
VYSOKÉ UČENÍ
TECHNICKÉ
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NEXT GENERATION VUT: Zvyšování kvality a relevance vzdělávání na VUT CZ.02.02.XX/00/23_022/0009052

Kinematics

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MINISTRY OF EDUCATION, YOUTH AND SPORTS

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Physics

Physics from Greek φυσικός (fysikos) means natural (based on φύσις – fysis: nature).

Physics is a science that studies and describes natural phenomena. It is based on the experimental study of various phenomena and its task is to formulate laws that explain these phenomena.

Physical quantities and units

Physical quantities express the physical properties of objects.

The laws of physics then express the objective dependence between physical quantities. A unit of a physical quantity is a suitably selected, constant and precisely defined value of the quantity with which the measured quantity is compared.

The most widely used system of units in the world is the SI system (an abbreviation of the French – Systѐme International, English – International System of Units).

The International System of Units (SI) includes:

- **Base units**

Meter, m (length)

Kilogram, kg (mass)

Second, s (time)

Ampere, A (electric current)

Kelvin, K (thermodynamic temperature)

Mol, mole (substance quantity)

Candela, cd (luminosity)

- **Additional units**

Radian, rad (plane angle)

Steradian, sr (solid angle)

- **Derived units**

Derived units are derived using definition unit equations from basic or already derived units, or also by means of additional units.

For example, for a force acting on a material point, $\vec{F} = m\vec{a}$,

where m is the mass of the mass point, $\vec{a}$ is its acceleration.

Then the unit of force $[F] = [m]\cdot[a]$ = kg.m.s^{-2}.

In the products of unit symbols, we use a multiplying dot between units.

For practical reasons, we give some derived units special names and tags. Examples of some derived units are shown in the following table.

Unit name	**Size**	**Symbol and expression**
newton	force	$N = kg.m.s^{-2}$
pascal	pressure	$Pa = N.m^{-2}$
joule	energy, work	$J = N.m$
watt	power	$W = J.s^{-1}$
hertz	frequency	$Hz = s^{-1}$
coulomb	electric charge	$C = A.s$
volt	electrical voltage, electric potential	$V = J.C^{-1}$
ohm	electrical resistance, impedance, reaktance	$\Omega = V.A^{-1}$

- **Multiples and submultiples of units**

Since some units are too large for practical purposes and others are too small, we create multiples and submultiples of units by multiplying or dividing by a suitable power of ten. Multiples and submultiples of units are formed mainly in a series with a coefficient10^3.

Prefix	Symbol	Factor	Prefix	Symbol	Factor
deka	da	10^1	deci	d	10^{-1}
hekto	h	10^2	centi	c	10^{-2}
kilo	k	10^3	mili	m	10^{-3}
mega	M	10^6	mikro	μ	10^{-6}
giga	G	10^9	nano	n	10^{-9}
tera	T	10^{12}	piko	p	10^{-12}
peta	P	10^{15}	femto	f	10^{-15}
exa	E	10^{18}	atto	a	10^{-18}

- **Secondary units**

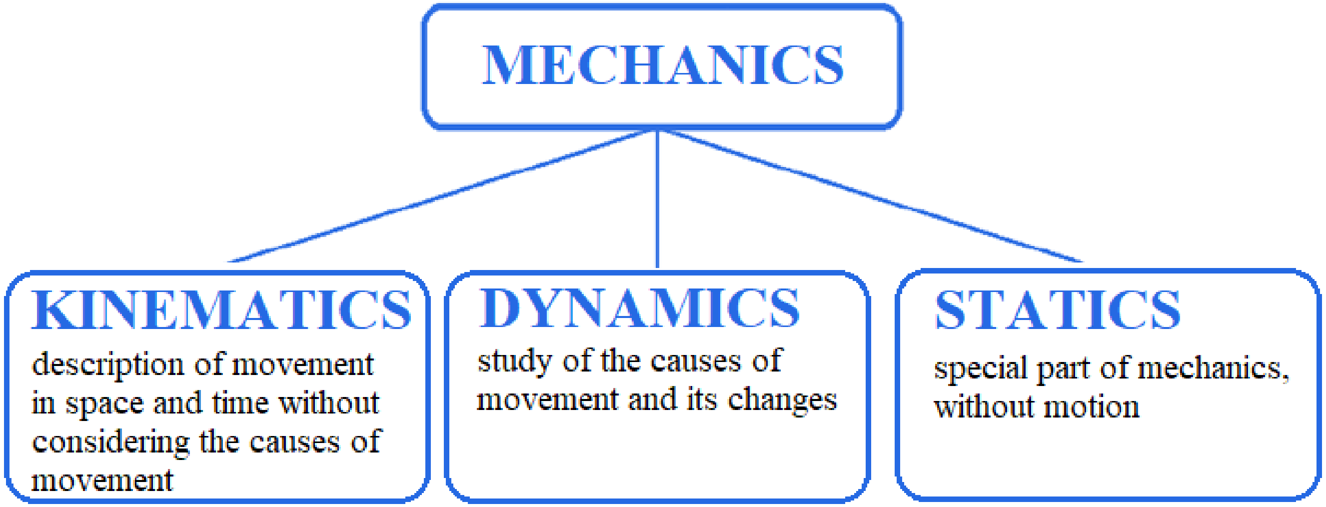
Some units of time, angle, mass, volume, and other older units that are not SI units have a long history of use. Most of them require conversion coefficients that are not powers of ten to convert to the corresponding SI unit.

Quantity	Unit name and symbol	Conversion factor
time	minute (min)	1 min = 60 s
	hour (h)	1 h = 60 min = 3600 s
	day (d)	1d = 24 h = 86400 s
angel	angular degree (°)	1° = (π/180) rad
	angular minute (′)	1′ = (1/60)°
	angular second (″)	1″ = (1/60)′
area	hectare (ha)	1 ha = 1 hm^2 = 10^4 m^2
volume	litre (l)	1 l = 1 dm^3 = 10^{-3} m^3
mass	tuna (t)	1 t = 1000 kg
temperature	Celsius (°C)	1 K = 1°C
optical power	dioptre (D)	1 D = 1 m^{-1}
energy	electron-volt (eV)	1 eV = $1{,}602 \cdot 10^{-19}$ J

Mechanics

Mechanics studies the laws of mechanical motion of bodies and their interaction. Mechanical motion is a change in the position of a body or a part of it with respect to the chosen frame of reference as a function of time.

The most commonly used is the so-called laboratory system – a right-handed Cartesian system firmly connected to the Earth (the right-handed direction is counterclockwise).



Kinematics

When solving some mechanical problems, we do not have to take into account the size of the body and its shape. Then we can replace the given body with a mass point (MP).

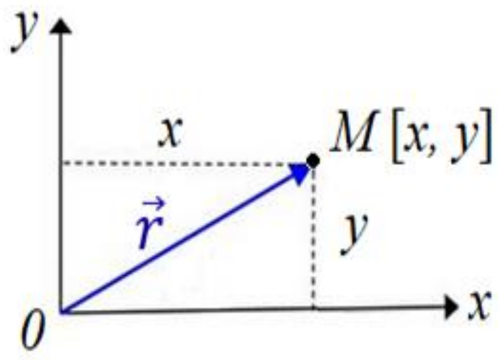
Mass point is a model of a body whose dimensions and shape are negligible with respect to the kinematic quantities describing its motion. We are only interested in the mass of the body.

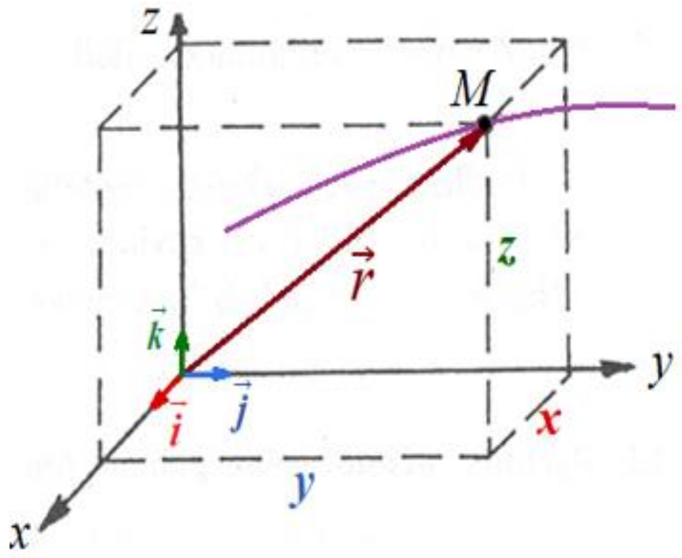
In order to unambiguously determine the position of a body and the change in this position, we need to know the fundamental kinematic quantities at every moment:

- position, i.e. the position vector $\vec{r}$,
- velocity $\vec{v}$,
- acceleration $\vec{a}$. (The increase in velocity over time is called acceleration.)

Kinematics

Position vector $\vec{r}$ we define as a vector whose origin is at the origin of the coordinate system and its endpoint is at the point where the position is determined. The coordinates of the position vector are identical to the coordinates of the given point.





Determining the position in the plane and in space.

Kinematics

In a rectangular coordinate system with unit vectors $\vec{i}$, $\vec{j}$, $\vec{k}$ in the direction of the coordinate axes, the position of the point M with coordinates x, y, z is given by the position vector $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, whose size is $r = \sqrt{x^2 + y^2 + z^2}$. The unit of magnitude of a position vector is the meter – m.

A **trajectory** or **path** is the sum of all the positions that a mass point passes through during its movement. The trajectory of the motion of a body can be a straight line or generally a curve (circle, part of a parabola, etc.).

The length of the trajectory that a mass point describes over a certain period of time is called the length of the path. It is a scalar quantity and its unit is the meter. Very often we call the length of the path simply „the path“ and we refer to it as *s*.

Kinematics

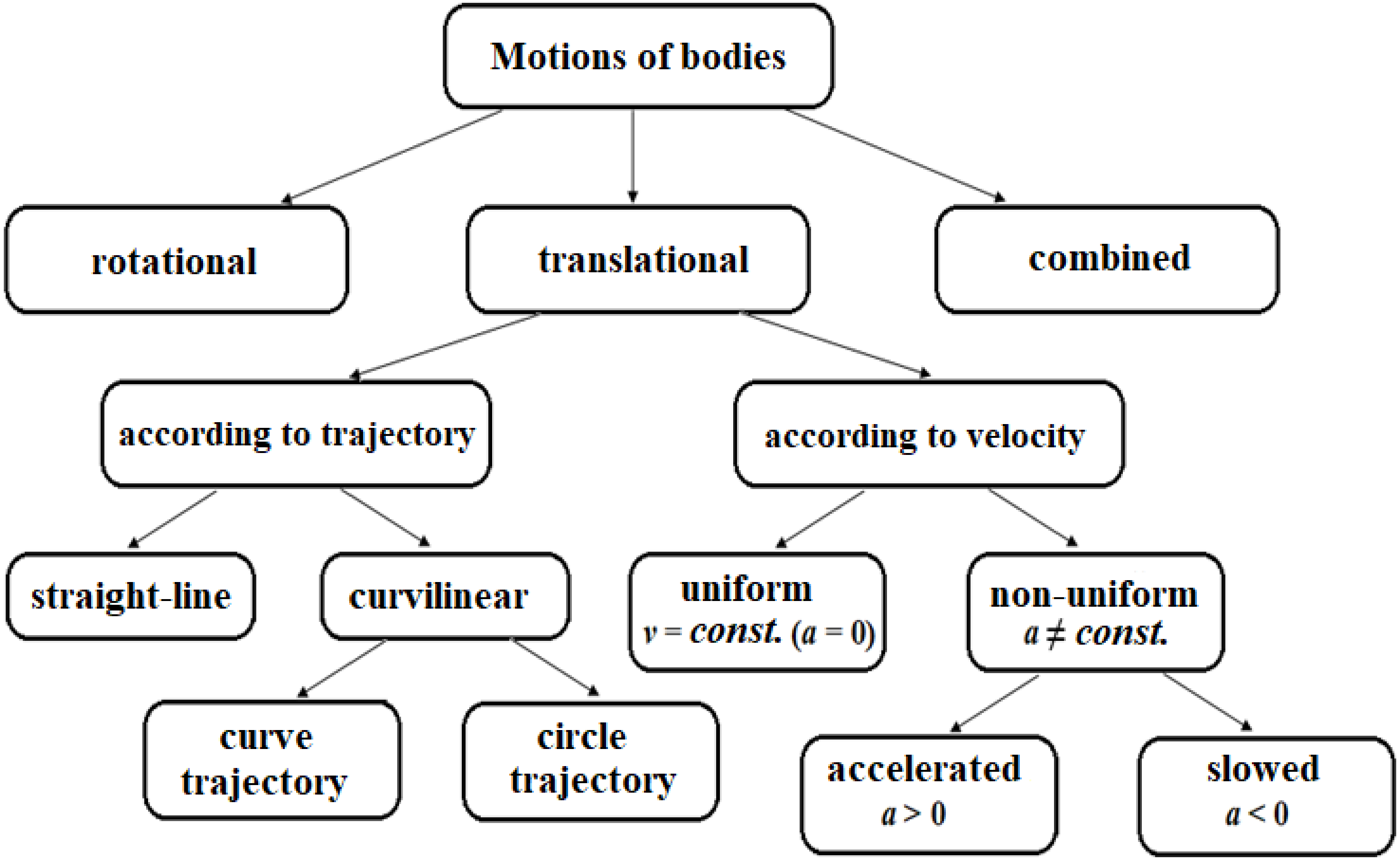
To unambiguously describe the motion, we need to know, in addition to the geometric shape of the trajectory, also the description of the dependence of the position on time.

The mathematical notation of position as a function of time can be determined using parametric equations, where the parameter is the independent variable time t.

The way in which it determines how the position of a body changes with respect to time is one of the basic characteristics of motion and is called velocity $\vec{v}$ (unit is $m.s^{-1}$). Velocity is a vector quantity.

Kinematics

Motions of bodies can be divided into types according to the figure.



Kinematics

A motion is called the straightforward motion if the displacement, velocity, and acceleration vectors have the same direction. The trajectory is a straight line.

In fact, most movements are curvilinear – in general, their trajectory is a curve. Motion in a circle is a special case of curvilinear motion (the trajectory is a circle).

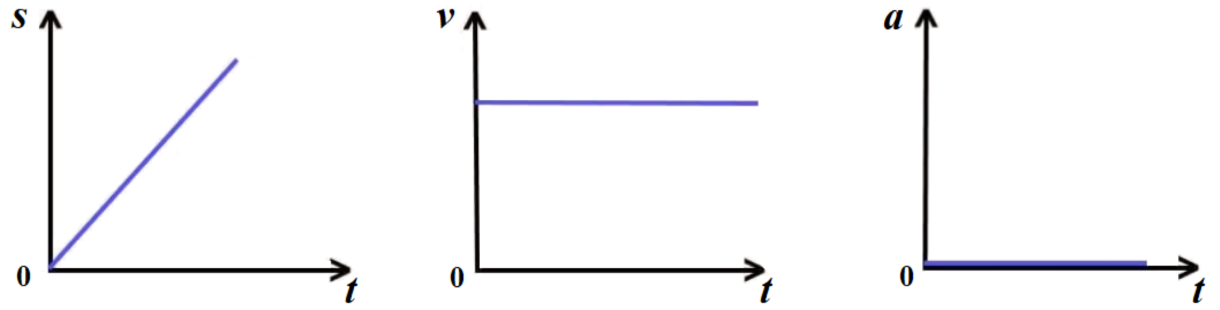
Uniform straightforward motion

Definition: A body moves in a straight line at a constant velocity, i.e., the velocity does not change over time, it is constant, it does not accelerate or slow down.

Path (in meters – m): $s = v \cdot t$, where t is the time (in seconds – s), v is the velocity.

Velocity: v = const. (the vector still has the same direction and magnitude).

Acceleration (m.s^{-2}): $a = 0$.



Graphs of the dependence of path, velocity and acceleration on time for uniform straightforward motion.

Uniformly accelerated / slowed straightforward motion

Definition: A motion in a straight line in which the velocity changes uniformly, i.e., constant acceleration/deceleration.

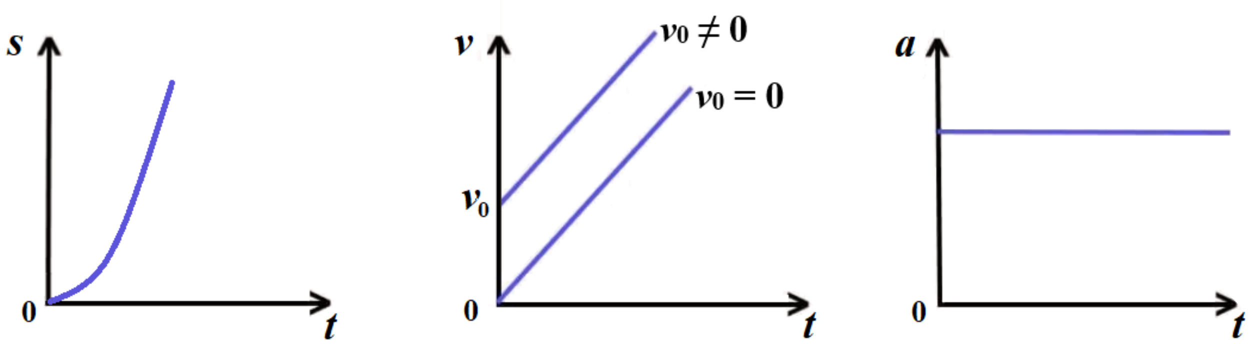
Time-dependent path: $s = v_0 t + \frac{1}{2} a t^2$, where v_0 is the initial velocity.

Velocity: $v = v_0 + at$ (speed increases in direct proportion to time).

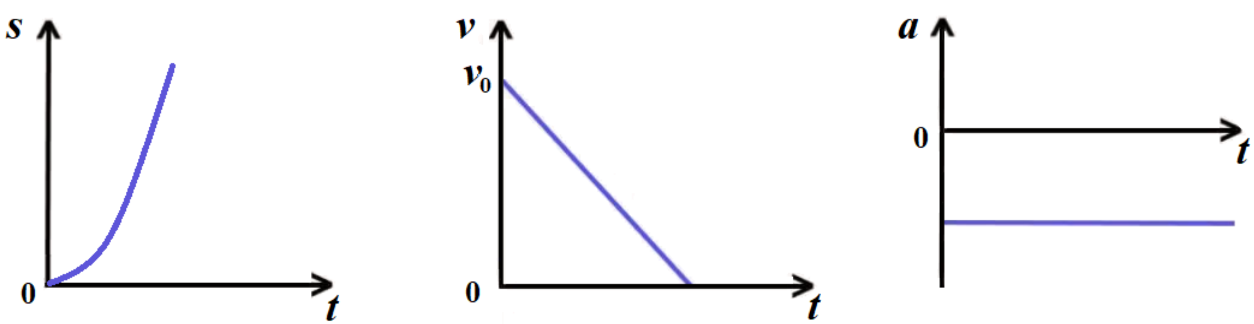
Acceleration: a = const.

Positive ($a > 0$) is acceleration or negative ($a < 0$) is braking.

Graphs of the dependence of path, velocity and acceleration on time for uniformly accelerated straightforward motion.



Graphs of the dependence of path, velocity and acceleration on time for uniformly slowed down straightforward motion.



Summary

- If the acceleration is constant a = const., the motion is called uniformly accelerated ($a > 0$) or uniformly slowed down ($a < 0$).
- If the acceleration changes over time, it is a motion with variable acceleration $a = a\,(t)$.
- In the case where the acceleration is zero $a = 0$, we call such a movement uniform. For this movement, the following applies: v = const.

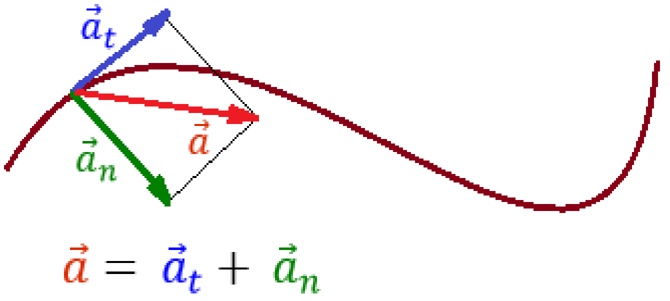
Summary Table

Motion	**Acceleration**	**Velocity**	**Position**
Uniform straightforward	$a = 0$	$v_0 = \text{const.}$	$x = x_0 + v_0 t$
Uniformly accelerated straightforward	$a = \text{const.}$, $a > 0$	$v = v_0 + at$	$x = x_0 + v_0 t + \frac{1}{2} a t^2$
Uniformly slowed straightforward	$a = \text{const.}$, $a < 0$	$v = v_0 - at$	$x = x_0 + v_0 t - \frac{1}{2} a t^2$

Curvilinear motion

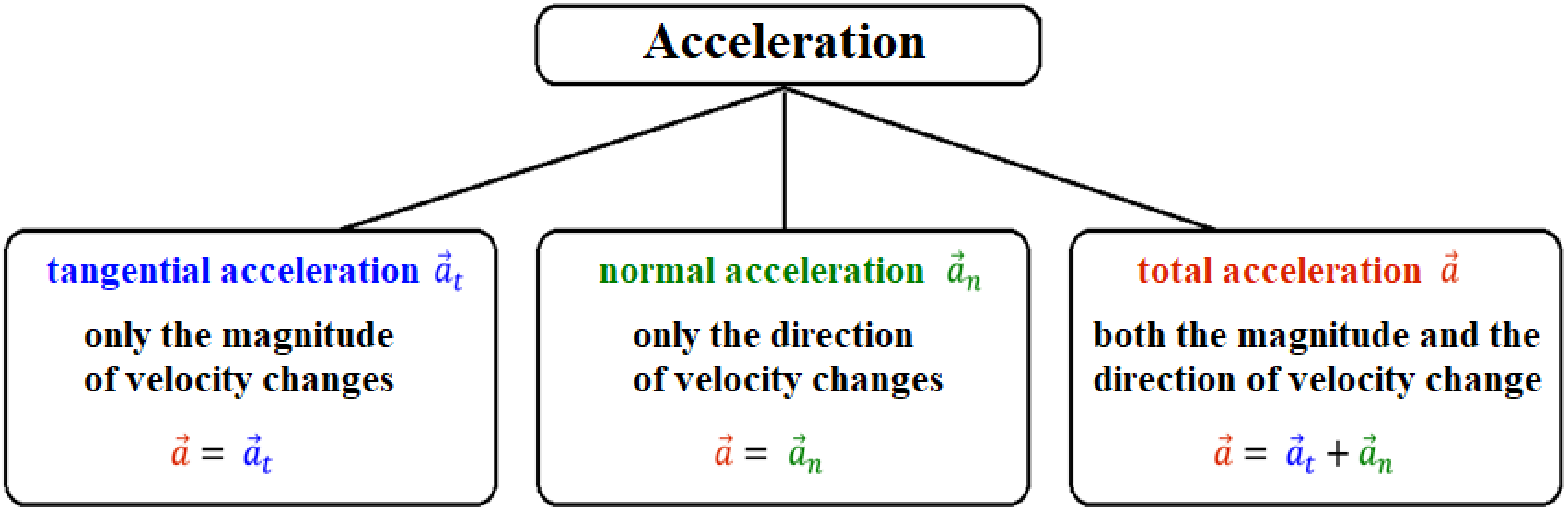
Definition: Motion in which the direction of velocity changes (velocity vector curves). If the magnitude of the velocity is constant and its direction changes, the body has a non-zero acceleration (the so-called centripetal acceleration). In the curvilinear motion of a mass point in space, the trajectory is generally a spatial curve.

We choose the direction of the tangent to the trajectory and the direction of the perpendicular (normal) to this tangent.
This normal points to the center of the arc that forms the trajectory at a given point.



For curvilinear movements, the following applies:

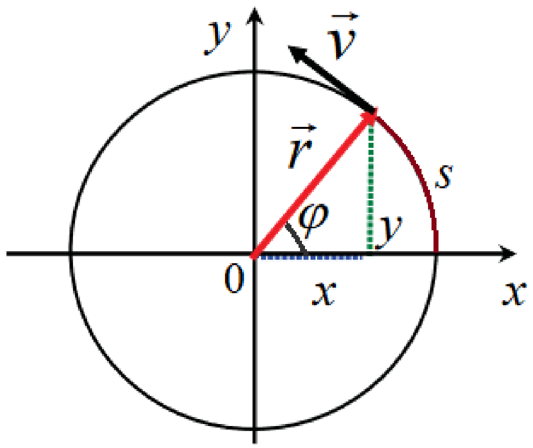
- Instantaneous velocity is tangent to trajectory.
- Acceleration can be divided into: tangential acceleration – changes the magnitude of velocity and normal (centripetal) acceleration – changes the direction of velocity.



Circular motion

Circular motion is a curvilinear motion whose trajectory is a circle of radius *r*.

The position of a mass point can be determined using the rectangular coordinates [*x*, *y*] in the plane of the circle.



In the case of circular motion, it is advisable to use polar coordinates. Converting polar coordinates to Cartesian coordinates:

$$x = r\cos\varphi,\ y = r\sin\varphi.$$

We write the position vector as

$$\vec{r} = x\vec{i} + y\vec{j} = (r\cos\varphi)\vec{i} + (r\sin\varphi)\vec{j}.$$

Circular motion

The circumferential velocity (m.s^{-1}) is the velocity of the point according to the relation:

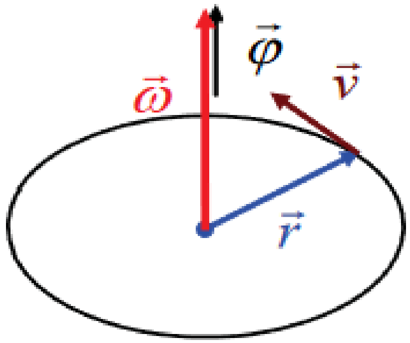
$$v = \omega \cdot r,$$

where ω is the angular velocity (rad.s^{-1}), r is the radius of the circle (in meters). Generally, the circumferential velocity $\vec{v}$ is a vector, which has a direction tangent to the trajectory. At a constant angular velocity, a mass point orbits faster the farther it is from the axis of rotation.

Angular velocity ω expresses how fast rotates the vector $\vec{r}$ at the end of which is a mass point performing a circular motion. If a mass point moves in the positive sense of an angle φ, the angular velocity has a positive sign, otherwise the angular velocity is negative.

Circular motion

The angular path (angular coordinate) $\vec{\varphi}$ can also be assigned a vector that lies in the axis of rotation. It follows that even the angular velocity vector lies in the axis of rotation and has the same orientation as the angular path.



The circumferential velocity vector is then

$$\vec{v} = \vec{\omega} \times \vec{r}, \vec{v} \perp \vec{\omega} \perp \vec{r}.$$

For acceleration, the following applies:

$$\vec{a} = \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times \vec{v} = \vec{a}_t + \vec{a}_n,$$

where $\varepsilon = \dfrac{\Delta\omega}{\Delta t}$ is the magnitude of angular acceleration.

The unit of angular acceleration is rad.s^{-2}.

Circular motion

For acceleration components, the following applies:

• tangent component of acceleration $\vec{a}_t = \vec{\varepsilon} \times \vec{r}$, has a direction tangent to the trajectory. The size of the tangent component is $a_t = \pm\varepsilon \cdot r$, where the top sign holds for $\omega > 0$ and lower for $\omega < 0$.

• normal component of acceleration $\vec{a}_n = -\omega^2 \vec{r}$, has a direction to the center of a circular trajectory, and hence it is called centripetal acceleration.

The size of the normal component is $a_n = \frac{v^2}{r} = \frac{\omega^2 r^2}{r} = \omega^2 r.$

From the previous equations, it follows that the magnitude of acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = r\sqrt{\varepsilon^2 + \omega^4}.$$

a) Uniform circular motion

Angular velocity(rad.s^{-1}): ω = const.

Angular coordinate or angular path (in radians – rad):

$\varphi = \varphi_0 + \omega \cdot t,$ kde φ_0 is the initial angular coordinate.

The time course of the motion of a mass point is usually described by the traveled path *s*.

Arc length (in meters): $s = \varphi \cdot r$.

The following quantities can be defined for uniform circular motion. If in time $t = 0$ a mass point is in the initial position $\varphi_0 = 0$, then its angular path is $\varphi = \omega \cdot t$.

a) Uniform circular motion

This motion is periodic, that is, it is constantly repeated.

The basic characteristic of periodic motion is the time of one orbit – period T. The unit of period is the second.

For time $t = T$, the angular path is equal to $\varphi = \omega \cdot T = 2\pi$, circumferential path is equal to the length of the circle (circumference of the circle) $s = 2\pi r$.

From this it follows that the period is $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi r}{v}$, when using the relationship between circumferential and angular velocity $\omega = \dfrac{v}{r}$.

Frequency is the number of revolutions per second (the reciprocal of the period) $f = 1/T$. The frequency indicates how many times per unit of time the entire motion is repeated. The unit of frequency is s^{-1} = Hz (hertz).

By combining two equations, we get the relation $\omega = 2\pi f$.

b) Uniformly accelerated circular motion

Angular acceleration: ε = const.

Angular velocity: $\omega = \omega_0 + \varepsilon \cdot t,$ where ω_0 is the initial angular velocity.

Angular coordinate: $\varphi = \varphi_0 + \omega_0 t + \frac{1}{2}\varepsilon \cdot t^2.$

Summary

The definitions of the angular quantities φ, ω, ε are analogous to the definitions of the translation quantities s, v, a. Therefore, they lead to analogous results.

There is an analogy between the relations for straightforward (translation) motion and circular motion. For example, for uniformly accelerated motion, the following applies:

Straightforward motion		**Circular motion**	
acceleration	$a = const.$	**angular acceleration**	$\varepsilon = const.$
velocity	$v = v_0 + a \cdot t$	**angular velocity**	$\omega = \omega_0 + \varepsilon \cdot t$
path	$s = v_0 t + \frac{1}{2} a \cdot t^2$	**angular path**	$\varphi = \omega_0 t + \frac{1}{2} \varepsilon \cdot t^2$

Basic types of motions

Acceleration / Motion	**normal** (changes the direction of velocity)	**tangential** (changes the magnitude of velocity)
straightforward uniform	$a_n = 0$	$a_t = 0$
straightforward non-uniform	$a_n = 0$	$a_t \neq 0$
straightforward uniformly accelerated (uniformly slowed)	$a_n = 0$	$a_t = const. \neq 0$, $a > 0$ $(a < 0)$
curvilinear uniform	$a_n \neq 0$	$a_t = 0$
curvilinear non-uniform	$a_n \neq 0$	$a_t \neq 0$
circular uniform	$a_n = const. \neq 0$	$a_t = 0$
circular non-uniform	$a_n = const. \neq 0$	$a_t \neq 0$

Examples: Uniform straightforward motion

1. A car travels at a speed v = 60 km/h on the highway without acceleration. What distance does the car move in 10 seconds?

Solution:

As a first step, you need to convert the speed unit from km/h to the basic units of the system SI – m/s:

$$v = 60\,\text{km/h} = \frac{60 \cdot 1000}{3600}\ \text{m/s} = \frac{50}{3}\ \text{m/s} = 16{,}67\ \text{m/s}.$$

The motion is uniform without acceleration (the speed is constant 60 km/h). We assume that the motion is straightforward.

Therefore, we can use the relation to calculate the length of the path traveled by a car during 10 s: $s = v \cdot t$.

We substitute the specified values $s = 16{,}67 \cdot 10 = 166{,}7$ (m).

Answer: A car travels the distance 166,7 m in 10 seconds.

Examples: Uniformly accelerated straightforward motion

2. The car starts from a standstill and accelerates with constant acceleration $2\ \mathrm{m/s^2}$. What distance it travels in 5 seconds and what speed it has at this moment?

Solution:

The car starts from a standstill, which means that the initial speed is $v_0 = 0\ \mathrm{m/s}$ and moves with acceleration $a = 2\ \mathrm{m/s^2}$, i.e., the movement of the car is uniformly accelerated.

In 5 seconds, the speed and trajectory are:

$$v = v_0 + at = 0 + 2 \cdot 5 = 10\ (\mathrm{m/s}),$$

$$s = s_0 + v_0 t + \frac{1}{2} a t^2 = 0 + 0 \cdot 5 + \frac{1}{2} \cdot 2 \cdot 5^2 = 25\ (\mathrm{m}).$$

Answer: The car travels a distance of 25 m in 5 s and its speed is at this point10 m/s.

Examples: Uniform circular motion

3. A stone tied to a string rotates evenly in a circle of radius *r* = 1 m, with a velocity *v* = 4 m/s. What is the period of rotational motion and what is its centripetal acceleration?

Solution:

It is a uniform circular motion where the total acceleration is $\vec{a} = \vec{a}_t + \vec{a}_n$. At the same time, the magnitude of tangential acceleration is zero (the motion is uniform, the magnitude of the circumferential velocity does not change).

The vector of the normal component of acceleration points to the center of the circular trajectory. Therefore, the magnitude of the normal (centripetal) component of acceleration can be calculated from the relation $a_{\mathrm{d}} = \frac{v^2}{r} = \frac{4^2}{1} = 16\ (\mathrm{m}/s^2)$.

Period is $T = \frac{s}{v} = \frac{2\pi r}{v} = \frac{2\ \pi \cdot 1}{4} \approx 1{,}57\ (\mathrm{s})$.

Answer: The period of rotational motion is 1,57 s and its centripetal acceleration is $16\ \mathrm{m}/s^2$.

Literature

1. Svoboda, E. a kolektiv. Přehled středoškolské fyziky. SPN, Praha, 1991.
2. Halliday, D., Resnick, R., Walker, J. Fyzika, VUTIUM, Brno, 2000.
3. Urgošik, B. Fyzika, STNL, Praha, 1987.
4. Lepil, O., Bednařík, M., Hýblová, R. Fyzika pro střední školy. Prometheus, Praha, 1993.
5. Lepil, O., Houdek, V., Pecho, A. Fyzika pro III. Ročník gymnázií, SPN, Praha, 1986.
6. Hradilová, E., Uhdeová, N. Fyzika Přijímací zkoušky na vysoké školy, VUT v Brně, Brno, 2003.
7. Mechlová, E., Košťál, K. a kol. Výkladový slovník fyziky pro základní vysokoškolský kurz fyziky. Prometheus, Praha, 2001.

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