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Gravitation

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MINISTRY OF EDUCATION, YOUTH AND SPORTS

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Gravitation

Gravitation is a fundamental force of nature that affects the motion of celestial bodies.

Here we will describe the definitions and key concepts related to gravitation, Newton's law of gravitation, and the motions of bodies in the Earth's gravitational field.

Gravitation is a natural phenomenon that manifests itself as the mutual attraction (interaction) of all objects that have mass or energy.

It is one of the four fundamental interactions in physics (along with the electromagnetic, strong nuclear and weak nuclear forces). Although gravitation is the weakest of the fundamental forces, it has a dominant influence over long distances and structures, such as planets, stars, galaxies, and entire universes.

Gravitation

Definition: Gravitation is the universal force of attraction between bodies, described by Newton's law of gravitation, which states that force is directly proportional to the product of masses and inversely proportional to the square of the distance between them.

The properties of gravitational forces were clarified by Isaac Newton. Based on the study of the motion of the Moon around the Earth and the motion of the planets around the Sun, in 1687 he came to the expression of the general law of gravitation, which is one of the most important laws of physics.

Newton's law of gravitation

Newton's law of gravitation is: Every two bodies attract each other by the gravitational force of F_g, the size of which depends on the mass of both bodies and the square of the distance between their centers.

$$F_g = \kappa \frac{m_1 m_2}{r^2},$$

where $\kappa = 6{,}67 \cdot 10^{-11}$ N.m^2.kg^{-2} (or m^3.kg^{-1}. s^{-2}) is the gravitational constant,

m_1 a m_2 are the masses of two bodies,

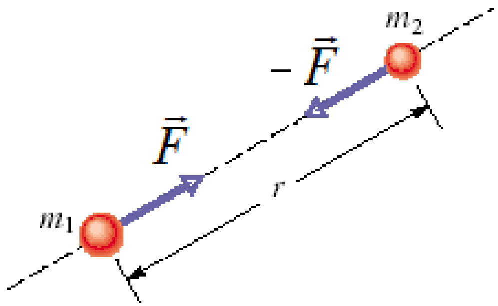
r is the distance between the centers of these objects.

Newton's law of gravitation

Versatility: The law applies to all bodies and explains why an apple falls to the Earth and why the Earth revolves around the Sun.

Effects: The gravitational force is directed towards the center of the Earth.

The gravitational force is always **two-sided** – a falling stone is attracted to the Earth by a force of the same magnitude, but of opposite orientation as the Earth is attracted to the stone.

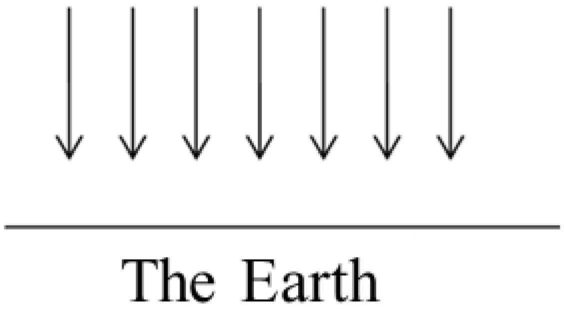


Earth's gravitational field

The Earth's gravitational field is the region around the Earth in which the gravitational force acts. Most of the motions in our surroundings, from a falling apple to the orbit of the Moon, are controlled by this field.

The Earth's gravitational field is central (radially directed towards the center of the Earth) and its intensity decreases with increasing altitude, and thus the gravitational acceleration decreases.

Near the Earth's surface, the gravitational field is almost uniform (homogeneous).



Weight

Weight F_G or G is the force that acts on bodies on the Earth's surface:

$$F_G = m \cdot g,$$

where g is the free-fall acceleration and m is the mass of the body.

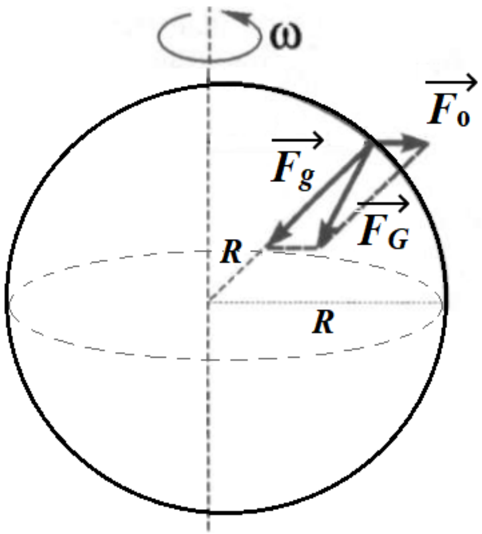
The weight is the resultant of the gravitational force of the Earth and the centrifugal force produced by the rotation of the Earth around its axis

$$\overrightarrow{F_G} = \overrightarrow{F_g} + \overrightarrow{F_o}.$$

The difference between weight and gravitational force is not very large and can be neglected in ordinary cases.

Weight

The gravitational force is directed towards the center of the Earth, the weight in most cases is directed slightly outside the center of the Earth. However, the weight determines the direction that we perceive as perpendicular to the Earth's surface (vertical).



$\overrightarrow{F_g}$ is the gravitational force,

$\overrightarrow{F_o}$ is the centrifugal force that arises when the Earth rotates around its axis with a constant angular velocity ω,

$\overrightarrow{F_G}$ is the weight.

R is the radius of the Earth.

Weight

The weight vector is used to determine the vertical direction. The weight gives all bodies in the system connected to the Earth's surface a free-fall acceleration *g*, i.e. the acceleration of free fall at a given location.

The *g*-value varies slightly depending on latitude and altitude. The smallest free-fall acceleration is in the region of the equator and is greatest at the geographic poles.

In Czech latitudes (49° to 51°), the value of free-fall acceleration on Earth is approximately 9,81 $m.s^{-2}$.

Examples

1. Calculate the magnitude of force that two mass points, each of mass of 100 g, will exert on each other, 50 cm apart.

Solution:

At the beginning, we need to convert the units to the basic units of the SI system:

body masses $m_1 = m_2 = 100\ \text{g} = 100 \cdot 10^{-3}\ \text{kg} = 0{,}1\ \text{kg}$,

distance $r = 50\ \text{cm} = 50 \cdot 10^{-2}\ \text{m} = 0{,}5\ \text{m}$.

Examples

To calculate the force, we use Newton's law of gravitation:

$$F_g = \kappa \frac{m_1 m_2}{r^2}.$$

We substitute the specified quantities and the gravitational constant:

$$F_g = \kappa \frac{m_1 m_2}{r^2} = 6{,}67 \cdot 10^{-11} \frac{0{,}1 \cdot 0{,}1}{0{,}5^2} = 2{,}668 \cdot 10^{-12} \text{ (N)}.$$

Answer: The magnitude of the force exerted by two material points on each other is $2{,}668 \cdot 10^{-12}$ N.

Examples

2. A stone of mass of 250 g is found on the surface of the Earth. Determine their mutual force effect.

Solution:

At the beginning, we need to convert the units to the basic units of the SI system:

stone mass $m_1 = 250\ \mathrm{g} = 250 \cdot 10^{-3}\ \mathrm{kg} = 0{,}25\ \mathrm{kg}$.

The distance r is the radius of the Earth (we use the tabular quantity)

$r = R_Z = 6378\ \mathrm{km} = 6378 \cdot 10^3\ \mathrm{m}$.

The mass of the Earth is also a tabular quantity $M_Z = 5{,}98 \cdot 10^{24}\ \mathrm{kg}$.

Examples

According to Newton's law of gravitation, the magnitude of force is:

$$F_g = \kappa \frac{m_1 m_2}{r^2} = 6{,}67 \cdot 10^{-11} \frac{0{,}25 \cdot 5{,}98 \cdot 10^{24}}{(6378 \cdot 10^3)^2} = 2{,}45 \text{ (N).}$$

By this force, therefore, the stone draws up the Earth as strongly as the Earth pulls down the stone.

Answer: The magnitude of the force exerted by the stone and the Earth on each other is 2,45 N.

Unsolved examples

3. The body is located on the surface of the Earth and acts on it with a force of 35 N. Calculate the mass of the body. Assume that the radius of the Earth and the mass of the Earth are $R_Z = 6378$ km, $M_Z = 5{,}98 \cdot 10^{24}$ kg.

[3,57 kg]

4. Two bodies of mass of 5 kg and 7 kg exert a force of 0.12 N on each other. Determine the distance between the bodies.

[$1{,}39 \cdot 10^4$ m]

5. At what height above the surface of the Earth will a body of mass of 1 kg be located if the force acting between the body and the Earth is 10 N?

[6315583,9 m]

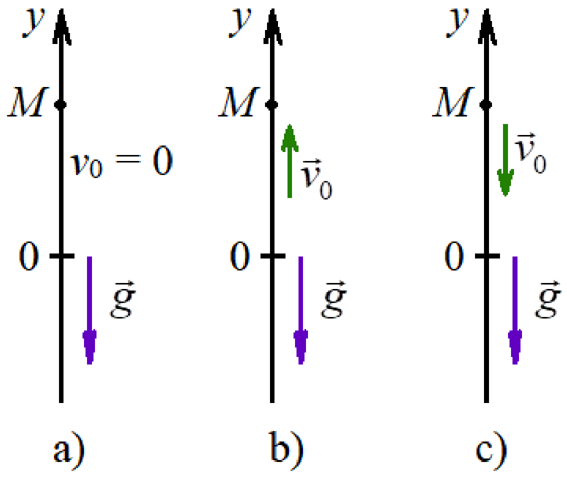
Motions of bodies in the Earth's gravitational field

Distribution of motions according to the initial velocity vector:

a) **Free fall** ($v_0 = 0$, $a = -g$),

b) **Vertical throw upwards** ($v_0 > 0$, $a = -g$),

c) **Vertical throw downwards** ($v_0 < 0$, $a = -g$).



Motions of bodies in the Earth's gravitational field

Each of the motions can be described using the equations given in the following table.

Motion	Acceleration	Velocity	Position
Free fall	$a = -g = \text{const.}$	$v_0 = 0,$ $v = -gt$	$y = y_0 - \frac{1}{2}gt^2$
Vertical throw upwards	$a = -g = \text{const.}$	$v_0 > 0,$ $v = v_0 - gt$	$y = y_0 + v_0 t - \frac{1}{2}gt^2$
Vertical throw downwards	$a = -g = \text{const.}$	$v_0 < 0,$ $v = -v_0 - gt$	$y = y_0 - v_0 t - \frac{1}{2}gt^2$

Motions of bodies in the Earth's gravitational field

Projectile motion is a motion with an initial velocity v_0 and an elevation angle of α.

In general, we do not consider the resistance of the environment.

The result of the motion is a curvilinear trajectory, which is called a parabola.

The principle of superposition applies:

- in the x-axis: uniform movement with constant velocity $v_x = v_{0x}$,
- in the y-axis: uniformly accelerated motion with initial velocity v_{0y} and acceleration $a_y = -g$.

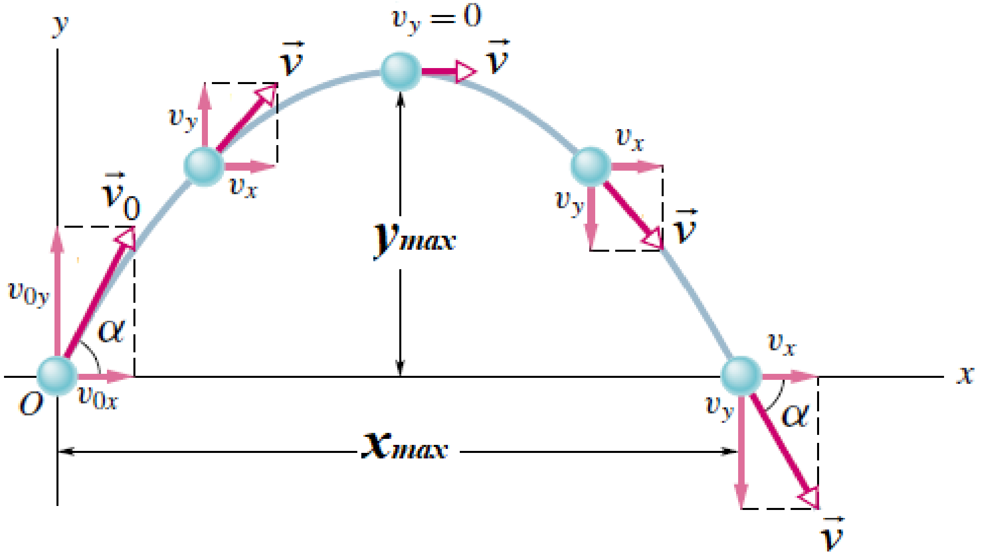
Motions of bodies in the Earth's gravitational field

Thus, the equations for a projectile motion have the form:

Acceleration $a_x = 0,$ $a_y = -g.$

Velocity $v_x = v_0 \cos\alpha,$ $v_y = v_0 \sin\alpha - gt.$

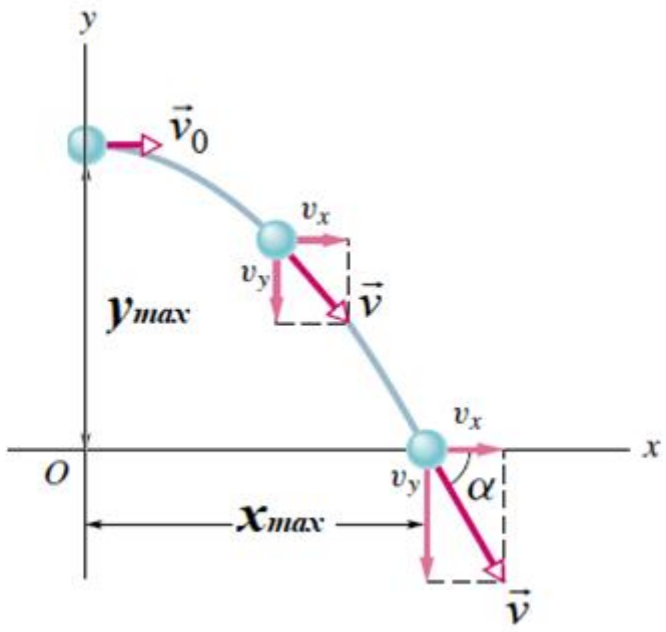
Position $x = x_0 + v_0 \cos\alpha\, t,$ $y = y_0 + v_0 \sin\alpha\, t - \frac{1}{2} g t^2.$



Motions of bodies in the Earth's gravitational field

Horizontal throw – It is solved in the same way as a projectile motion with an elevation angle of 0.

Sometimes it is advantageous to orient the vertical *y*-axis downwards.



Equations for horizontal throw:

Acceleration $a_x = 0, a_y = -g.$

Velocity $v_x = v_0, v_y = -gt.$

Position $x = x_0 + v_0 t,$

$$y = y_0 - \frac{1}{2} g t^2.$$

Summary Table

Motion	Acceleration	Velocity	Position
Free fall	$a = -g =$ const.	$v_0 = 0, v = -gt$	$y = y_0 - \frac{1}{2}gt^2$
Vertical throw upwards	$a = -g =$ const.	$v_0 > 0,$ $v = v_0 - gt$	$y = y_0 + v_0 t - \frac{1}{2}gt^2$
Vertical throw downwards	$a = -g =$ const.	$v_0 < 0,$ $v = -v_0 - gt$	$y = y_0 - v_0 t - \frac{1}{2}gt^2$
Horizontal throw	$a_x = 0,$ $a_y = -g$	$v_x = v_0,$ $v_y = -gt$	$x = x_0 + v_0 t,$ $y = y_0 - \frac{1}{2}gt^2$
Projectile motion	$a_x = 0,$ $a_y = -g$	$v_x = v_0 \cos\theta,$ $v_y = v_0 \sin\theta - gt$	$x = x_0 + v_0 t \cos\theta,$ $y = y_0 + v_0 t \sin\theta - \frac{1}{2}gt^2$

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Thank you for your attention

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