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Statistical Estimate of Uniaxial Compressive Strength of Rock Based on Shore Hardness

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Abstract

The paper presents the use of advanced stochastic simulation techniques for estimating the strength behavior of rock materials. The Shore Rebound hardness was measured on fifty rock specimens coming from eleven different geological localities in Czech Republic. The dry unit weight of every tested rock material was determined also. Uniaxial compressive strength of rock was evaluated then by conducting the compression test on every specimen. Empirical distribution of Shore hardness and dry unit weight variables obtained from laboratory tests was approximated by the best fitted theoretical probability distribution. The stochastic simulation using Latin Hypercube Sampling was conducted based on those distributions. Two different equations used for estimating the compressive strength of rock on the basis of Shore hardness in practice was used as model functions. Comparison and statistical evaluation of uniaxial compressive strength of rock determined by compression test and those obtained as a result of stochastic simulation is discussed. The description of probability distribution of uniaxial strength is obtained as a result of introduced analysis, which can be used as input for fully probabilistic design models of rock materials.

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*Keywords:*Rock mechanics; Shore Scleroscope; Rebound hardness; Uniaxial compressive strength of rock; Stochastic simulation techniques; Latin Hypercube Sampling; Test of goodness of fit; Probability distribution

1. Introduction

Several studies using Shore hardness have been done to estimate strength parameters of intact rock. For example: [1], [2] or [3]. Extension of referenced knowledge and practical experience in using of Shore hardness parameter from region of Czech Republic is presented in this paper. The authors collected relatively wide range of rock types,

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hence the results of examination of currently used correlations are general valid. Shore hardness of rock specimens was measured firstly. Uniaxial compressive strength was tested on the same specimens so obtained results can be correlated and reliably compared. Two basic formulas for estimation of UCS from Shore hardness were chosen for detailed analysis. Results of measurements were then used as inputs for stochastic simulation method and sensitivity analysis to evaluate the possibility of estimation of rock material strength in this way.

2. Description of tested rocks and laboratory tests

2.1. Locations of rock sampling

Samples of the rocks come from 11 different locations around Czech Republic, see Table 1. Places are concentrated in Northern Bohemia, in surroundings of Prague and in Moravia. Thus, selected rocks are covering considerable part of the country and they consist of wide variety of different rock types. The data contained in this paper give interesting opportunity to analyse possible correlations of characteristics among various rock types.

Table 1. Description of locations and rock types.

No.	Name of locality	Rock type	Origin
1	Dolní Kounice	Granodiorite	igneous
2	Ústi nad Labem	Trachyte	igneous
3	Velké Opatovice	Sandy marlite	sedimentary
4	Hrob	Paragneiss	metamorphic
5	Čertovy schody	Limestone	sedimentary
6	Dolní Žleb	Sandstone	sedimentary
7	Vlastějovice	Orthogneiss	metamorphic
8	Hanušovice	Amphibolite	metamorphic
9	Vilémov	Phyllite / Quartzite	metamorphic
10	Vrané nad Vltavou	Tuffite	sedimentary
11	Štěchovice	Shale	sedimentary

2.2. Laboratory tests

There were 5 samples of rock tested coming from each locality. Exceptions are locality no. 4 – Hrob where only two samples were possible to prepare and locality no. 7 – Vlastějovice with four extracted samples. The test specimens were prepared from drill cores therefore they had cylindrical shape with diameter 44 mm and height approx. 75 mm. The density was identified according precise measurement of dimensions and weight for each sample.

Further there was tested scleroscopic hardness with apparatus Shore–type D (manufacturer: The Shore Instrument & Mfg. Co. N. Y.). There were always 10 rebounds recorded on down and up base. It is 20 rebounds for each specimen totally. Obtained values were statistically processed afterwards and the uniaxial compressive strength was calculated according correlations (11) and (12). Finally, the uniaxial compressive strength was tested directly in hydraulic press Advantest 9 (manufacturer: Controls) in technological centre AdMaS. The speed of loading was set to approx. 0.3 MPa/s until failure. Moisture of samples during the test was equal to laboratory environment.

3. Statistical analysis of measured values of tested characteristics of rock

3.1. Theoretical probability distribution of input parameters

In the case of rock properties, a normal distribution often already shows an adequate compliance. For rock parameters, which show typically a large scatterings the lognormal distribution is preferable according [4]. The normal distribution (or Gaussian) of dry unit weight of rock was presupposed first. The normal distribution N (μ, σ^2) is a continuous probability distribution described by parameters μ and σ^2. It is defined by the probability density function (PDF) [5]:

$$f_{(x)} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{1}$$

The parameter μ is the mean of the distribution and the parameter σ^2 is its variance. The sample variance $\bar{X}$ was used as an estimator of the mean μ. The Shore hardness (with higher CoV) was approximated by Lognormal distribution $ln\ N(\mu,\sigma^2)$, which is a continuous probability distribution of random variable, whose logarithm is normally distributed. In other words, if the random variable X is lognormally distributed, then $Y=log(X)$. Distribution is described by parameters λ and ζ. It is defined by the probability density function (PDF).

$$f_x(x;\lambda;\zeta) = \frac{1}{x\zeta\sqrt{2\pi}} e^{-\frac{(\ln x-\lambda)^2}{2\zeta^2}} \tag{2}$$
$$0 < x < \infty; 0 < \lambda < \infty; \zeta^2 > 0$$

3.2. Test of goodness of fit

The Anderson–Darling test is a statistical test of whether a given sample of data is drawn from a given probability distribution. The Anderson-Darling (AD) test was used to decide if a data in samples of dry unit weights and Shore hardness comes from a population with presupposed distributions. The null hypothesis “The data follow the presupposed distribution” is tested.

The Anderson-Darling statistic was given by the formula:

$$AD = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1) * \left[ln\, F(X_i) + ln\big(1 - F(X_{n-i+1})\big)\right] \tag{3}$$

where n is sample size, F(X) is cumulative distribution function for the tested distribution and i is the i^{th} sample, when the data is sorted in ascending order. The adjusted value of statistic is given by:

$$AD* = AD\left(1 + \frac{0,75}{n} + \frac{2,25}{n^2}\right). \tag{4}$$

P-value for adjusted AD statistic was possible to determine and use to conclude, if the test was significant in case of this study. The p-value is the probability of getting a more extreme result if the null hypothesis is true. If the p-value was higher than the significance level $\alpha = 0.05$, the null hypothesis was accepted. There are different equations for calculation p-value depending on the value of AD* [6]. AD test for the Lognormal distribution was implemented by transforming the data using a logarithm and using above test for normality. Description of probability distributions based on above described analysis is summarized in two tables below:

Table 2. Dry unit weight of rock *γ* [kg/m³] – Normal probability distributions.

No.	Name of locality	Rock type	Mean [kg/m³]	Std	COV
1	Dolní Kounice	Granodiorite	2618	12	0.005
2	Ústi nad Labem	Trachyte	2423	24	0.010
3	Velké Opatovice	Sandy marlite	2152	74	0.045
5	Čertovy schody	Limestone	2669	4	0.001
7	Vlastějovice	Orthogneiss	2579	16	0.006
8	Hanušovice	Amphibolite	2869	69	0.024
10	Vrané nad Vltavou	Tuffite	2626	13	0.057
11	Štěchovice	Shale	2690	16	0.006

Table 3. Shore hardness *Sh* [-] – Lognormal probability distributions.

No.	Name of locality	Rock type	Mean [-]	Std	COV
1	Dolní Kounice	Granodiorite	68	7	0.097
2	Ústi nad Labem	Trachyte	67	6	0.087
3	Velké Opatovice	Sandy marlite	37	5	0.139
5	Čertovy schody	Limestone	46	11	0.109
7	Vlastějovice	Orthogneiss	67	8	0.119
8	Hanušovice	Amphibolite	59	10	0.172
10	Vrané nad Vltavou	Tuffite	74	7	0.095
11	Štěchovice	Shale	65	5	0.081

3.3. Analysis of small samples

The analysis of small samples is not reliable and results contains unusually high rate of uncertainty [7]. For number of samples n 4 ≤ 20 a procedure based on order statistics was introduced by Horn [8, 9]. This approach is based on a depth which corresponds to the sample quartiles. The pivot depth is expressed by

$$H = int \frac{(n+1)/2}{2} \tag{5}$$

Or

$$H = int \frac{\frac{(n+1)}{2}+1}{2} \tag{6}$$

according to which H is an integer. The Lower pivot is

$$x_L = x(H) \tag{7}$$

And the upper pivot is

$$x_U = x(n+1-H) \tag{8}$$

The estimate of the parameter of location is then expressed by the pivot halfsum

$$P_L = \frac{x_L + x_U}{2} \tag{9}$$

The results are summarized in Table 4.

Table 4. Ultimate compressive strength for rock in uniaxial compression σ_c [MPa] – values from uniaxial compression test.

No.	Name of locality	Rock type	Pivot halfsum [MPa]
1	Dolní Kounice	Granodiorite	66
2	Ústi nad Labem	Trachyte	68
3	Velké Opatovice	Sandy marlite	52
5	Čertovy schody	Limestone	45
7	Vlastějovice	Orthogneiss	71
8	Hanušovice	Amphibolite	49
10	Vrané nad Vltavou	Tuffite	72
11	Štěchovice	Shale	28

3.4. Transformation of random variables using simulation methods

The situation arises in the case of determining the ultimate compressive strength of rock in uniaxial compression where it is necessary to find the probability distribution of the random variable *Y* (σ_c), which is a function of the vector *X* of random variables (γ, *Sh*), whose probability distributions are known:

$$Y = h(X) \tag{10}$$

where *h* is a real function of two real variables defined on the field of values of a random vector *X*. It can be said, that the random variable *Y* is transformation of random vector *X* [10]. The transformation model function was considered alternatively by two equations. Equation (11) uses both parameters of Shore hardness *Sh* and unit weight γ [11]:

$$\sigma_1 = 10^{(0,00066\ .\gamma\ .Sh + 3,62)}.6,895 \tag{11}$$

And equation (12) [12], in where the ultimate compressive strength depends only on the Shore hardness *Sh*

$$\sigma_1 = 3,54.(Sh - 12) \tag{12}$$

Simulation methods such as Monte Carlo or variance reduction techniques such as LHS method can be advantageously used to accelerate and facilitate the transformation and to calculation of statistical moments of PDF assigned to random variables *Y* in this specific example focused on determination of ultimate compressive uniaxial strength [13]. A random vector *X* (with description of probability distribution and moments characteristics of its components calculated in the previous paragraphs) serves as the random input for further processing, which consists of the following steps:

- Defining the transformation function $Y = h(X)$;
- Calculating realization of function $Y = h(X)$ for all generated realizations of vector *X*;

- Assigning the most suitable probability distribution to the set of data consists of the realizations of the function $Y = h(X)$ and calculating its statistical central moments.

Latin Hypercube Sampling technique LHS–mean with 10e5 simulations was used for generating realizations of two-dimensional vector *X*. The parameters space was described by probability distributions summarized in the paragraph 2.1. The advantage of this method consists in the fact, that it requires less number of simulations while conserving significance estimates of statistical parameters usually [14]. Software tool Freet was used for performing the described analysis [15].

4. Results

Table 5. Results – estimate of uniaxial compressive strength of rock calculated according equation (11).

No.	Name of locality	Rock type	Mean [MPa]	Std	COV
1	Dolní Kounice	Granodiorite	156	26	0.164
2	Ústi nad Labem	Trachyte	134	18	0.132
3	Velké Opatovice	Sandy marlite	61	7	0.110
5	Čertovy schody	Limestone	91	11	0.121
7	Vlastějovice	Orthogneiss	152	28	0.182
8	Hanušovice	Amphibolite	150	43	0.285
10	Vrané nad Vltavou	Tuffite	185	33	0.180
11	Štěchovice	Shale	153	21	0.138

Table 6. Results – estimate of uniaxial compressive strength of rock calculated according equation (12).

No.	Name of locality	Rock type	Mean [MPa]	Std	COV
1	Dolní Kounice	Granodiorite	197	23	0.118
2	Ústi nad Labem	Trachyte	193	21	0.106
3	Velké Opatovice	Sandy marlite	90	18	0.206
5	Čertovy schody	Limestone	119	18	0.148
7	Vlastějovice	Orthogneiss	196	28	0.144
8	Hanušovice	Amphibolite	168	36	0.216
10	Vrané nad Vltavou	Tuffite	220	25	0.113
11	Štěchovice	Shale	188	19	0.100

The result of above presented analysis is the set of probability distributions described by their moment parameters summarized in Table 5 and Table 6. The design parameters (e. g. lover 5 % quantile, the upper 95 % quantile, the mean etc. according to the actual design situation) can be determined from these distributions via common statistical methods. Data in Table 5 and Table 6 were approximated by the Lognormal (3 par) probability distribution.

Additionally, the relative effect of each basic variable on the response of model function was measured using the partial correlation coefficient between each basic input variable and the response variable (uniaxial strength). The nonparametric rank-order statistical correlation was expressed by the Spearman correlation coefficient. A high positive correlation coefficient, in range < 0.9; 1.0 >, was observed for variable *Sh* for both model alternatives – equation (11) and equation (12). Opposite, the response of model equation (11) seemed not to be so sensitive on variable γ, which correlation coefficients was close to zero in case of every tested rock types.

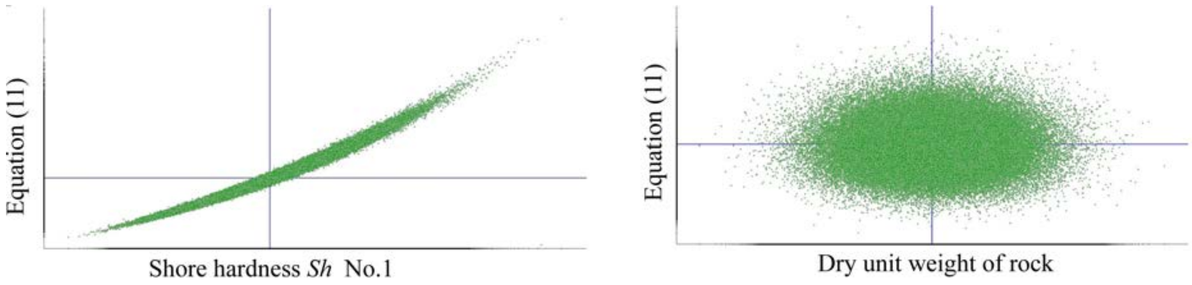

Fig. 1. Sensitivity of Equation (11) response on Shore hardness *Sh* (left) and on Dry unit weight of rock *γ* (right) for rock No.1 Granodiorite Dolní Kounice. Adapted from Freet graphical results.

Figure 1 (left picture) shows in cartesian coordinates the strong positive sensitivity of response of equation (11) on the Shore hardness *Sh*. The right picture illustrates the small sensitivity of the same model function on the dry unit weight of rock *γ*. Both pictures are related to the rock samples No. 1 – Granodiorite from Dolní Kounice.

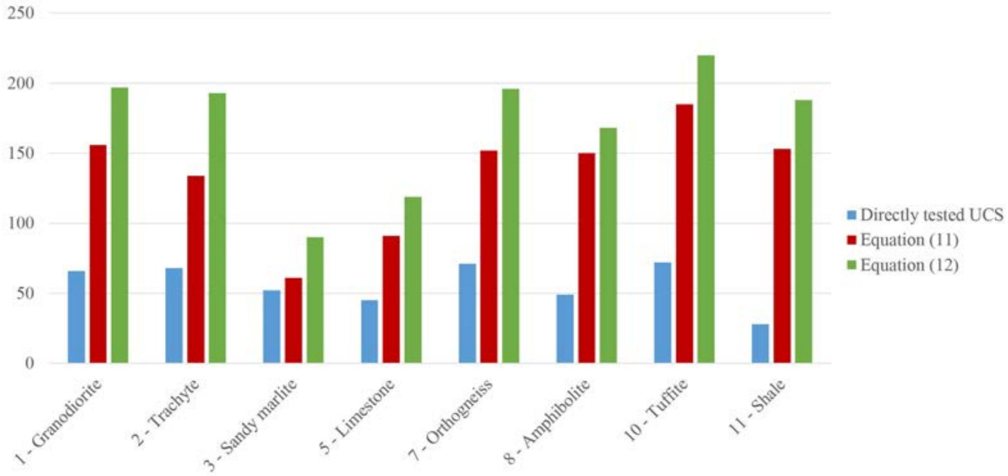

Fig. 2. Chart comparing results of Mean UCS [MPa] obtained from various methods.

5. Discussion

Stochastic simulation techniques were employed to make an estimate of the uniaxial compressive strength of variety of rock types originating from Czech Republic area. Two empirical equations formulating relationship between shore hardness and uniaxial compressive strength that are commonly used in practice were examined. Values of the uniaxial strength resulting from those relationships were compared to the values of uniaxial strength measured directly on rock samples tested in hydraulic press. There can be found some conclusions coming from comparison (see Fig. 1) of directly and indirectly assessed uniaxial strength of rock material:

- Both commonly used equations overestimate the uniaxial strength of rock. The relationship (12) according to [12] overestimates the strength more than the equation (11) presented in [11].
- Bigger difference can be seen between the directly and indirectly determined strength in case of rock materials with the higher heterogeneity (e. g. Granodirite loc. 1) and orthotropy (e. g. Shales loc. 11).
- As the dynamic - elastic Shore scleroscope hardness test procedure was originally developed for testing of steel in metallurgy, the values of strength of relatively homogeneous materials (e. g. limestone loc. 5 or sandy marlite loc. 3) determined by this technique are apparently in good accordance with those measured directly.

Based on results of conducted measurement and calculations it can be recommended to use the shore scleroscope for estimating the uniaxial strength of rock with highest caution and with taking the information about origin of the tested rock into account. The correlations between shore hardness and uniaxial compressive strength should be continuously revised and improved to achieve their usability for practice. Results of analysis summarized in this paper can also be used for this purpose. Secondary outcome of completed work is the set of fully described probability distributions of dry unit weight of rock that can be eventually used in fully probabilistic design methods.

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