
VYSOKÉ UČENÍ
TECHNICKÉ
V BRNĚ

VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ

NEXT GENERATION VUT: Zvyšování kvality a relevance vzdělávání na VUT CZ.02.02.XX/00/23_022/0009052

Dynamics

doc. RNDr. Milada Bartlová, Ph.D.,
doc. Ing. Vlasta Sedláková, Ph.D.,
Mgr. Naděžda Bogatyreva, Ph.D.

Co-funded by the European Union

MINISTRY OF EDUCATION, YOUTH AND SPORTS

Contents

- Dynamics of a Mass Point
- Basic quantities of dynamics
- Newton's laws of motion
- Forces in nature
- Examples of application of Newton's laws
- Mechanical work
- Mechanical power
- Mechanical energy
- Impulse
- Law of conservation of linear momentum
- Examples

Dynamics of a mass point

The study of the connections between the interaction of bodies and changes in their state of motion is the subject of dynamics.

Material objects interact with each other in different ways, we say that they interact with each other. In classical mechanics, this interaction is understood as the interaction of bodies that leads to a change in their state of motion or to their deformation, or to both phenomena at the same time.

The interaction of bodies can be realized basically either by touching each other or by means of physical fields.

Dynamics of a mass point

The mutual interaction of material objects is evaluated by a physical quantity called **force**. Therefore, force cannot exist independently of material objects.

Thus, dynamics studies the connections between motion and the forces that cause it.

The basic concepts of dynamics are kinematic concepts, i.e. position vector, velocity and acceleration, as well as the concepts of **mass**, **linear momentum** and **force**.

Basic quantities of dynamics

Mass is a measure of the inertia and gravitational properties of a body.

It is a fundamental property of all material objects.

Mass is a positive scalar quantity.

The unit of mass is the kilogram – $[m] = \mathrm{kg}$.

Linear momentum

The instantaneous state of motion of a mass point is determined by its position and velocity from a kinematic point of view. From a dynamic point of view, the effects of movement, e.g. when hitting a wall, are determined by mass and speed. Therefore, we introduce the quantity **linear momentum** $\vec{p}$ as a dynamic measure of motion,

$$\vec{p} = m \cdot \vec{v}.$$

Linear momentum is a vector quantity.

The unit of linear momentum is $[p] = \mathrm{kg.m.s^{-1}}$.

Basic quantities of dynamics

Force is a measure of the interaction between material objects.

This action can result in a change in the state of motion, deformation, or both.

All forces acting on a material object can be divided into:

- forces arising from direct contact of bodies (compressive forces, tensile forces, frictional forces),
- forces caused by the physical field (gravitational forces, electric forces, magnetic forces).

Basic quantities of dynamics

Force is a vector physical quantity.

The unit of force is [F] = N (Newton).

If several forces act on a material point, the resulting force $\vec{F}$ is given by their vector sum according to the principle of superposition

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n = \sum_{i=1}^{n} \vec{F}_i .$$

Newton's laws of motion

Dynamics is based on three fundamental laws published in 1687 by Isaac Newton (1642–1727):

1. Law of Inertia

Every body remains in a state of rest or uniform straightforward motion unless it is forced by external forces to change this state.

2. The Law of Force

The acceleration of a mass point is directly proportional to the magnitude of the applied force and inversely proportional to the mass, and has the same direction as the force.

3. The Law of Action – Reaction

Every action always causes an equally large reaction in the opposite direction, i.e. the interactions of two bodies are of equal magnitude and in the opposite direction.

Newton's first law

Newton's first law (Law of Inertia) guarantees the existence of preferred reference frames, inertial frames.

Inertial reference frames (inertial frames) are those in which Newton's first law applies.

If a system is at rest or in uniform straightforward motion with respect to another inertial frame, this frame is also inertial.

For example, if a system accelerates, decelerates, turns or rotates relative to another inertial frame, it is a noninertial reference frame.

Newton's first law

We cannot prove the law of inertia – we cannot realize the state of a body when no forces act on it. In nature, there is neither absolute rest nor absolute uniform straightforward motion.

Rest and uniform straightforward motion are relative and depend on the choice of the coordinate system that is associated with a particular reference body. The reference body (most often the Earth) can move.

From the point of view of describing motion, rest and uniform motion are equivalent (for example, a person on a train, in an elevator).

Newton's first law can also be expressed as: **if a body is at rest or moves uniformly in a straight line, the resultant of all external forces acting on the body is zero.**

Newton's second law

The mutual relationship between cause (i.e. the acting force) and the effect (i.e. the change in the state of motion of a material point) is given by Newton's second law of motion:

The temporal change in lineasr momentum is proportional to the external force and has the same direction with it.

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t},$$

where $\vec{p} = m \cdot \vec{v}$ is the linear momentum of a material point and $\vec{F} = \sum_{i=1}^{n} \vec{F}_i$ is the vector sum of all forces acting on it.

Newton's second law can also be expressed using mass and acceleration. If the mass m is independent of time, then

$$\vec{F} = m \cdot \vec{a},$$

where $\vec{a}$ is the acceleration of a material point, caused by the acting force $\vec{F}$.

Some particular forces

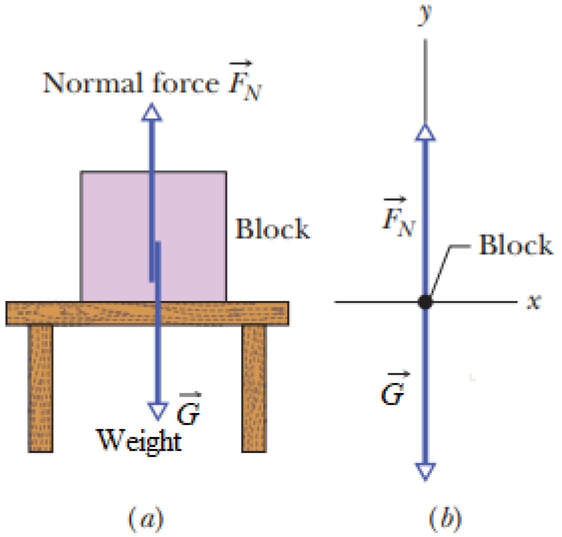
Weight

Weight $\overrightarrow{F_G}$ or $\vec{G}$ is the result of gravitational and centrifugal force, $\overrightarrow{F_G} = m \cdot \vec{g}$, where $\vec{g}$ is the free-fall acceleration (see Gravitation).

Perpendicular compressive force (normal force)

It exists only if the body is on a surface.

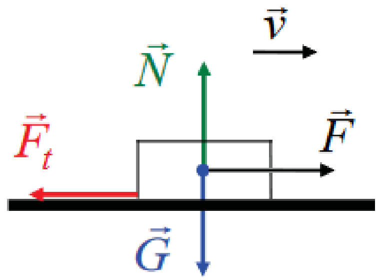
The surface exerts a compressive (normal) force $\vec{N}$ or $\overrightarrow{F_N}$ on the body.
This force is always perpendicular to the surface.



Some particular forces

Friction

The frictional force acts against the direction of motion and is parallel to the surface.



There are static and kinetic frictions. Static friction – the body is at rest, the static frictional force exactly balances the relevant component of the external force that tries to set it in motion.

Kinetic (dynamic) friction – during motion.

For the magnitude of the frictional force, the following applies

$$F_t = \mu \cdot N,$$

where μ is the coefficient of friction, N is the magnitude of the normal force.

Some particular forces

Drag force

It is formed when a body moves in a liquid or gas.

Acts against the direction of motion.

The magnitude of the drag force is

$$F_o = a v + b v^2$$

where a, b are quantities dependent on the shape and size of the body and the properties of the environment.

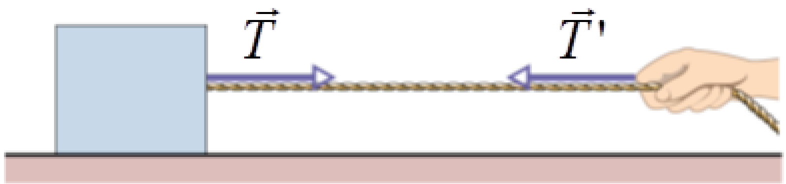
At low speeds, the first term prevails, at higher speeds the second.

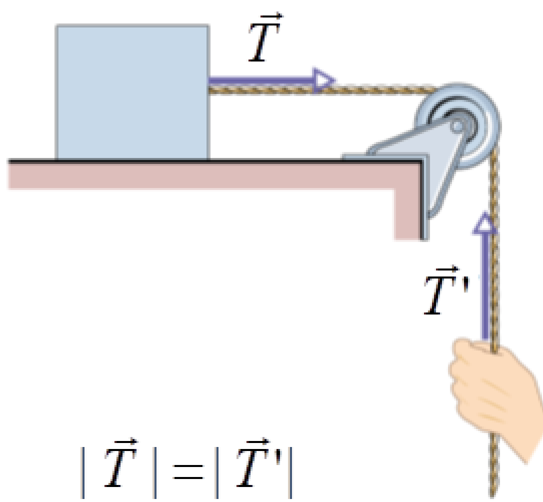
The drag force can have a direction other than speed (an example is an airplane).

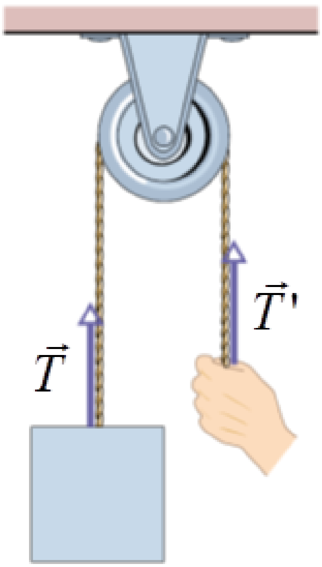
Some particular forces

Tension force

Tension force $\vec{T}$ is the force exerted by the cord on the body during its pull.







$$|\vec{T}| = |\vec{T}'|$$

Examples of application of Newton's laws

All the examples below can be solved also with frictional forces $\overrightarrow{F_{\mathrm{t}}}$, or drag forces $\overrightarrow{F_{\mathrm{o}}}$. Weight is $\overrightarrow{G} = m \cdot \vec{g}$.

When solving examples, it is necessary to:

- Draw a picture ilustrating the situation described.
- Draw the acting forces.
- Show the direction of motion and the direction of acceleration.
- Write the equation of motion in vector form.
- Select a coordinate system.
- Write the equation of motion in components.

Solving the equation of motion

The equation of motion has the form

$$\vec{F} = m\vec{a},$$

where the left side is the vector sum of all forces acting on the body $\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$

and the right side $m\vec{a} = ma_x\vec{i} + ma_y\vec{j} + ma_z\vec{k}$.

The component equations are

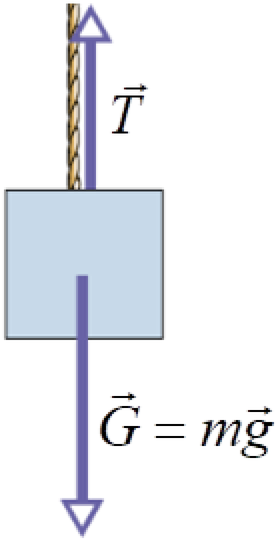
$$F_x = ma_x,$$

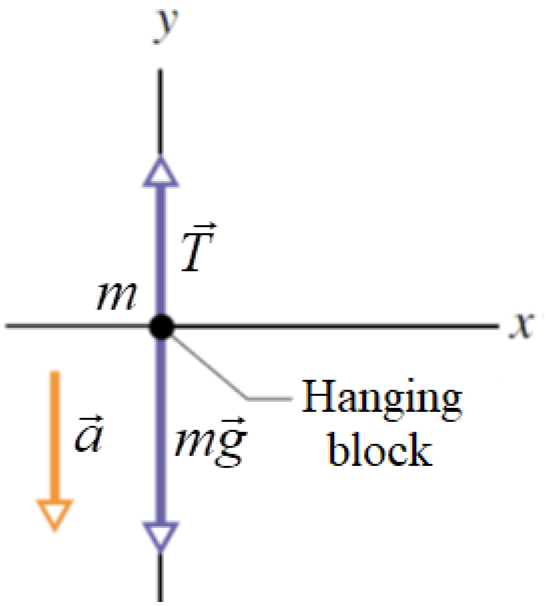
$$F_y = ma_y,$$

$$F_z = ma_z.$$

Examples of application of Newton's laws

Hanging body

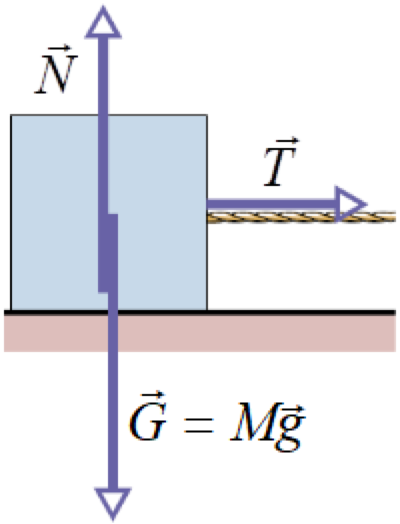


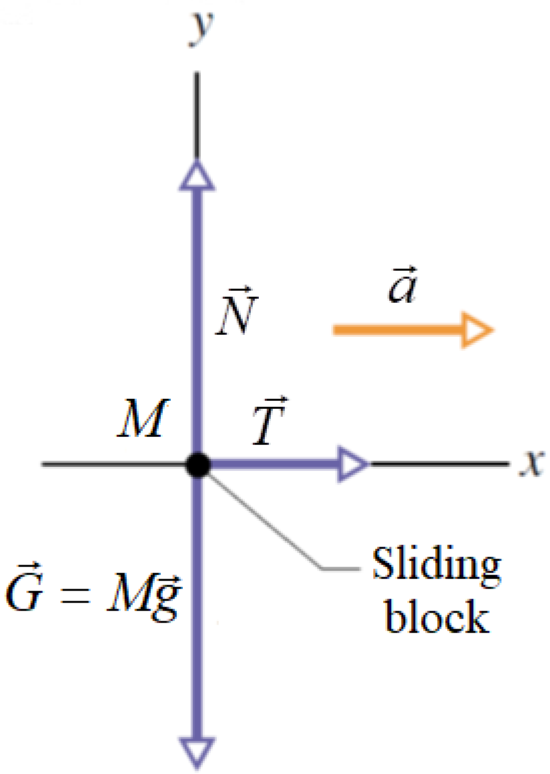


$$\vec{G} + \vec{T} = m\vec{a}, \text{ resp. } \vec{G} + \vec{T} + \vec{F}_o = m\vec{a}.$$

Examples of application of Newton's laws

Sliding body

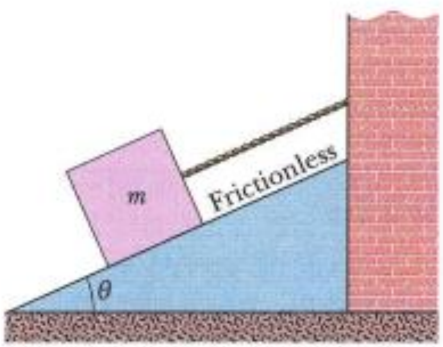


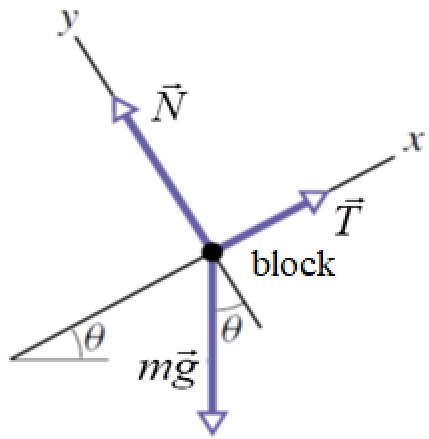


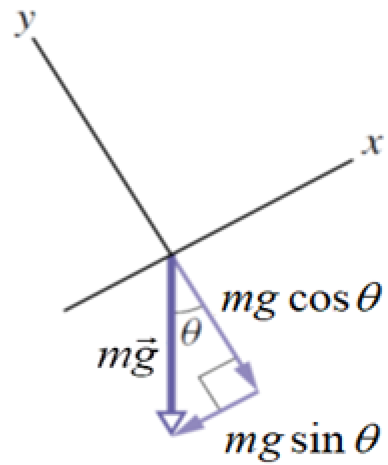
$$\vec{G} + \vec{N} + \vec{T} = M\,\vec{a}, \text{ resp. } \vec{G} + \vec{N} + \vec{T} + \vec{F}_t = M\,\vec{a}.$$

Examples of application of Newton's laws

Inclined plane







$$\vec{G} + \vec{N} + \vec{T} = m\vec{a}, \text{ resp. } \vec{G} + \vec{N} + \vec{T} + \vec{F}_{\mathrm{t}} = m\vec{a}.$$

Mechanical work

By acting a force on a body, its state of motion generally changes. The scalar quantity that expresses the path effect of a force is called the **work** W.

Constant force work

If a constant force of magnitude F acts on a body, and the body moves along the line in which the force acts, then the work done by this force is defined by the product

$$W = Fs,$$

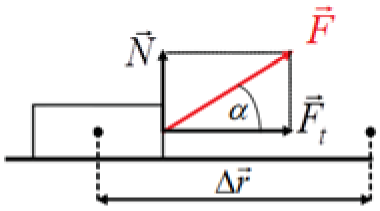
where s is the path that the body has made by the action of this force.

The unit of work is $[W]$ = N.m = joule = J.

Mechanical work

If a constant force $\vec{F}$ acts on a body, its size and direction in space do not change. The force $\vec{F}$ causes a straightforward displacement of the body by the vector $\Delta\vec{r}$ (displacement). At the same time, this constant force makes an angle α with the straight-line trajectory of the body.

The work W done by the constant force $\vec{F}$ is defined as the product of the magnitude of the tangent component of the force $F_t = F\cos\alpha$ (i.e. the component of the force $\vec{F}$ in the direction of displacement) and the magnitude of the displacement Δr

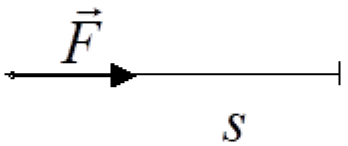
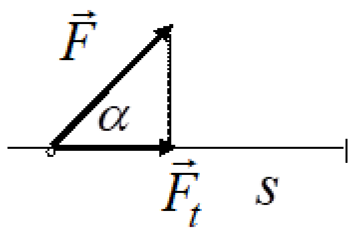
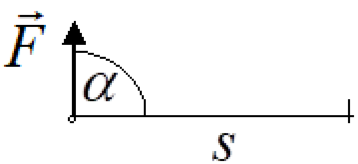
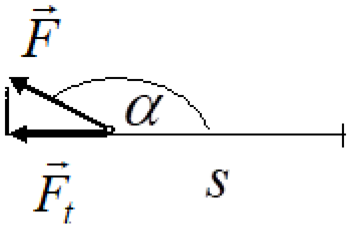


Work of constant force

$$W = F_t\,\Delta r = F\cos\alpha\,\Delta r.$$

In this case, the work W can also be expressed as the scalar product of the vectors $W = \vec{F}\cdot\Delta\vec{r}$.

Work when moving a body in a straight line by a distance *s* in the direction to the right

	$W = F \cdot s \cdot \cos 0° = F \cdot s$ $W > 0$	Force does work
	$W = F \cdot s \cdot \cos\alpha$ $W > 0$	Force does work
	$W = F \cdot s \cdot \cos 90°$ $W = 0$	Force does not do work
	$W = F \cdot s \cdot \cos\alpha$ $W < 0$	Force consumes work

Mechanical power

Work of the same magnitude will always produce the same effect, regardless of the time for which it has been done. Nevertheless, the time for which the work was performed is also an important characteristic.

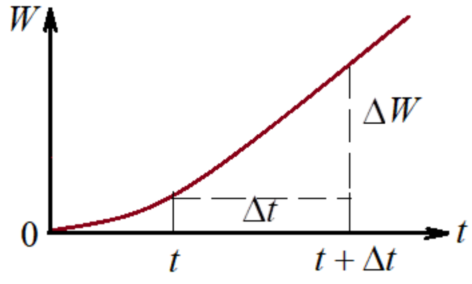
Power is a scalar quantity that characterizes how quickly mechanical work is performed.

The power P is directly proportional to the work and inversely proportional to the time it takes for that work to be done, i.e., the power is numerically equal to the work done per unit of time.

If a given force $\vec{F}$ has done work ΔW in the time interval $< t, t + \Delta t >$, the mean value of the power in this interval is equal to

$$P_s = \frac{\Delta W}{\Delta t}$$

The unit is $[P] = \mathrm{J.s^{-1}} = \mathrm{watt} = \mathrm{W}$.



Mechanical power

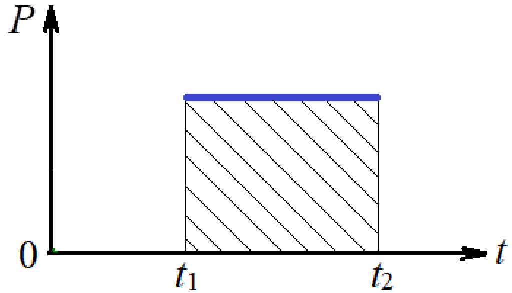
If the force F is constant over a certain time interval t, the power can be calculated

$$P = \frac{W}{t} \text{ or } P = \vec{F} \cdot \vec{v} = Fv \cos\alpha,$$

where α is the angle that the force makes with the direction of displacement (the direction of velocity $\vec{v}$).

In the case of constant power P = const. we can determine the work that is done in a time interval $< t_1, t_2 >$. Therefore,

$$W = P \cdot (t_2 - t_1).$$



Machine efficiency

The efficiency of a machine η is the proportion of the useful work of the machine and the total work delivered to the machine

$$\eta = \frac{W}{W_0}.$$

In practice, it is expressed as the ratio of power and input

$$\eta = \frac{P}{P_0}.$$

The efficiency of the machine is given as a percentage and it is true that $\eta < 100$ %.

Mechanical energy

Energy is a scalar physical quantity that expresses the ability of bodies to do work.

The unit of energy is $[E] = [W]$ = joule = J.

We assume that an external body can change the position and velocity of a mass point by applying a force $\vec{F}$. In doing so, it performs work.

Kinetic energy

Kinetic (motion) energy is a dynamic scalar quantity that is related to the motion of a body and that changes when we do work on a body.

The kinetic energy of a state with velocity v is then given by the relation

$$E_k = \frac{1}{2} m v^2.$$

Mechanical energy

Potential energy

Potential (positional) energy is a scalar quantity that characterizes the ability of a body to perform work when changing its position.

If the work is done by the forces of the gravitational field at the surface of the Earth, we speak of potential gravitational energy. In a homogeneous gravitational field, the potential energy

$$E_p = mgh,$$

where m is the mass of the mass point, g is the free-fall acceleration, and h is its height above the zero potential energy level (i.e., usually above the ground, above the pad, etc.).

Law of conservation of mechanical energy

If only internal forces act in the system, the increase in the kinetic energy of a material point due to the work of these internal forces is equal to the decrease in its potential energy, and vice versa.

For an isolated system, the law of conservation of mechanical energy applies

$$E_k + E_p = \text{const.}$$

The total mechanical energy of the isolated system is constant.

The law of conservation of mechanical energy is a special case of the law of conservation of energy.

This law can also be expressed as: Energy cannot be lost anywhere, only one form of energy changes to another.

Impulse

If the resulting force $\vec{F}$ acts on a body, its state of motion changes. This change is reflected in a change in its position and speed, and therefore in linear momentum. The change in linear momentum depends not only on the force $\vec{F}$, but also on the time for which the force has been exerted.

The time effect of force is called the impulse of force. The impulse $\vec{I}$ of force acting on a body in the time interval Δt is equal to the change in its linear momentum $\Delta\vec{p}$ in this time interval.

$$\vec{I} = \Delta\vec{p}.$$

The change in linear momentum of a body is equal to the impulse of force that caused the change.

The impulse is a vector quantity that describes the effects of force over a time interval.

The unit of impulse is $[I] = [F]\cdot[t]$ = N.s.

Impulse

According to Newton's second law of motion $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$, the force $\vec{F}$ acting on a mass point of mass m in the time interval Δt causes a change in linear momentum

$$\Delta\vec{p} = \vec{F}\cdot\Delta t.$$

The change in momentum in the time interval $< t_1, t_2 >$ then be

$$\Delta\vec{p} = \vec{p}_2 - \vec{p}_1 = m\vec{v}_2 - m\vec{v}_1 = m(\vec{v}_2 - \vec{v}_1) = m\cdot\Delta\vec{v}.$$

The impulse $\vec{I}$ is generally equal to the change in linear momentum over a given time interval $< t_1, t_2 >$, i.e.

$$\vec{I} = \Delta\vec{p} = m\cdot\Delta\vec{v}.$$

If the applied force is constant for Δt, the impulse of the force can be expressed

$$\vec{I} = \Delta\vec{p} = \vec{F}\cdot\Delta t = \vec{F}\cdot(t_2 - t_1).$$

Law of conservation of linear momentum

We consider an isolated system of two particles acting on each other with forces. A force $\overrightarrow{F_1}$ acts on particle 1, a force $\overrightarrow{F_2}$ acts on particle 2. According to Newton‘s 3rd law of motion these are action – reaction forces

$$\overrightarrow{F_1} = -\overrightarrow{F_2}.$$

From this, the law of conservation of linear momentum for an isolated system can be derived: $\overrightarrow{F_1} + \overrightarrow{F_2} = 0$ or also $\frac{\Delta\vec{p}_1}{\Delta t} + \frac{\Delta\vec{p}_2}{\Delta t} = 0$, then $\frac{\Delta(\vec{p}_1 + \vec{p}_2)}{\Delta t} = 0$,

$$\vec{p}_1 + \vec{p}_2 = \text{const.}$$

The law of conservation of linear momentum for an isolated system is of general application. For more points, generalize $\sum_{i=1}^{n} \vec{p}_i = \text{const.}$

The total linear momentum of an isolated system does not change (it preserves).

Examples

1. With what acceleration does a train with the mass of 800 t start moving if it is subjected to a traction force of 160 kN from the locomotive?

Solution:

At the beginning, we need to convert the units to the basic SI units: mass m = 800 t = 800000 kg.

To calculate the acceleration, we use Newton‘s second law $F = m\,a$.

$$a = \frac{F}{m} = \frac{160000}{800000} = \frac{1}{5} = 0{,}2\ (\mathrm{m.s^{-2}}).$$

Answer: The magnitude of acceleration is 0,2 m.s^{-2}.

Examples

2. The experimental rocket sled can be uniformly accelerated from rest to a speed of 1600 km.h^{-1} in 1,8 seconds. Find the magnitude of the average force required if the sled mass is 500 kg.

Solution:

At the beginning, we need to convert the units to the basic SI units: velocity v = 1600 km.h^{-1} = $\frac{1600 \cdot 1000}{3600} \frac{\mathrm{m}}{\mathrm{s}} \approx 444{,}44$ m.s^{-1}.

According to the task, the sled starts from rest, i.e. the initial speed is $v_0 = 0$. The motion is uniformly accelerated, therefore the acceleration can be calculated according to the relation (see Kinematics)

$$a = \frac{v - v_0}{t} = \frac{444{,}44 - 0}{1{,}8} \approx 246{,}91\ (\mathrm{m.s^{-2}}).$$

To calculate the force, we use Newton‘s second law

$$F = m \cdot a = 500 \cdot 246{,}91 = 123456{,}79\ (\mathrm{N}).$$

Answer: The magnitude of the average force is 123456,79 N.

Examples

3. By pedaling, the cyclist generates a force that acts on the bicycle in the direction of its motion with an average force of 50 N. A frictional force and a drag force of air of 10 N act against its motion. Find the acceleration of the cyclist if his mass including the bike is 80 kg.

Solution:

At the beginning, we need to calculate the resultant force as a vector sum of the forces applied and then using Newton‘s second law, we find the acceleration.

The following forces act on the body (cyclist and bicycle):

$\vec{G}$ is the weight (the motion is on the surface of the Earth, the direction of the force is vertical).

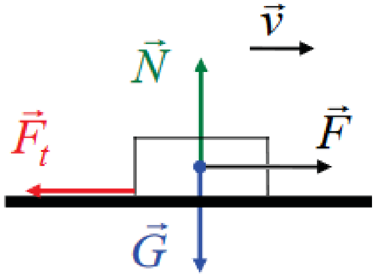
$\vec{N}$ is the normal force (the body is located on the pad – on the surface of the Earth, the direction of the force is perpendicular to the pad).

Examples

$\vec{F}$ is the force generated by the cyclist (acting in the direction of motion).

$\vec{F}_\mathrm{t}$ is the frictional and resistive force (acting against the direction of motion).

Equation of motion in the vector form: $\vec{G} + \vec{N} + \vec{F} + \vec{F}_\mathrm{t} = m\,\vec{a}$.



We divide the forces into horizontal (x-axis) and vertical (y-axis) direction and write the component equations.

x-axis: $F - F_\mathrm{t} = ma_x$, the motion takes place on a horizontal plane with acceleration $a_x = a$.

y-axis: $N - G = ma_y$, in this direction, the body does not move, so the acceleration is $a_y = 0$.

Examples

Then $N - G = 0$, i.e. $N = G = mg$. Thus, the normal force and weight are compensated.

From the equation for the *x*-axis, we calculate the acceleration of the cyclist

$$a = \frac{F - F_t}{m} = \frac{50 - 10}{80} = \frac{1}{2} = 0{,}5\ (\mathrm{m.s^{-2}}).$$

Answer: The magnitude of the cyclist's acceleration is 0,5 m.s^{-2}.

Examples

4. Find what work we do when pulling out a nail 5 cm long if we apply a net force of 130 N to it.

Solution:

At the beginning, we need to convert the units to the basic SI units:

$$l = 5\ \mathrm{cm} = 5 \cdot 10^{-2}\ \mathrm{m} = 0{,}05\ \mathrm{m}.$$

To calculate, we use the relation $W = F \cdot s$, where F is the average force we exert and s is the path, i.e. the length of the nail l. After substituting the values, we get

$$W = 130 \cdot 0{,}05 = 6{,}5\ (\mathrm{J}).$$

Answer: When pulling out the nail, we need to do the work of 6,5 J.

Examples

5. The car with the mass of 1200 kg increased its speed from 72 km.h^{-1} to 90 km.h^{-1} in 10 seconds. Find:

a) the magnitude of the force that caused this change,

b) the distance the car travels at increasing speed,

c) change in the kinetic energy of the car.

Solution:

At the beginning, we need to convert the units to the basic SI units:

$$v_0 = 72\ \mathrm{km.h^{-1}} = \frac{72 \cdot 1000}{3600}\ \frac{\mathrm{m}}{\mathrm{s}} = 20\ \mathrm{m.s^{-1}},$$

$$v_1 = 90\ \mathrm{km.h^{-1}} = \frac{90 \cdot 1000}{3600}\ \frac{\mathrm{m}}{\mathrm{s}} = 25\ \mathrm{m.s^{-1}}.$$

Examples

a) The motion is uniformly accelerated, i.e. the velocity changes over time according to the relation $v = v_0 + at$, therefore, the acceleration can be calculated from the relation

$$a = \frac{v - v_0}{t} = \frac{25 - 20}{10} = \frac{1}{2} = 0{,}5\ (\mathrm{m.s^{-2}}).$$

To calculate the force, we use Newton‘s second law

$$F = m \cdot a = 1200 \cdot 0{,}5 = 600\ (\mathrm{N}).$$

b) The distance traveled by the car at increasing speed, i.e., during uniformly accelerated motion over a period of 10 s, is

$$s = s_0 + v_0 t + \frac{1}{2} a t^2 = 0 + 20 \cdot 10 + \frac{1}{2} \cdot \frac{1}{2} \cdot 10^2 = 225\ (\mathrm{m}).$$

Examples

c) Kinetic energy can be calculated according to the relation $E_k = \frac{1}{2}mv^2$. The change in kinetic energy is

$$\Delta E_k = E_{k1} - E_{k0} = \frac{1}{2}m{v_1}^2 - \frac{1}{2}m{v_0}^2 = \frac{1}{2}m({v_1}^2 - {v_0}^2)$$

$$\Delta E_k = \frac{1}{2} \cdot 1200 \cdot (25^2 - 20^2) = 135000 \text{ (J)}.$$

Answer: a) the magnitude of the force that caused the change is 600 N, b) the distance traveled by the car at increasing speed is 225 m, c) the change in the kinetic energy of the car is 135 kJ.

Examples

6. The aircraft with the mass of 60 tons climbed from an altitude of 1000 m to the altitude of 3000 m, increasing its speed from 160 m.s^{-1} to 200 m.s^{-1}. Find:

a) change in the kinetic energy of the aircraft,

b) change in the potential energy of the aircraft,

c) the size of the work done by the aircraft's engines. Do not consider air resistance.

Solution:

At the beginning, we need to convert the units to the basic SI units: aircraft mass m = 60 t = 60000 kg.

We assume: $h_0 = 1000$ m, $h_1 = 3000$ m,

$v_0 = 160$ m.s^{-1}, $v_1 = 200$ m.s^{-1}.

The motion is near the Earth's surface, therefore we consider the free-fall acceleration g = 9.81 m.s^{-2}.

Examples

a) The relation for kinetic energy is $E_k = \frac{1}{2}mv^2$. Then the change in kinetic energy is

$$\Delta E_k = E_{k1} - E_{k0} = \frac{1}{2}m{v_1}^2 - \frac{1}{2}m{v_0}^2 = \frac{1}{2}m({v_1}^2 - {v_0}^2)$$

$$\Delta E_k = \frac{1}{2} \cdot 60000 \cdot (200^2 - 160^2) = 432000000 = 432 \cdot 10^6 \text{ (J)}.$$

b) The relation for potential energy is $E_p = mgh$. Then the change in potential energy is

$$\Delta E_p = E_{p1} - E_{p0} = mgh_1 - mgh_0 = mg(h_1 - h_0)$$

$$\Delta E_p = 60000 \cdot 9{,}81 \cdot (3000 - 1000) = 1177200000 \text{ (J)} = 1177{,}2 \cdot 10^6 \text{ (J)}.$$

Examples

c) According to the definition, energy is a scalar physical quantity that expresses the ability of bodies to do work. Thus, the amount of work done by the engines of the aircraft is equal to the total change in energy.

$$W = \Delta E_k + \Delta E_p = 432 \cdot 10^6 + 1177{,}2 \cdot 10^6 = 1609{,}2 \cdot 10^6 \text{ (J)}.$$

Answer: a) the magnitude of the change in kinetic energy is 432 MJ, b) the magnitude of the change in potential energy is 1177,2 MJ, c) the magnitude of the work done by the engines of the aircraft is 1609,2 MJ.

Examples

7. The elevator motor transports a load of the mass of 250 kg in a uniform motion to a height of 18 m in 30 seconds. Find:

a) what work the engine will do,

b) engine power.

Solution:

a) The motor lifts the load to a certain height, i.e. the potential energy of the load changes. Therefore, we can say that the motor does a work as great as the change in potential energy.

In the homogeneous gravitational field of the Earth, the potential energy is

$$\Delta E_p = m \cdot g \cdot \Delta h, \text{ pak } W = \Delta E_p = 250 \cdot 9{,}81 \cdot 18 = 44145 \text{ (J)}.$$

b) Power is numerically equal to the work done per unit time, i.e., the power of the engine is $P = \frac{W}{t} = \frac{44145}{30} = 1471{,}5$ (W).

Answer: a) the motor has done the work of 44145 J, b) the motor power is 1471,5 W.

Unsolved examples

8. Find the magnitude of the resultant of the forces acting on a car with the mass of 1900 kg that starts with an acceleration of 3,1m.s^{-2}.

[5890 N]

9. A motorcycle with the mass of 225 kg reaches the speed of 90 km.h^{-1} from the rest in 5.0 seconds. Find:

a) the magnitude of the motorcycle's acceleration (we consider the acceleration to be constant),

b) the amount of resulting force accelerating the motorcycle,

c) change in the kinetic energy of the motorcycle.

[a) 5 m.s^{-2}, b) 1125 N, c) 70321,5 J]

Unsolved examples

10. A train starts moving along a horizontal track with an acceleration of 0,5 $m.s^{-2}$. Tension force of the locomotive is 40 kN. Calculate the work done by the locomotive in 1 minute.

[36 MJ]

11. The car with the mass of 1 t reduced its speed from 70 $km.h^{-1}$ to 60 $km.h^{-1}$ in 5 seconds. The motion of the car was straightforward and uniformly decelerated. Find:

a) the change of linear momentum of the car during braking,

b) magnitude of braking force.

[a) −2777,78 $kg.m.s^{-1}$, b) 555,56 N]

Unsolved examples

12. Calculate the change in potential energy when lifting a body of the mass of 15 kg to a height of 7 m.
[1030,05 J]

13. A cyclist rides on a horizontal road at a steady speed, overcoming a wind force of 20 N. Calculate:
a) the work he does on the 5 km track,
b) the power he has during a half-hour drive.

[a) 100 kJ, b) 55,56 W]

Literature

1. Svoboda, E. a kolektiv. Přehled středoškolské fyziky. SPN, Praha, 1991.
2. Halliday, D., Resnick, R., Walker, J. Fyzika, VUTIUM, Brno, 2000.
3. Urgošik, B. Fyzika, STNL, Praha, 1987.
4. Lepil, O., Bednařík, M., Hýblová, R. Fyzika pro střední školy. Prometheus, Praha, 1993.
5. Lepil, O., Houdek, V., Pecho, A. Fyzika pro III. Ročník gymnázií, SPN, Praha, 1986.
6. Hradilová, E., Uhdeová, N. Fyzika Přijímací zkoušky na vysoké školy, VUT v Brně, Brno, 2003.
7. Mechlová, E., Košťál, K. a kol. Výkladový slovník fyziky pro základní vysokoškolský kurz fyziky. Prometheus, Praha, 2001.

Thank you for your attention

Co-funded by
the European Union

MINISTRY OF EDUCATION,
YOUTH AND SPORTS