	1,34	0,41	<b>9,28</b>
		3 m	2 960	1,00	1,00	1,36	0,55	2,42	2,27	0,00	2,25	5,91	22,61	1,96	0,22	<b>4,93</b>
		4 m	3 960	1,00	1,00	1,36	0,55	1,81	1,69	0,00	1,82	3,57	22,61	2,52	0,14	<b>3,12</b>
	2 Pointloads in L/3	2 m	1 960	1,00	1,00	1,10	0,51	3,65	3,42	0,00	2,67	10,63	22,61	1,46	0,36	<b>8,09</b>
		3 m	2 960	1,00	1,00	1,10	0,51	2,42	2,27	0,00	1,88	4,95	22,61	2,14	0,19	<b>4,21</b>
		4 m	3 960	1,00	1,00	1,10	0,51	1,81	1,69	0,00	1,52	2,98	22,61	2,75	0,12	<b>2,64</b>
	End moments	2 m	1 960	1,00	1,00	1,00	0,00	3,65	3,42	0,00	3,79	15,05	22,61	1,23	0,46	<b>10,50</b>
		3 m	2 960	1,00	1,00	1,00	0,00	2,42	2,27	0,00	2,62	6,89	22,61	1,81	0,25	<b>5,63</b>
		4 m	3 960	1,00	1,00	1,00	0,00	1,81	1,69	0,00	2,07	4,06	22,61	2,36	0,16	<b>3,51</b>

### 2.3. Verification of substitute cross-section by FEM calculations

For the verification of the theoretical model based on the substitute cross-section the LTB analyses are being performed in the FEM software developed by J. Brodniansky from STU Bratislava [5]. This software is based on the solutions of differential equations describing the loss of stability during LTB in elastic range and allows the analysis on the members with variable cross-section. Therefore the software is suitable also for the calculations of the beams with the web holes. In the Table 2 are listed the elastic critical moments calculated for the full section, the weakened section, the substitute section and the variable section (parts between the web holes are modelled with full section and parts over the width of hole are modelled with weakened section).

Table 2. Calculated bending resistances.

Torsion supports	Load	Beam length L	$L_{SUPP}$ [mm]	Elastic critical moment $M_{cr}$ [kNm]					Difference *)
				EC Substitute section	Full section	Weakened section	Substitute section	Variable section	
Freely supported in torsion ( free warping at both ends )	Continuous load	2 m	1 960	11,35	11,72	10,68	11,38	11,36	-0,1%
		3 m	2 960	5,28	5,46	4,93	5,29	5,28	-0,1%
		4 m	3 960	3,18	3,30	2,65	3,18	3,18	-0,1%
	Pointload in L/2	2 m	1 960	12,68	12,98	11,85	12,61	12,60	0,7%
		3 m	2 960	5,91	6,07	5,49	5,88	5,87	0,8%
		4 m	3 960	3,57	3,67	3,29	3,55	3,54	0,8%
	2 Pointloads in L/3	2 m	1 960	10,63	10,96	10,00	10,65	10,61	0,2%
		3 m	2 960	4,95	5,12	4,62	4,96	4,94	0,1%
		4 m	3 960	2,98	3,09	2,76	2,99	2,98	0,3%
	End moments	2 m	1 960	15,05	15,53	14,05	15,05	15,02	0,2%
		3 m	2 960	6,89	7,12	6,40	6,89	6,87	0,2%
			4 m	3 960	4,06	4,21	3,76	4,06	4,05

Calculation for the substitute cross-section acc. to ČSN EN 1993-1-1 and Czech National Annex NB.3

Results of LTB analysis determined based on the FEM software for member with uniform cross-section

Results of LTB analysis determined based on the FEM software for member with variable cross-section

\*) Differences between values determined by LTB analysis performed in FEM software for member with variable cross-section and values determined by Eurocode calculations considering the cross-section properties of substitute cross-section.

From the results comparison of EC calculations using the substitute cross-section properties and LTB analyses performed in FEM software with variable cross-section follow negligible differences which verify the theoretical model with substitute cross-section.

### 3. Experimental verification

The beams of the lengths 2m (LB2), 3m (LB3) and 4m (LB4) were tested [6]. Simply supported specimens were loaded with two concentrated loads in the third of the span (see Figure 5). The beams are bended upwards (the top flange is under tension, bottom flange is under compression). The tension force from electrohydraulic cylinder is being spread via stiff member (couple of I profiles) into the loading frames situated in the third of the beam span.

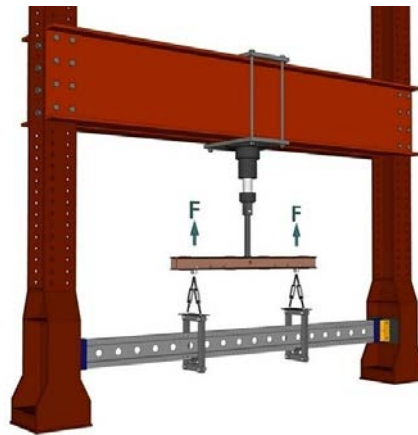


Fig. 5. Test setup visualization.

The load is introduced into the specimen via edge pressing on the U case mounted by means of adjustable bolts to the bottom flange of tested profile. The adjustable bolts allow set the position of the edge horizontally aligned with the position of shear center (Figure 6 right). In total 6 tests were performed (for each beam length 2 tests). The vertical deflection and the rotation of the section were continuously monitored in dependence on actual bending moment in the middle of the beam span (see figure 7 left).

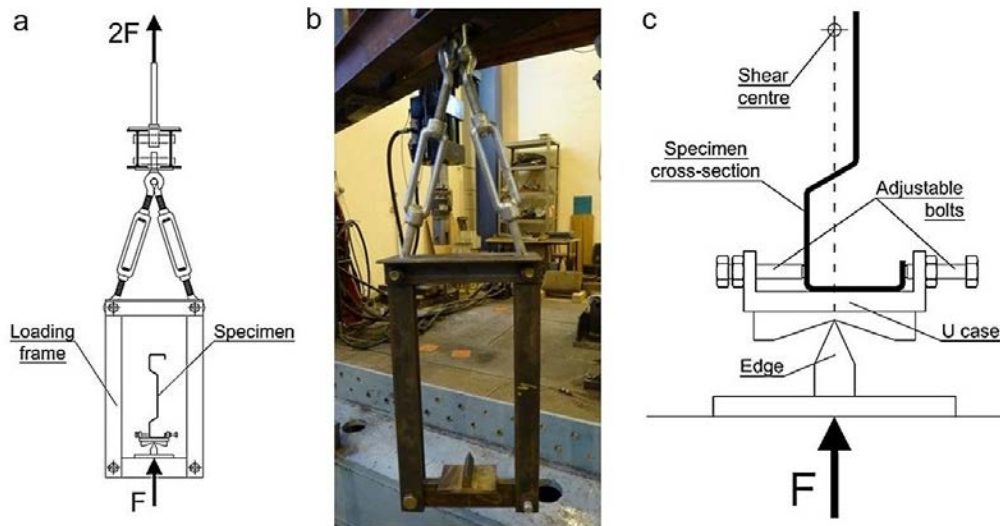


Fig. 6. Loading frame: (a) scheme; (b) realization; (c) detail U case.

The loading was ended when the beam achieved excessive lateral and torsional deflections, just before the contact of deformed specimen with the loading frame (see figure 7 right). The Graph 1 summarized the comparison of experimentally verified ultimate bending resistances of the perforated Sigma beams with resistance calculated according to EC design procedures using the substitute cross-section properties. This comparison is performed in the form of graph with relation between the relative slenderness  $\bar{\lambda}$  and reduction factor  $\chi_{LT}$  due to the lateral torsional effect. The EC resistance is represented by buckling curves defined for 3 different imperfection factors  $\alpha$  - for buckling

curve b (0,34) , c (0,49) and d (0,76). Experimentally verified values are statistically evaluated for each beam length according to EN 1990 – Annex D. The characteristic values (determined by means of the statistical evaluation of these three identical tests for each beam length) are plotted in the graph 1. From the comparison follows that the experimentally verified ultimate resistances and their characteristic values determined by the statistical evaluation are in full compliance with the resistances calculated acc. Eurocode design procedures considering substitute cross-section and buckling curve b, which is recommended for thin-walled sections.

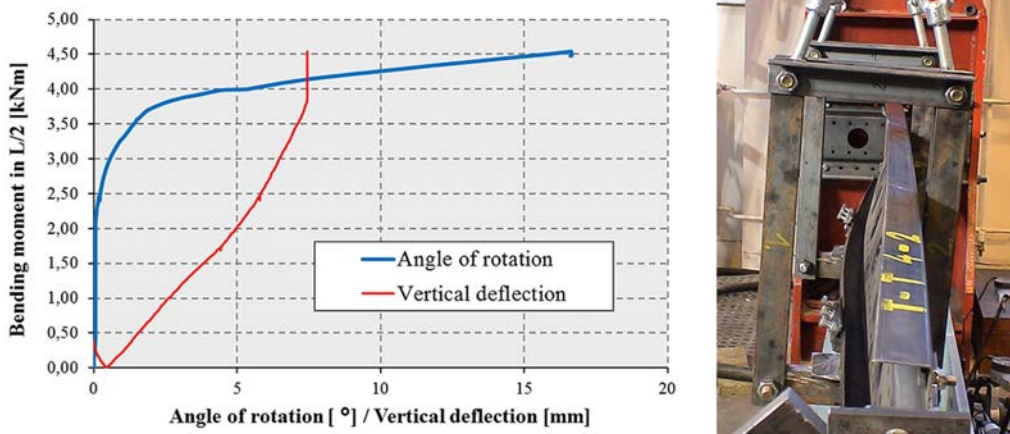


Fig. 7. Beam deformations in the middle of beam span.

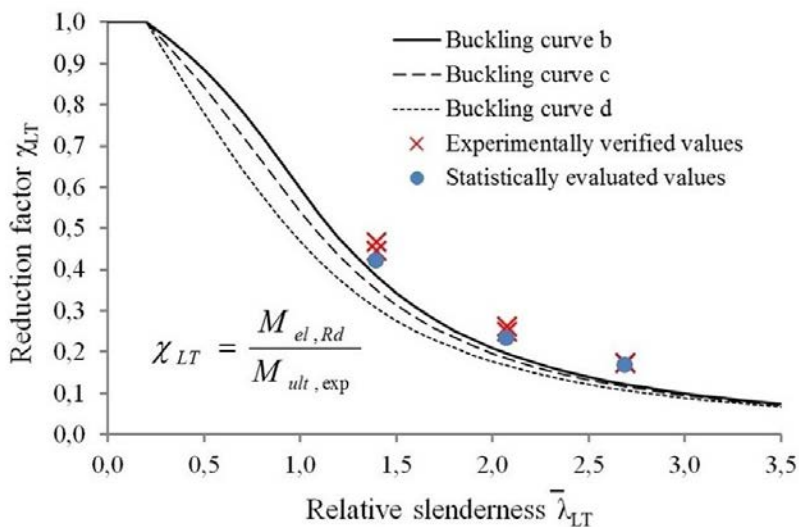


Fig. 8. Comparison of test results with EC calculations.

#### 4. Conclusions

This paper summarizes the determination of bending resistances of thin-walled steel beams with web holes with respect to LTB. The calculation of such resistance according to the design procedure listed in European standard EN 1993-1-1 and Czech national annex NB.3 using the substitute cross-section properties is verified by the FEM

calculations and by testing in laboratory conditions. The FEM calculation conforms the precise approximation of beam with web holes by the member with the substitute cross-section, which is defined as a weighted average of its cross-section properties of the full section and weakened section. The results of the experimental verification show reliable design of perforated beams based on the EC design procedures considering the substitute cross-section and buckling curve b, which is recommended for the thin-walled section.

### Acknowledgements

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