DEPENDENCY OF THE CONVERGENCE RATE MEAN EXTENT OF VARIATION ON THE REPETITIONS NUMBER IN STRONGLY CONNECTED TOPOLOGIES

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Abstract: This paper deals with the stochastic distributed algorithm – the push-sum protocol. We examine the effect of experiments repetitions on the mean of the convergence rates quantities. The main goal of the executed experiments is to show how many repetitions of the push-sum protocol are necessary to achieve a statistically credible representative of the obtained set of data. Within this paper, we have focused on strongly connected structures.

Keywords: Distributed computing, The push-sum protocol, The convergence rate mean extent of variation

1. INTRODUCTION

In the last decades, the evolution within the computer science has been strongly affected by a novel manner of communication among computers – so-called the distributed way [1]. The centralized systems, dominating in the past, are being substituted by the systems exploiting this novel computation manner. The systems executing distributed computing are labelled as the distributed systems and are usually formed by a set of the agents whose awareness of the other elements (as well as the system as the whole) is significantly limited.

In this paper, the authors‘ attention is focused on the stochastic distributed algorithm – the push-sum protocol. We examine the effect of experiments repetations on the mean of the convergence rates quantities. Even thought the experiment with more repetitions results in more statistically credible data, too extensive execution poses a demanding computing process. Thus, the motivation of this paper is to show how the number of the repetitions of the same experiment affects the credibility of the obtained results. The experiments is executed on a fully-connected topology, which is a representative of a strongly connected network.

2. THE PUSH-SUM PROTOCOL

The push-sum protocol is classified as a multi-purpose gossip-based aggregation algorithm, whose functionality is based on an iterative pair-wise distribution of the aggregated values among particular agents. Its execution is not deterministic; therefore, repetitions of an experiment may result in various outputs in spite of the constant inputs. As mentioned, the push-sum protocol can solve more problems after a smaller modification. Below is described the execution of the protocol [1]:

- The initial inner states of all the nodes represent the values from which the average is calculated. The initial weights are set to 1 for each node.

- Each node sends a half of its current inner state and a half of its weight to one of its neighbors chosen uniformly at random. The same value is stored in the inner memory and participates in further computation. This is repeated at each iteration.
- The sum of the inner state and the inner states sent by the adjacent nodes is the inner state for the next iteration. The sum of the weight stored in the inner memory and the weights sent by the adjacent nodes is the weight for the next iteration.
- The ratio of these two sums (the sum of the inner states is the numerator) poses the estimation of the average.
- The previously described procedure is repeated until the consensus is reached

3. THE EXAMINED TOPOLOGY

Within this paper, we focus our attention on the fully-connected mesh. As mentioned, the choice of this particular topology was related to its character. It is maximally connected structure, where the protocol is assumed to achieve fast convergence rates. Within this paper, a mesh topology with the size of eight agents is assumed.

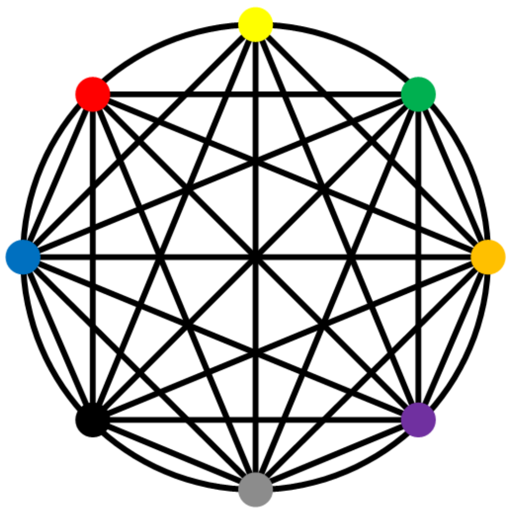

Figure 1: The examined fully-mesh topology

4. EXPERIMENTS

In this chapter, the results obtained within the numerical experiments are presented. The goal of these experiments is to show how the convergence rate mean extent of variation [2] of a data set varies when the number of repetitions changes. We change the number of the repetitions and repeat the experiment five times for each instance. In each figure, one of the repetitions is highlighted by a solid line in order to show the character of the obtained data.

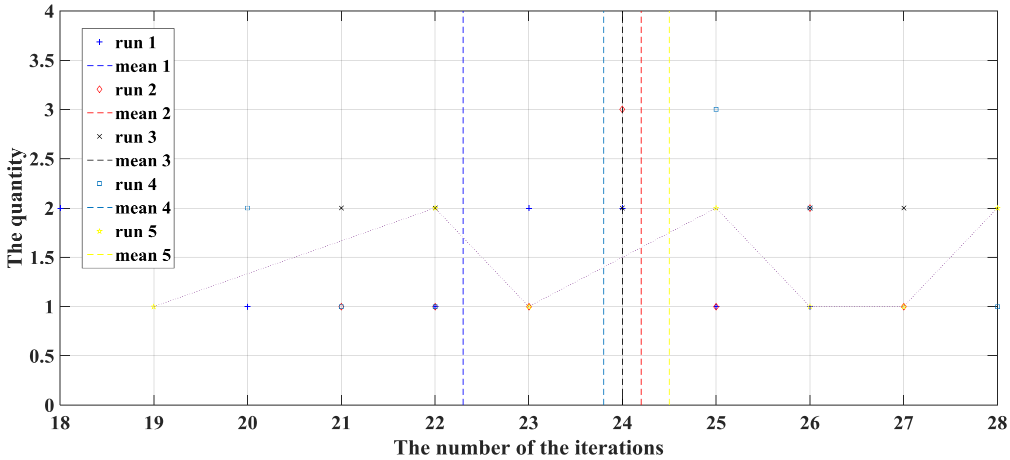

Figure 2: The results for 10 repetitions

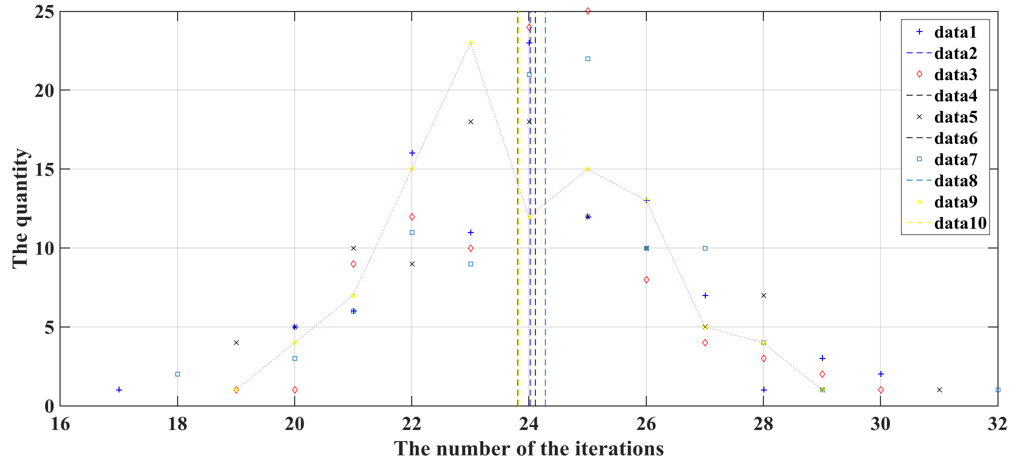

Figure 3: The results for 100 repetitions

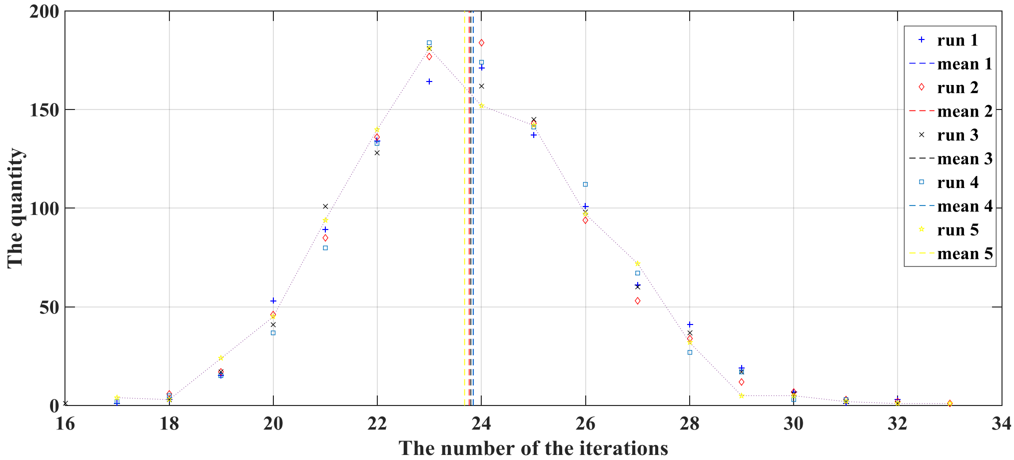

Figure 4: The results for 1 000 repetitions

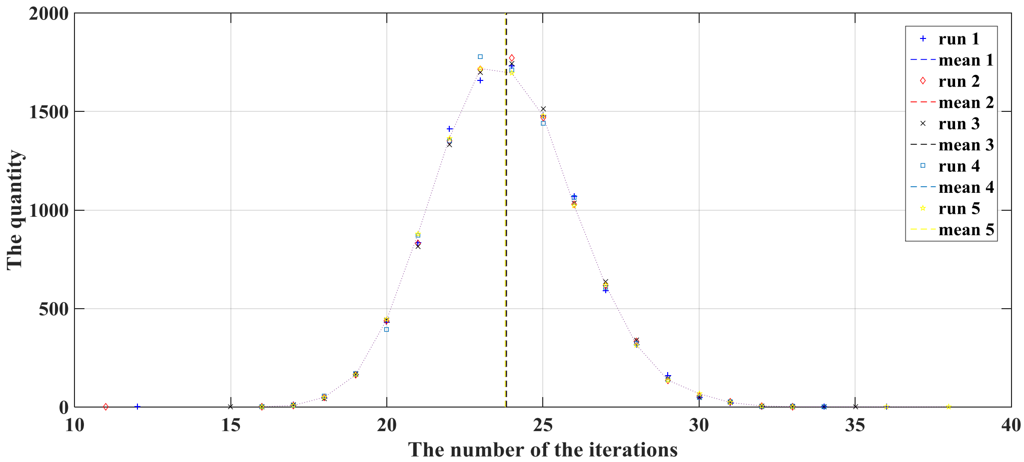

Figure 5: The results for 10 000 repetitions

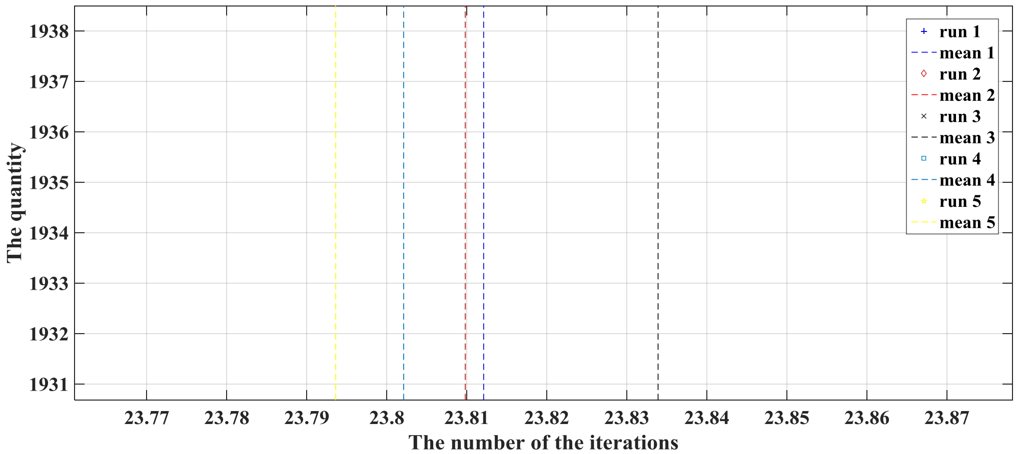

Figure 6: A detailed view of the results for 10 000 repetitions

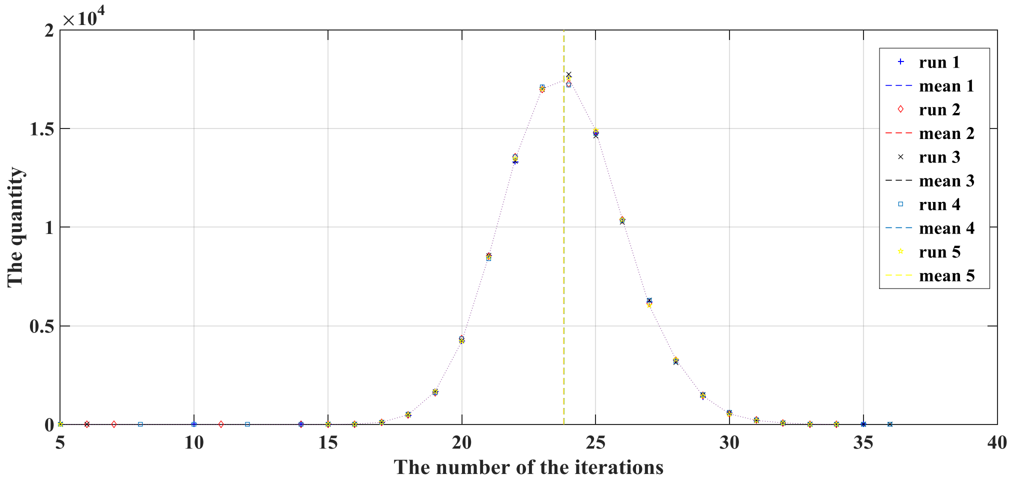

Figure 7: The results for 100 000 repetitions

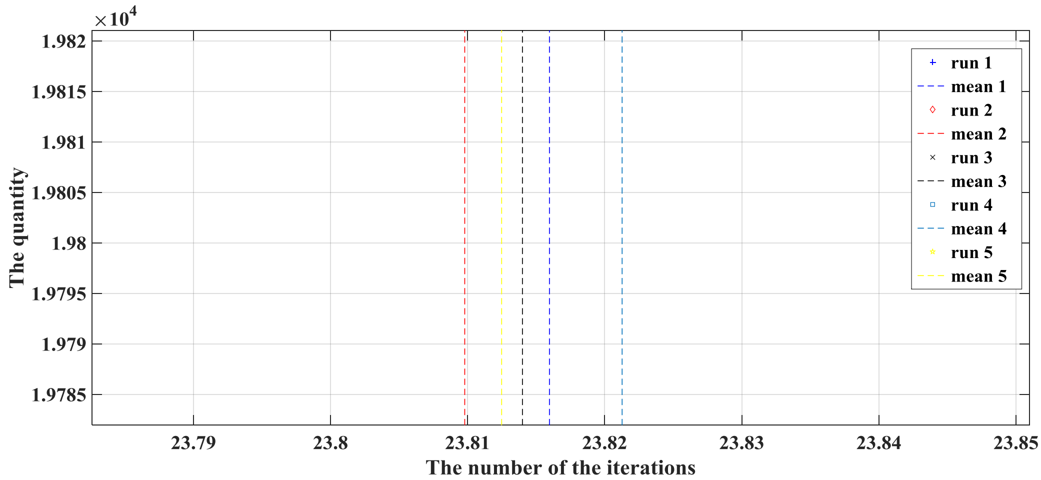

Figure 8: A detailed view of the results for 100 000 repetitions

	10	100	1 000	10 000	100 000
Run 1	22.3	24.02	23.843	23.8121	23.8160
Run 2	24.2	24.11	23.765	23.8098	23.8098
Run 3	24	23.81	23.797	23.8339	23.8140
Run 4	23.8	24.28	23.841	23.8021	23.8213
Run 5	24.5	23.82	23.677	23.7936	23.8125
Extent of variation	2.2	0.47	0.166	0.0403	0.0115

Table 1: The convergence rate mean extent of variations for the particular numbers of the repetitions

As we can see from the results, more repetitions result in a smaller extent of variation. Thus, as expected, a higher number of the repetitions ensures more credible results from the statistical point of view.

5. CONCLUSION

In this paper, the impact of the numbers of the push-sum protocol repetitions on the convergence rate mean extent of variation has been investigated. The experiments were executed on a fully-connected mesh topologies with the size of eight agents. The obtained results show that more repetitions ensure more credible results with a smaller convergence rate mean extent of variation.

ACKNOWLEDGEMENT

Research described in this paper was financed by the National Sustainability Program under grant LO1401. For the research, infrastructure of the SIX Center was used.

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