ANALYSIS OF EQUIVALENT THERMAL CONDUCTIVITY OF WINDING USING FEM-BASED MODEL

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Abstract: This paper deals with analysis of equivalent thermal conductivity of winding. Analytical method from literature is described first. Then, the FEM-based model is described, which was created in order to verify the functionality of the analytical method. Results of the FEM simulations showed that the analytical method works well.

Keywords: Electric machine, fill factor, slot, thermal analysis, thermal conductivity, winding

1 INTRODUCTION

From a temperature point of view, the most critical part of an electrical machine is usually a winding. The high temperature in the winding can significantly affect a winding insulation and thus a motor lifetime. Each thermal class of winding insulation has prescribed maximum permitted temperature rise (not a hot spot temperature) for which an insulation lifetime is declared (usually 20 000 h) [1]. Exceeding this temperature rise by approximately 8-10 K yields to shortening the insulation lifetime by half, see [1].

Winding temperature affects not only the insulation lifetime but it has also impact on an electric resistance of the winding. It is important to know the winding electric resistance (and thus its temperature) in many control strategies of electric machines, e.g., in an induction motor control strategy achieving maximum efficiency in a wide range of speed and torque, see [2]. For these reasons, it is necessary to know the winding temperature both, i.e., during a designing process of the machine and during machine operation.

Thermal modeling of winding can be quite difficult because it is composed of several parts with different thermal properties, namely of conductors with very good thermal conductivity (TC), conductor insulation with very low TC and impregnation which also has very low TC, see the Fig. 1. In thermal models of electrical machines, the winding is usually replaced by a homogeneous space with an equivalent thermal properties in individual directions of coordinate system, i.e. the equivalent homogenous material is orthotropic. Analytical equations for the calculating of an equivalent thermal conductivity (ETC) of the winding can be found in the literature, see [3]. The goal of this paper is to compare the analytical method with a numerical solution. There was created a thermal model based on the finite element method (FEM) in the Ansys software. This model is described in the paper.

2 ANALYTICAL METHOD FOR CALCULATING THE EQUIVALENT THERMAL CONDUCTIVITY OF WINDING

As previously mentioned, the resulting ETC of the winding depends on the thermal conductivities of the individual parts of the winding. But the resulting ETC also strongly depends on a fill factor of the slot. There are different definitions of the fill factor. Papers dealing with calculation of dependence of

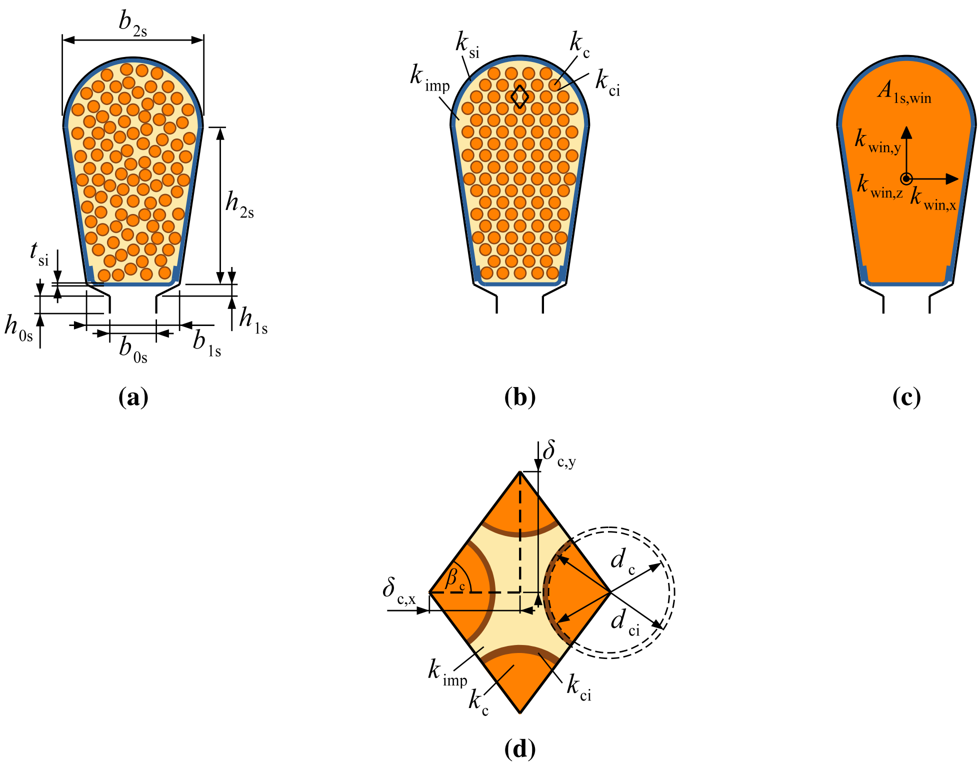

Figure 1: (a) Random distribution of wires in the slot and definition of slot dimensions. (b) Aligned distribution of wires for the FEM model. (c) Homogenized winding and equivalent thermal conductivities in individual directions. (d) Detail of the aligned distribution.

the ETC on the fill factor unfortunately do not usually define how the fill factor should be calculated. In this article, the fill factor is taken as ratio of the area of conductors (without their insulation) to the slot area in which the winding is concentrated, which is mathematically written as follows

$$K_{\mathrm{f,Cu}} = \frac{\pi d_{\mathrm{c}}^2 N_{\mathrm{c}}}{8 A_{\mathrm{1s,win}}}, \tag{1}$$

where $K_{\mathrm{f,Cu}}$ is the fill factor of winding, d_{c} is diameter of conductor (without its insulation), N_{c} is number of conductors in one slot, and $A_{\mathrm{1s,win}}$ is a part of one slot area in which the winding is concentrated, which can be calculated according to approximate equation

$$A_{\mathrm{1s,win}} \approx A_{\mathrm{1s}} - p_{\mathrm{s}} t_{\mathrm{si}}, \tag{2}$$

where A_{1s} is total area of one slot, p_{s} is perimeter of the slot, and t_{si} is thickness of a slot insulation. The parameters A_{1s} and p_{s} can be calculated according to slot dimensions, see Fig. 1 (a), as follows

$$A_{\mathrm{1s}} = h_{\mathrm{2s}} \frac{b_{\mathrm{1s}} + b_{\mathrm{2s}}}{2} + \frac{\pi b_{\mathrm{2s}}^2}{8}, \tag{3}$$

$$p_{\mathrm{s}} = b_{\mathrm{1s}} + \frac{\pi b_{\mathrm{2s}}}{2} + 2\sqrt{\frac{(b_{\mathrm{2s}} - b_{\mathrm{1s}})^2}{4} + h_{\mathrm{2s}}^2}. \tag{4}$$

The analytical calculation of the ETC of winding is divided into two steps [3]. First, the equivalent thermal conductivity of homogenised conductor and conductor insulation k_{c}^* is calculated according to the equation [3]

$$k_{\mathrm{c}}^* = k_{\mathrm{ci}} \frac{k_{\mathrm{c}}(1+\chi_{\mathrm{c}}) + k_{\mathrm{ci}}(1-\chi_{\mathrm{c}})}{k_{\mathrm{c}}(1-\chi_{\mathrm{c}}) + k_{\mathrm{ci}}(1+\chi_{\mathrm{c}})}, \tag{5}$$

where k_c is the TC of conductor, k_{ci} is the TC of conductor insulation, and χ_c is the area ratio of the conductor cross-section to wire cross-section (including conductor insulation) calculated according to the equation [3]

$$\chi_c = \left(\frac{d_c}{d_{ci}}\right)^2, \tag{6}$$

where d_{ci} is outer diameter of conductor insulation.

Subsequently, the required ETC of homogenised winding $k_{win,x,y}$ can be calculated according to the equation [3]

$$k_{win,x,y} = k_{imp}\frac{k_c^*(1+K_{f,Cu})+k_{imp}(1-K_{f,Cu})}{k_c^*(1-K_{f,Cu})+k_{imp}(1+K_{f,Cu})}, \tag{7}$$

where k_{imp} is the TC of winding impregnation. The symbols "x" and "y" in the subscript indicate that the calculated ETC corresponds to the x- and y-direction, see the Fig. 1 (c). Graphical interpretation of the equation (7) can be seen in the Fig. 3 (b) where the results are also compared with the FEM-based model results.

Calculation of the ETC of winding in z-direction is much easier than in the x or y ones. Assuming that $k_c \gg k_{ci}$ and $k_c \gg k_{imp}$, the ETC of homogenised winding in z-direction can by calculated simply as follows

$$k_{win,z} = k_c K_{f,Cu}\frac{A_{1s,win}}{A_{1s}}. \tag{8}$$

The equation (8) assumes that the cross-sectional area of the homogenised winding in XY-plane equals to A_{1s}.

3 FEM-BASED MODEL FOR CALCULATING THE EQUIVALENT THERMAL CONDUCTIVITY OF WINDING

To verify the analytical method presented in the previous section, a thermal model based on the finite element method was created, which is described in this section. The model was created in the well-known Ansys software.

3.1 MODEL DESCRIPTION

Individual wires are in produced motor usually distributed randomly as shown in the Fig. 1 (a). Parametrization of such an arrangement would be very complicated, therefore in the FEM-based model, it is assumed that the wires are distributed in rhombus alignment, see the Fig. 1 (b), (d). The advantage is that this layout can be fully parametrizable. This approach is used for example in [4].

Created geometry can be seen in the Fig. 2. There are totally $15 \times 15 = 225$ wires in this geometry. The geometry has outer dimensions L_x, L_y, L_z in the respective directions. According to the Fig. 1 (d), there were derived equations for calculating the distances between the individual wires depending on the fill factor. In this case, the fill factor is taken as the ratio of the conductor area to the rhombus area, i.e., $K_{f,Cu} = (\pi d_c^2)/(8\delta_{c,x}\delta_{c,y})$. The distance of the conductors can be then calculated according to the following equations

$$\delta_{c,y} = d_c\sqrt{\frac{\pi\tan(\beta_c)}{8K_{f,Cu}}}, \tag{9}$$

$$\delta_{c,x} = \frac{\delta_{c,y}}{\tan(\beta_c)}, \tag{10}$$

where $\delta_{c,x}$ is the distance of adjacent conductors in the x-direction, $\delta_{c,y}$ is the distance of adjacent conductors in the y-direction, and β_c is the angle between the centres of adjacent conductors. The outer dimensions of the created geometry are calculated according to the equations

$$L_x = \delta_{c,x}(N_{c,x}+1), \tag{11}$$

$$L_y = \delta_{c,y}(N_{c,y}+1), \tag{12}$$

where $N_{c,x}$ is the number of conductors in x-direction and $N_{c,y}$ is the number of conductors in y-direction. The dimension L_z should be as small as possible to ensure a small number of elements. It is obvious according to the Fig. 2 that created geometry does not meet conditions of a consistent fill factor near the boundaries, but with a sufficiently large number of conductors, this effect is negligible.

According to the Fig. 1 (d), it is also possible to derive the maximum theoretical fill factor

$$K_{f,Cu,max} = \frac{\pi}{8\sin(\beta_c)\cos(\beta_c)}\left(\frac{d_c}{d_{ci}}\right)^2. \tag{13}$$

Graphical interpretation of the equation (13) can be seen in the Fig. 3 (a).

3.2 Boundary Conditions and Simulation Results

Using the FEM model, the ETCs $k_{win,x}$ and $k_{win,y}$ were evaluated. The boundary conditions that were used in the simulations are shown in the Fig. 2. The individual ETCs were determined separately, so the shown boundary conditions were not used simultaneously (the solutions are one-dimensional).

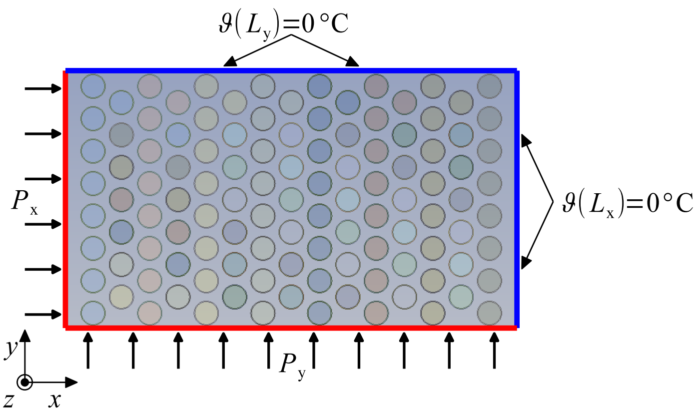

Figure 2: The geometry of the FEM-based model and definition of boundary conditions (during the simulations, the boundary conditions for x- and y-direction are not applied simultaneously).

By applying mentioned boundary conditions, the maximum temperature ϑ_{max} is at the side where the heat flow (P_x or P_y) enters. Then it possible to determine the required ETCs according to the equations

$$R_x = \frac{L_x}{L_y L_z k_{win,x}} = \frac{\vartheta_{max}}{P_x} \Rightarrow \boxed{k_{win,x} = \frac{L_x}{L_y L_z}\frac{P_x}{\vartheta_{max}}}, \tag{14}$$

$$R_y = \frac{L_y}{L_x L_z k_{win,y}} = \frac{\vartheta_{max}}{P_y} \Rightarrow \boxed{k_{win,y} = \frac{L_y}{L_x L_z}\frac{P_y}{\vartheta_{max}}}. \tag{15}$$

A total of 75 simulations with different values of the parameters $K_{f,Cu}$, β_c, and k_{imp} were performed (dimensions of the FEM geometry were recalculated according to the equations (9)-(12)). For the all simulations: $d_c = 1$ mm, $d_{ci} = 1.07$ mm, $k_c = 380$ W/(m·K), and $k_{ci} = 0.25$ W/(m·K). The FEM model results are shown in the Fig. 3 (b).

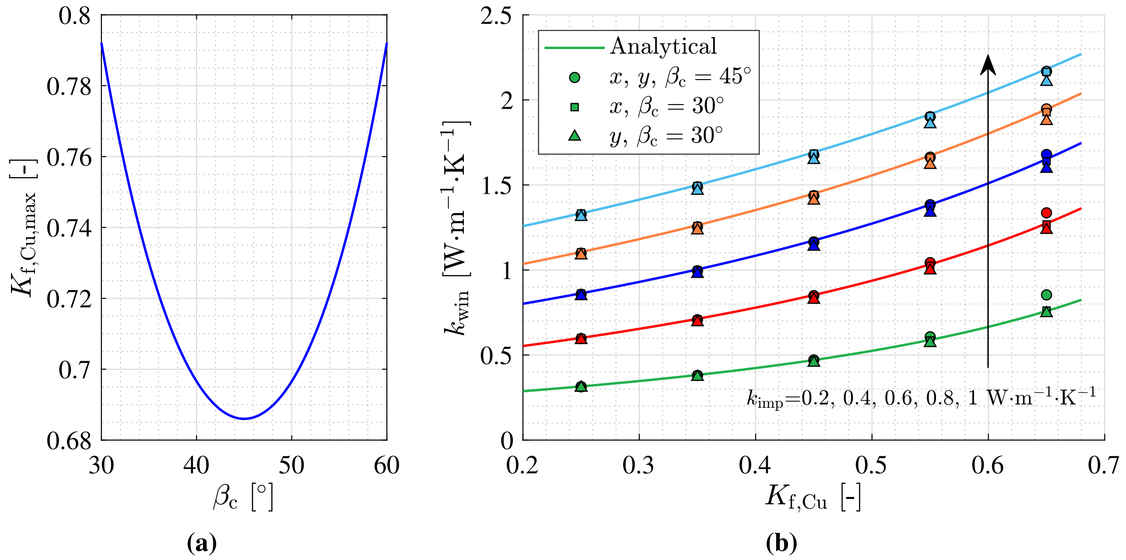

Figure 3: (a) Maximum theoretical fill factor depending on the angle between the centres of adjacent conductors. (b) Comparison of analytical calculations with FEM simulation results.

4 CONCLUSION

The aim of the paper was to verify the analytical method for calculating the equivalent thermal conductivity of winding. The FEM-based model was created for this purpose. The comparison of the analytical and numerical results can be seen in the Fig. 3 (b). The differences in the results are very small. It was also found that the equivalent thermal conductivities in different directions do not differ much depending on the arrangement of the conductors.

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