# EFFECTS OF APRIORY GIVEN HOMOGENOUS COORDINATES NULLSPACE CONSTRAINTS ON SLAM CONVERGENCE

**Jan Klečka**

Doctoral Degree Programme (3), FEEC BUT

E-mail: xkleck01@stud.feec.vutbr.cz

Supervised by: Karel Horák

E-mail: horak@feec.vutbr.cz

**Abstract**: The paper aims at the topic of SLAM (Simultaneous Localization and Mapping) algorithms convergence. Standard SLAM algorithms process observations composed of elements which are considered to be independent of each other. Following pages deals with changes that occur on convergence by the apriory assumption that some or all observation elements are bound by some mathematical model. In this case, the considered models are given by nullspace constraints in homogenous coordinates – in other words observed points lies on the same beforehand unknown line or plane. All described experiments are just simulations.

**Keywords**: SLAM, maximum likelihood, apriory gave constraints

## 1 INTRODUCTION

Simultaneous Localization and Mapping (SLAM) algorithms are an approximately three-decades-old scientific topic which has originally sprung from demands of mobile robotics. The basic concept is that observer i.e. robot moves and periodically makes observations of surrounding environment in order to get its position and virtual model of explored environment i.e. map. SLAM algorithms are then the methods used to the recurrent processing of environment observations into position and map. The methods recurrence is essential for algorithms real-time feasibility. [1]

Today's state-of-the-art SLAM algorithms are narrowly focused on single sensor type and based on a number of different heuristics. Efford I give to my research is on the other hand directed to the development of general SLAM algorithm framework or approach. Generalization leads me to exact statistical methods which are, however, computationally demanding. One thing I want to utilize to lower this demands is based on composing maps from low-level geometrical models (lines, planes, etc.). That means I want to process observations which are from beginning overdefined and I was not sure what effect this will have on SLAM algorithm so I realized this experiment.

## 2 MATHEMATICAL BACKGROUND

Let’s have a generally nonlinear observation model:

$$\mathbf{z}_i = \mathrm{h}(\mathbf{x}_i, \mathbf{m}) + \mathbf{v} \tag{1}$$

where $\mathbf{x}_i$ is observers state vector i.e. information about observers position and orientation in observation number $i$, $\mathbf{m}$ is vector representation of the environment and $\mathbf{v}$ is a stochastic variable which represents observation noise.

SLAM problem can then be defined as:

$$\begin{pmatrix} \hat{\mathbf{x}}_i \\ \hat{\mathbf{m}}_i \end{pmatrix} = \mathrm{f}(\mathbf{z}_i, \hat{\mathbf{m}}_{i-1}) \tag{2}$$

where $\hat{\mathbf{x}}_i$ is an estimate of $\mathbf{x}_i$ and $\hat{\mathbf{m}}_i$ is an estimate of $\mathbf{m}$ in step $i$.

Because of the aim of this experiment, let's in addition consider, it exists some set of homogenous equations which constraints $\mathbf{m}$:

$$\mathbf{C}\begin{pmatrix}\mathbf{m}\\1\end{pmatrix}=\mathbf{0} \tag{3}$$

Where $\mathbf{C}$ is constraints matrix which has apriory known shape, however some or all values unknown.

And so $\mathbf{m}\notin R^{\dim(\mathbf{m})}$ but it can be defined parameter vector $\mathbf{p}\in R^{\dim(\mathbf{p})}$ ($\dim(\mathbf{p})<\dim(\mathbf{m})$) and function $\mathrm{g}(\cdot)$ such that:

$$\mathbf{m}=\mathrm{g}(\mathbf{p}) \tag{4}$$

Because of this, the SLAM problem can be reduced to estimate only $\hat{\mathbf{p}}_i$ instead of $\hat{\mathbf{m}}_i$.

It exists several statistical methods (and many heuristical methods) used for process observations into parameters estimate. I choose the maximum likelihood method because it is widely considered as most efficient among exact statistical methods [2].

Maximum likelihood is a method based on maximization of likelihood function:

$$L(\mathbf{X},\mathbf{m};\mathbf{Z})=p(\mathbf{Z}\,|\,\mathbf{X},\mathbf{m}) \tag{5}$$

However, because of many advantageous, the maximum is usually searched in logarithmic likelihood function:

$$l(\mathbf{X},\mathbf{m};\mathbf{Z})=\log(L(\mathbf{X},\mathbf{m};\mathbf{Z})) \tag{6}$$

Considering that observation noise has a normal probability distribution ($\mathbf{v}\sim\mathrm{No}(\mathbf{0},\mathbf{\Sigma}_{obs})$) I developed three approaches for $(\hat{\mathbf{x}}_i\,|\,\hat{\mathbf{m}}_i)^T$ estimation in order to understand and described the differences in performance of SLAM algorithm with apriory given constraints and algorithm without it.

### 2.1 NON-CONSTRAINTS APPROACH

The first approach is based on the periodical recurrent processing of observation $\mathbf{z}_i$ and $\hat{\mathbf{m}}_{i-1}$ exactly as defined by the equation (2). The only deviation from this model is an initial step which uses $\mathbf{z}_0$ and initially given $\mathbf{x}_0$.

The log-likelihood functions are following:

$$l_0=-\frac{1}{2}(\mathbf{z}_0-\mathrm{h}(\mathbf{x}_0,\mathbf{m}))^T\mathbf{\Sigma}_{obs}^{-1}(\mathbf{z}_0-\mathrm{h}(\mathbf{x}_0,\mathbf{m})) \tag{7}$$

$$l_i=-\frac{1}{2}(\mathbf{z}_i-\mathrm{h}(\mathbf{x}_i,\mathbf{m}))^T\mathbf{\Sigma}_{obs}^{-1}(\mathbf{z}_i-\mathrm{h}(\mathbf{x}_i,\mathbf{m}))+(\hat{\mathbf{m}}_{i-1}-\mathbf{m})^T\mathbf{\Sigma}_{est}^{-1}(\hat{\mathbf{m}}_{i-1}-\mathbf{m}) \tag{8}$$

### 2.2 CONSTRAINTS APPROACH

The second approach takes into consideration constraints given by the equation (4) and recurrently estimates $(\hat{\mathbf{x}}_i\,|\,\hat{\mathbf{p}}_i)^T$ from observations $\mathbf{z}_i$.

The log-likelihood functions are following:

$$l_0 = -\frac{1}{2}(\mathbf{z}_0 - \mathrm{h}(\mathbf{x}_0, \mathrm{g}(\mathbf{p})))^T \mathbf{\Sigma}_{obs}^{-1}(\mathbf{z}_0 - \mathrm{h}(\mathbf{x}_0, \mathrm{g}(\mathbf{p}))) \tag{9}$$

$$l_i = -\frac{1}{2}(\mathbf{z}_i - \mathrm{h}(\mathbf{x}_i, \mathrm{g}(\mathbf{p})))^T \mathbf{\Sigma}_{obs}^{-1}(\mathbf{z}_i - \mathrm{h}(\mathbf{x}_i, \mathrm{g}(\mathbf{p}))) + (\hat{\mathbf{p}}_{i-1} - \mathbf{p})^T \mathbf{\Sigma}_{est}^{-1}(\hat{\mathbf{p}}_{i-1} - \mathbf{p}) \tag{10}$$

### 2.3 BUNDLE ADJUSTMENT APPROACH

The last approach is designed to be reference gold-standard algorithm. Technically it is not SLAM by definition because it is not recurrent. In every step, it process all so far captured observations and estimates not only $(\hat{\mathbf{x}}_i \,|\, \hat{\mathbf{p}}_i)^T$ but the whole trajectory. It also takes into consideration the given constraints. Due to all of this, it should be by the statistical point of view the best possible estimator.

The log-likelihood function is following:

$$l_i = -\frac{1}{2}\sum_{n=0}^{i}(\mathbf{z}_n - \mathrm{h}(\mathbf{x}_n, \mathrm{g}(\mathbf{p})))^T \mathbf{\Sigma}_{obs}^{-1}(\mathbf{z}_n - \mathrm{h}(\mathbf{x}_n, \mathrm{g}(\mathbf{p}))) \tag{11}$$

## 3 SIMULATIONS SETUP

To acquire information about the performance of each approach, I realize experimental simulation. I create 2D virtual environment composed of several points which lie on the same line.

The fact that all points lie on the same line causes constraints in the space of map parameters:

$$\begin{pmatrix} a & b & 0 & 0 & \cdots & 0 & 0 & c \\ 0 & 0 & a & b & \cdots & 0 & 0 & c \\ \vdots & & & & & & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & a & b & c \end{pmatrix} \begin{pmatrix} \mathbf{m} \\ 1 \end{pmatrix} = \mathbf{0} \tag{12}$$

where $(a \,|\, b \,|\, c)^T$ is a vector of line parameters. Let’s for the unique solution assume $a^2 + b^2 = 1$.

Under this constraints, the map representation can be defined by a function of parameter vector:

$$m = g(p) = g\begin{pmatrix} a \\ b \\ c \\ r_1 \\ \vdots \\ r_n \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} -b \\ a \end{pmatrix} r_1 - \begin{pmatrix} ac \\ bc \end{pmatrix} \\ \vdots \\ \begin{pmatrix} -b \\ a \end{pmatrix} r_n - \begin{pmatrix} ac \\ bc \end{pmatrix} \end{pmatrix} \tag{13}$$

To produce observations of this virtual environment, I defined four different observation functions with the ascendant amount of nonlinearities. The first function is just a translation, the second is rotation with translation. The third and fourth utilize the second function and add to it other nonlinearities. The third transforms the output of the second function into polar coordinates and by doing so it models the behavior of line lidar sensor. The fourth function models a line camera. Following definitions represent observations functions for a single observed point, for observing multiple points they are applied to each of observed point of environment respectively.

$$h_1(x,m)=\left(\begin{pmatrix}1 & 0 & -t_x\\ 0 & 1 & -t_y\end{pmatrix}\right)\begin{pmatrix}m_x\\ m_y\\ 1\end{pmatrix} \qquad h_2(x,m)=\left(\begin{pmatrix}x_c & x_s & -x_c t_x - x_s t_y\\ -x_s & x_c & x_s t_x - x_c t_y\end{pmatrix}\right)\begin{pmatrix}m_x\\ m_y\\ 1\end{pmatrix}$$

$$h_3(x,m)=\begin{pmatrix}\sqrt{h_2(x,m)^T h_2(x,m)}\\ \operatorname{atan2}\left(h_2(x,m)_2, h_2(x,m)_1\right)\end{pmatrix} \quad h_4(x,m)=\left(\frac{k\cdot h_2(x,m)_2}{h_2(x,m)_1}\right) \tag{14}$$

where $x=(t_x \mid t_y \mid x_c \mid x_s)^T$ and $x_c^{\,2}+x_s^{\,2}=1$

The last thing left to define is the algorithm for finding maximum likelihood. I used the Gauss-Newton algorithm with the nonlinear operator for applying increments described in [3]. Because the estimations vectors are unambiguous when $x_c^{\,2}+x_s^{\,2}=1$ and $a^2+b^2=1$ (or in general if there exist some different bounds between $x_c, x_s$ and $a$, $b$) it is better to calculate only angular increments and applied them via the rotation transformation.

## 4 RESULTS

I made series of experiments by processing 50 sets of 30 observations from the circle.

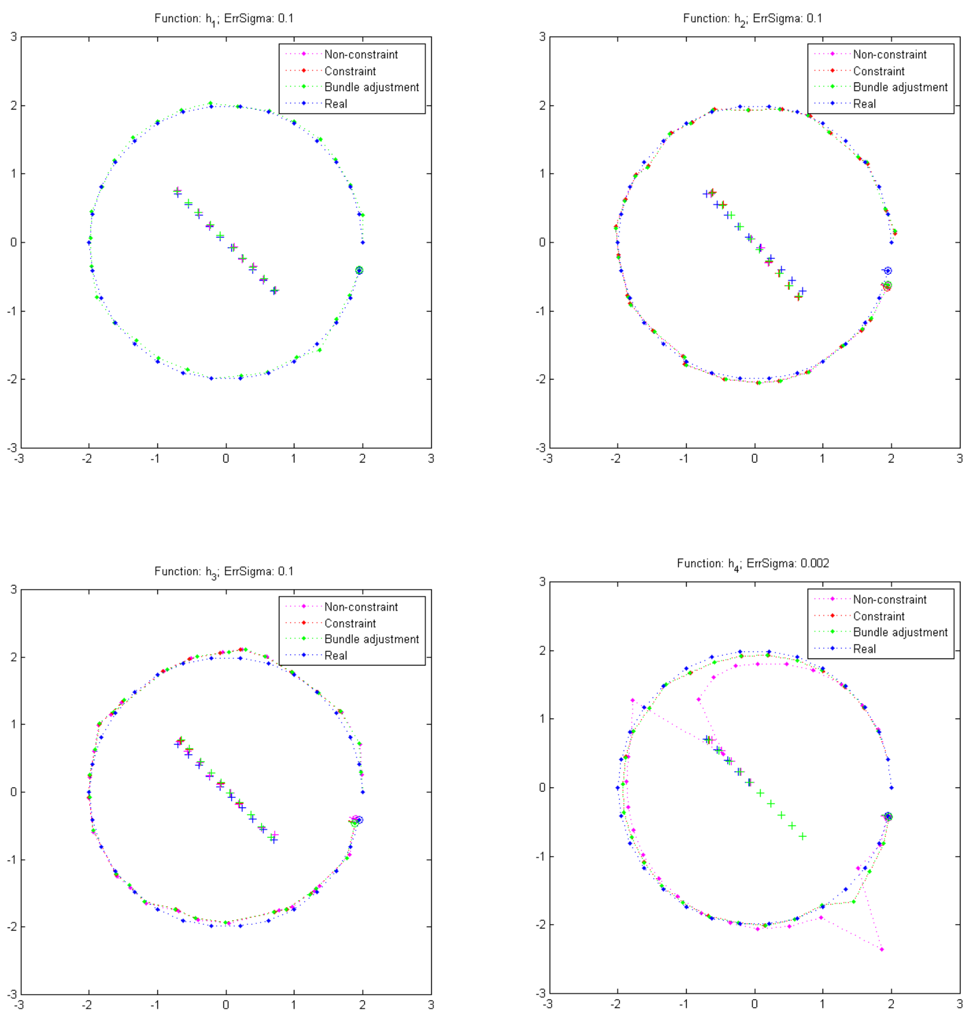


**Figure 1:** Resulting trajectories and maps of all three approaches for four observation functions

The first part of results is presented in the Fig. 1. It is a graphical representation of a processing single set of observations. However, to present the performance in multiple runs, I sum the $t_x, t_y$ estimation errors in every of the 50 runs and compute the average position estimation error. The results are following:

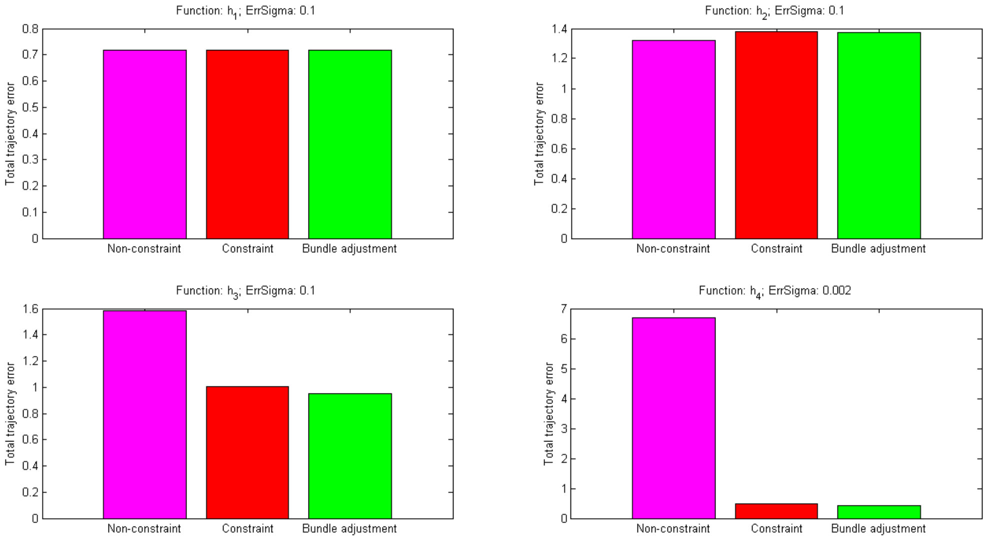


**Figure 2:** Total errors in estimation of observers position

## 5 CONCLUSIONS

In this paper, I present my experiment aimed at what differences are cast by processing apriory knowledge of the environment on a SLAM algorithm. For simple observation function, I didn't observe any significant differences. Significant differences are observable in the case of line camera observation model. This superior performance is probably caused by the fact that non-constraint processing is in this case badly balanced and maximal likelihood method comes not even close to its asymptotic properties.

These results prove that observations processing with awareness of dependencies between individual elements can under certain circumstances leads to significantly better performance. My work will continue with more complex simulations with different parametrizations.

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