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subject : review of Pavel Loucka thesis

This document is a review of the thesis of Pavel Lucka, supervised by Petr Vasik from IM FME BUT in Brno. The document is entitled “*Algorithms for Conics in Geometric Algebras*”.

This thesis explores Geometric Algebra and conics, an active research area, particularly focused on constructing conics from a minimal set of control points or through conic fitting techniques. It also addresses the geometrical interpretation and analysis of conics.

From a scientific view point, this work is clearly relevant. Conic section is still not completely clarified in Geometric Algebra. Conic section construction, fitting, intersection and the way it relates to improper points is in my opinion a clearly valuable research direction. Through this document, Pavel Loucka demonstrates his knowledge about this topic. Conics study can find applications in various domains, such as video games, computer vision or physics.

The dissertation clearly meets the objectives stated in the *doctoral thesis topic* presented last year, as well as the objectives claimed in the introduction.

The document consists of a 107 pages document organized into an introduction, three chapters and a conclusion:

- The first chapter presents GAC (*Geometric Algebra for Conics*), a foundational framework for representing and manipulating conics using Geometric Algebra.
- The second chapter focuses on conic fitting with respect to GAC. Pavel Lucka examines various algorithms tailored to geometric constraints such as centered, axis alignment, or both, or just general conics. This section includes an advanced method for fitting conics through a set of points with exact passage through selected waypoints, a significant achievement. Testing concludes the section, including tests about robustness against Gaussian noise.

- The third chapter addresses conic construction from a minimal set of control points. It begins with the traditional approach of the wedge of five points, permitting up to two points at infinity. Pavel Lucka then explores constructing a conic from the pencil of two conics intersecting at four points, leading to an interesting analysis on creating line-pairs or generalized parabolas from these pencils. This section also covers construction algorithms and their geometric interpretations.

Each section of the document is presented with clarity, and relevant bibliographic references are thoughtfully integrated. Regarding scientific contributions, certain aspects stand out as particularly innovative, such as the study of waypoints in the fitting process and the analysis of generalized parabolas derived from a pencil of two conics. In the fitting section, a comparison with existing methods and their respective objective functions would have been beneficial. Indeed, most methods fitting points in linear time minimize an algebraic error, but minimizing a geometric error would provide better results, at the price of being non-linear and long to compute. A comparison with other Geometric Algebra frameworks for conics would also have enriched the work.

Overall, I find that Pavel Lucka's thesis demonstrates his scientific expertise and his capacity for innovation in research. For these reasons, I recommend awarding him the PhD degree.

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