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THE EFFECT OF THE FREE SURFACE ON THE SINGULAR STRESS FIELD AT THE FATIGUE CRACK FRONT

OPLT Tomáš[1,2], POKORNÝ Pavel[2], NÁHLÍK Luboš[2], HUTAŘ Pavel[2]

[1] *Faculty of Mechanical Engineering, Brno University of Technology, Institute of Solid Mechanics, Mechatronics and Biomechanics, Technická 2896/2, Brno, Czech Republic*
[2] *Institute of Physics of Materials of the Academy of Sciences of the Czech Republic, Žižkova 22, Brno, Czech Republic, email: oplt@ipm.cz*

Abstract: Description of stress singularity in the vicinity of a free surface is presented. Its presence causes the retardation of the fatigue crack growth in that region and fatigue crack is being curved. Numerical model is used to study dependence of the stress singularity exponent on Poisson's ratio. Estimated values are compared to those already published. Experimentally measured angles of fatigue crack on SENB specimens confirm the relation between Poisson's ratio and the angle between crack front and free surface.

KEYWORDS: fatigue crack, stress singularity exponent, vertex singularity, crack front shape.

1 Introduction

The linear elastic fracture mechanics is typical approach for description of the fatigue crack behaviour. Stress field around a crack tip is described by stress intensity factor [1], [2], [3]. This approach is based on an assumption of two-dimensional singular stresses in the vicinity of the crack front with the stress singularity exponent equal to 0.5 [1]. In reality, the stress state near the crack tip is always three-dimensional. Square root singularity is always dominant in the middle of the body, where plain strain conditions prevail. In the intersection of the crack front with a free surface, so called vertex point, additional singularity appears [4], [5] Therefore, the stress field in the area near the free surface is much more complicated than in the middle of the body and its description is still open question for scientific community. In spite of the fact, that the effect of the free surface can be neglected for many applications, in some cases can have a strong influence on the fatigue crack behaviour.

During last few decades, several authors investigated effects of this vertex singularity, see e.g.[4]–[10]. Generally, they found that stress singularity exponent depends on Poisson's ratio. In case of a straight crack, the value of the stress singularity exponent in the intersection between crack front and free surface is always smaller then 0.5 [4], [5]. Lower value of the stress singularity causes a decrease of the fatigue crack propagation rate [6], [9]. Main effect of the free surface is that the fatigue crack does not grow as a straight line, but the crack front is typically curved. Despite all previous research and present knowledge, accurate definition of the area influenced by the free surface or an interaction between two free surfaces in case of thin structures are still open questions.

Aim of this paper is to describe the singular stress field of the fatigue crack front and explain typical curvature of the fatigue crack in the experimental specimens. First, the straight crack front stress field in the relatively thick Single Edge Notched Bend (SENB) specimen is accurately described by finite element simulation and stress singularity exponent was numerically estimated. Then, obtained data were compared with literature. The angle in the

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intersection between crack front and free surface was experimentally measured for EA4T steel and aluminium alloy to confirm numerical results experimentally.

2 Model

Numerical model of the standard SENB specimen was created in order to study the stress field in the region close to vertex point. Specimen (fig. 1) was of a thickness $2B$ = 20 mm, length $2L$ = 210 mm, height W = 50 mm and with a crack length of a = 15 mm. Material model was considered as isotropic linear elastic with Young's modulus E = 200 GPa and varying Poisson's ratio ν=0 – 0.499. Force, applied on the SENB to induce Mode I loading of the crack was F = 8000 N and corresponds to the experimental tests. Since the evaluation of the stress singularity exponent is very sensitive to the mesh of finite elements, very fine mapped mesh of linear SOLID185 elements was generated along the crack front. Elements were refined even more in the vicinity of the free surface. Given geometry allows to use an advantage of existing symmetry, only one-quarter of the specimen was modelled.

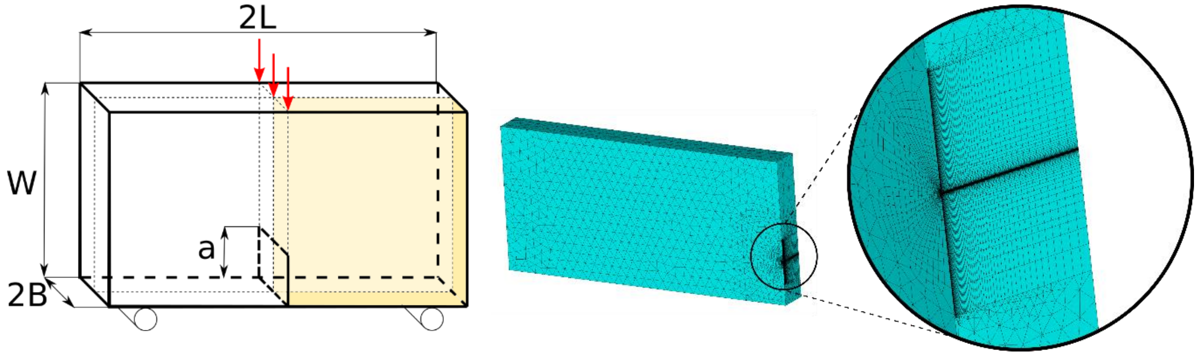

Fig. 1 Single Edge Notched Bend (SENB) specimen used
for numerical modelling and detail of refined mesh

3 Vertex point singularity

Fracture mechanic usually describes elastic stress field near the crack tip as [1]:

$$\sigma_{ij} = \frac{K_I}{r^{1/2}} f_{ij}\,(\theta), \tag{1}$$

where σ_{ij} are the elastic stress components, r and θ are polar coordinates with origin at the crack tip. K is the stress intensity factor (subscripts stands for denoting mode I) and $f_{ij}(\theta)$ is corresponding shape function. Concept of the stress intensity factor is based on the assumption of two-dimensional singular elastic stress field with the square-root singularity.

When the analysis is extended to the third dimension, the crack tip (point) becomes a crack front (line). Square-root singularity is valid in the middle of the body, where plane strain conditions exist. However, different singular field appears in the area where the crack front intersects the free surface, so called vertex point. Many authors put their effort to describe this singularity with different techniques, e.g. Bažant and Estenssoro [4] presented variational principles and Benthem [5] finite difference method. They found the vertex singularity is not of a power 0.5, but it changes. Based on their results, the power of the vertex singularity p is a function of Poisson's ratio and the value is in range between0.5 (for ν=0) and 0.33 (forν=0.5). The assumption of this solution is crack in semi-infinite plate.

According to paper [9], stress field along the crack front, vertex point included, can be described in each single plane perpendicular to the crack front by generalized stress intensity factor using relation generally approximated as 2D solution in the form:

$$\sigma_{ij} \approx \frac{H_I}{r^p} f_{ij}(p,\theta), \tag{2}$$

where H_I is the generalized stress intensity factor, p is the stress singularity exponent and $f_{ij}(p,\theta)$ is corresponding shape function. In this case, the stress and displacement components depend on the distance r from the crack front as $\sigma_{ij} \approx r^{-p}$ and $u_i \approx r^{1-p}$. Based on this relation, stress singularity exponent p can be estimated numerically. The methodology is presented on the fig. 2. An opening displacement u_x is plotted against the distance from the crack front r, both in logarithmic form. The stress singularity exponent is estimated by equation p = 1-A, where A is the slope of the line. As the mesh sensitivity of the direct method is high, very fine mesh along the crack front (especially near the free surface) must be employed.

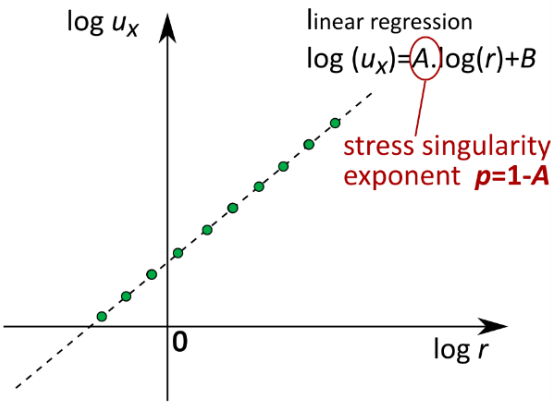

Fig. 2 Numerical estimation of stress singularity exponent

By performing presented method in every node on the crack front we captured change of the stress singularity exponents, see fig. 3. As it was mentioned above, in the middle of the specimen the stress singularity exponent is equal to 0.5. Close to the intersection between crack front and free surface stress singularity exponent decrease in dependence with Poisson's ratio, see fig. 3. In case of ν = 0, the stress singularity exponent p has constant value of 0.5 along the whole crack front. With increasing Poisson's ratio, p significantly decrease at the free surface (especially in the region 0-3 mm). However, behind thickness of about 3 mm, p becomes constant and equal to 0.5, regardless of Poisson's ratio.

Values of the stress singularity exponent for models with various Poisson's ratio in the vertex point were already published (tab. 1). Researchers applied different techniques, but got very similar results. Benthem [5] used for the evaluation finite difference, Bažant & Estenssoro [4] used variational principles and Burton et al [8] used FEM. The results obtained in this paper are in a very good agreement with those already published.

Tab. 1 Comparison of the singularity exponent at vertex point obtained by different techniques

Source	$\mu = 0$	$\mu = 0.15$	$\mu = 0.3$	$\mu = 0.4$
Finite difference by Benthem [5]	0.500	0.484	0.452	0.414
Variational principles by Bažant&Estenssoro [4]	0.500	0.484	0.452	0.413
FEM by Burton et al [8]	0.499	0.485	0.445	0.370
FEM of SENB	**0.497**	**0.480**	**0.449**	**0.414**

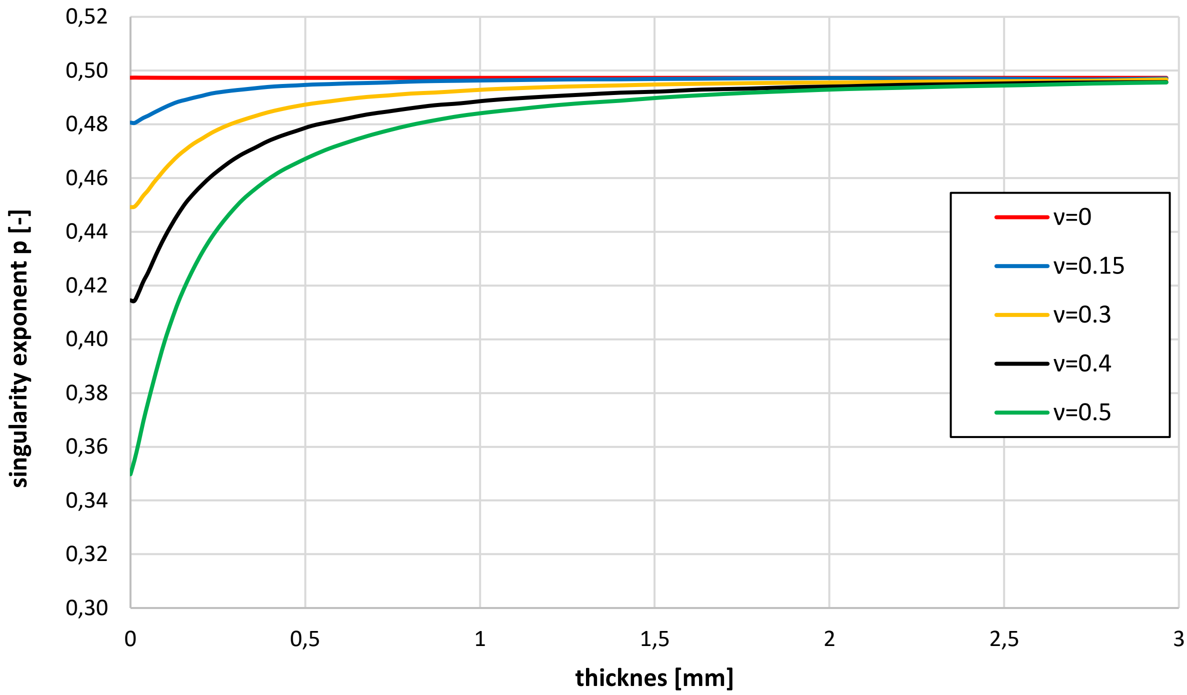

Fig. 3 Stress singularity exponent p for different Poisson's ratios (only range of 0 - 3 mm is shown, because the rest until the middle of the specimen at 10 mm is constant)

Question about the size of the area influenced by the free surface remains. If we take a look on fig. 4, there is a part of a FEM model with presented opening stress σ_x. In case of $\nu = 0$, the stress field is the similar through the thickness of the model. However, in case of $\nu = 0.3$, distribution of stress differs close to the free surface, while in the middle of the body is similar to the case of $\nu = 0$. The third figure shows the difference of both previous stress fields. One may notice, that most of the model has small difference of stresses. There is a very small area in the vicinity of the vertex point, where the difference of both stress fields is significant and where the crack propagation can be influenced. Size of this area is in agreement with area where stress singularity exponent differs from 0.5.

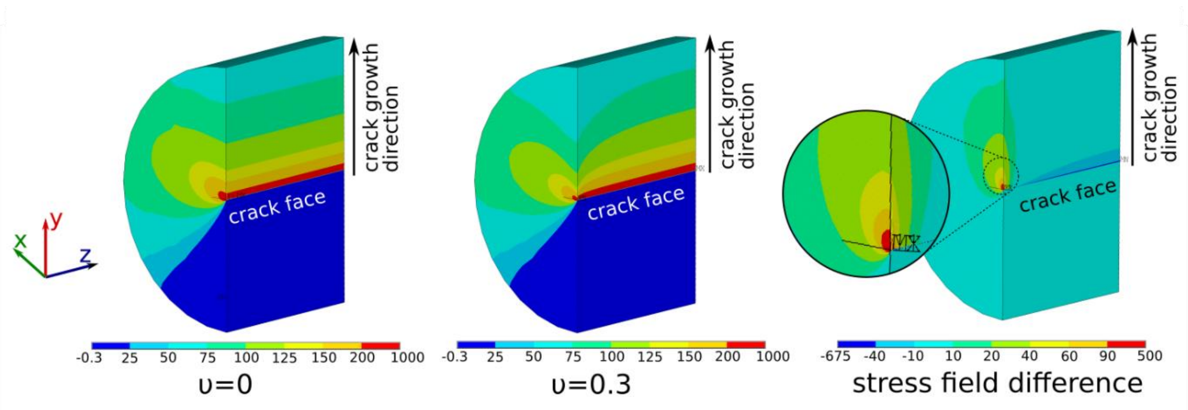

Fig. 4 Opening stress σ_x for models with $\nu = 0$, $\nu = 0.3$ and the difference of these stress fields

4 Fatigue crack front curvature

Presence of the vertex point singularity has an important effect in the crack shape formation. Let's assume a specimen with initial straight crack is loaded cyclically (see fig. 5). The vertex point singularity (at the free surface) causes the fatigue crack propagation rate (FCPR) decrease (in region of $p < 0.5$) [9]. The fatigue crack grows slower compared to the middle of the specimen (where $p = 0.5$) and characteristic curved shape with an angle γ between the crack front and the free surface is created. Once the angle γ_r is reached, FCPR is constant along the crack front. Also the square-root singularity is ensured along whole crack front, including the vicinity of vertex point. Characteristic angle can be determined by the Pook's empirical expression [7]

$$\gamma_r = 90° - \arctan\left(\frac{2-\upsilon}{\upsilon}\right). \quad (3)$$

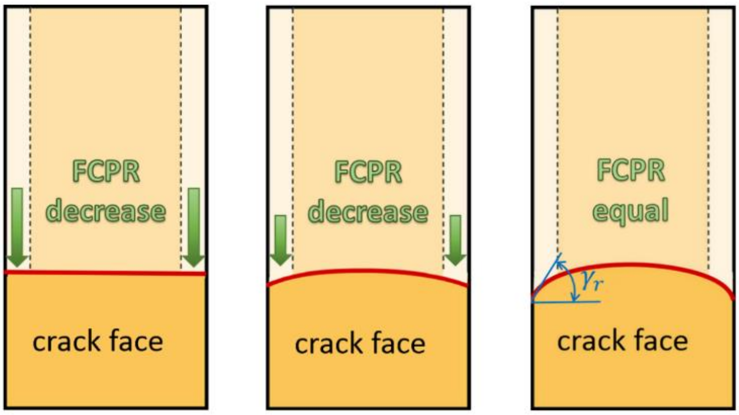

Fig. 5 Connection among the vertex point singularity, the fatigue crack propagation rate and the crack front shape

Set of experiments on SENB specimen of thickness $2B = 20$ mm subjected to cyclic loading with load ratio $R = 0.8$ was executed. Being in the range of only tension loading ensures that crack faces can not come into the contact with each other. It allows to eliminate the possibility of an appearance of the crack closure effect, which would influence the shape of the crack front.

Following figures show the fracture surface with beach marks, which allow a recognition of experimentally obtained crack front shapes. The first figure corresponds to a steel specimen (EA4T) with Poisson ratio $\upsilon = 0.3$, the second one corresponds to an aluminium alloy 7075 with Poisson's ratio $\upsilon = 0.395$. Chemical composition of steel and aluminium alloy may be seen in the tables below.

Tab. 2 Chemical composition of aluminium alloy 7075

component	**Al**	**Cr**	**Cu**	**Fe**	**Mg**	**Mn**	**Si**	**Ti**	**Zn**
min	87.1	0.18	1.2	0.0	2.1	0.0	0.0	0.0	5.1
max	91.4	0.28	2.0	0.5	2.9	0.3	0.4	0.2	6.1

Tab. 3 Chemical composition of steel EA4T

component	C	Si	Mn	P	S	Cr	Cu	Mo	Ni	V
min	0.22	0.15	0.5	0.00	0.000	0.90	0.0	0.0	0.0	0.0
max	0.29	0.40	0.8	0.02	0.015	1.20	0.3	0.3	0.3	0.06

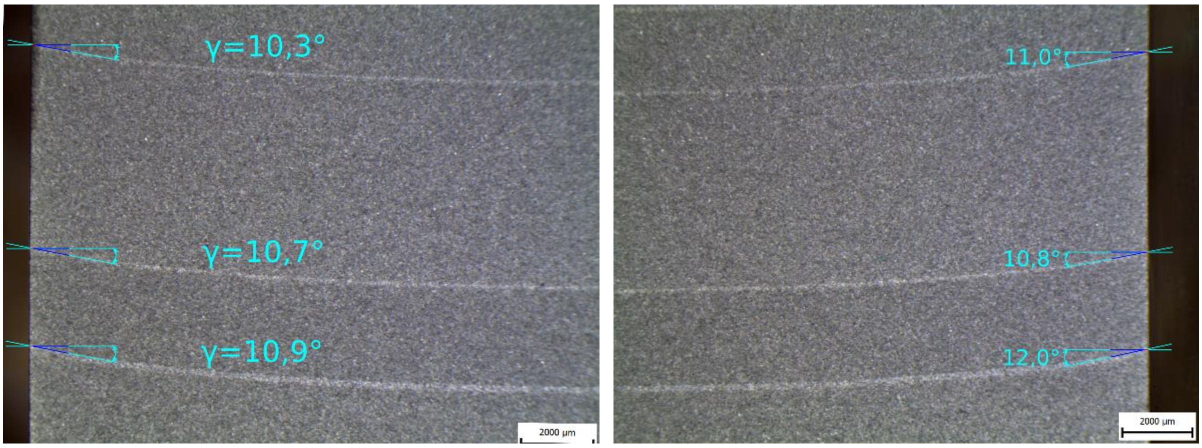

Fig. 6 Beach marks show the shape of the fatigue crack front on steel SENB specimen (ν = 0.3)

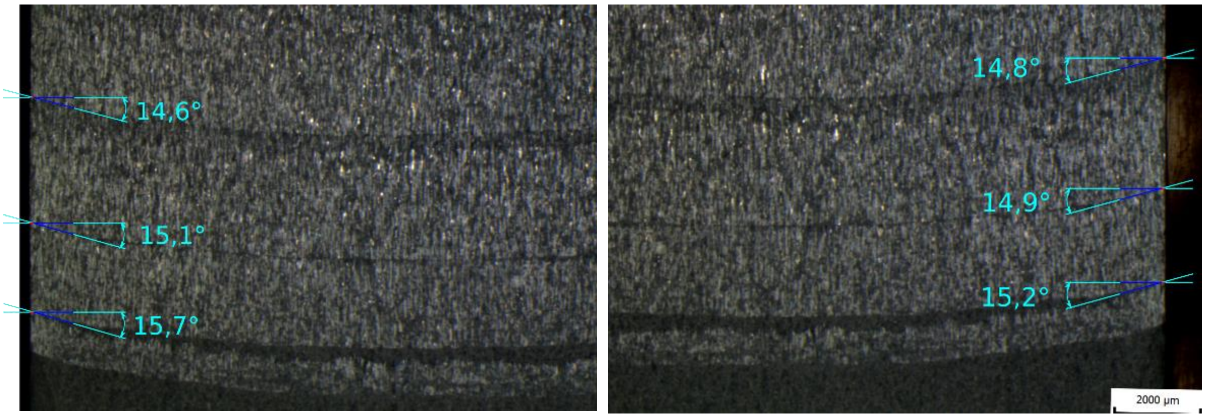

Fig. 7 Beach marks show the shape of the fatigue crack front on aluminium SENB specimen (ν = 0.395)

Same technique was used for the angle determination for both specimens. Line between two points was constructed and the angle between it and horizontal line was evaluated. Starting point lies 0.1 mm from the free surface, ending point lies 1 mm from the free surface.

Averages of measured angles of both materials lie in a range between Pook's [7] and Heyder's curve [10]. Fig. 6 shows steel specimen with average angle of crack front curvature γ = 11.0 °. Fig. 7 shows aluminium alloy specimen with averaged angle of crack front curvature γ = 15.1 °.

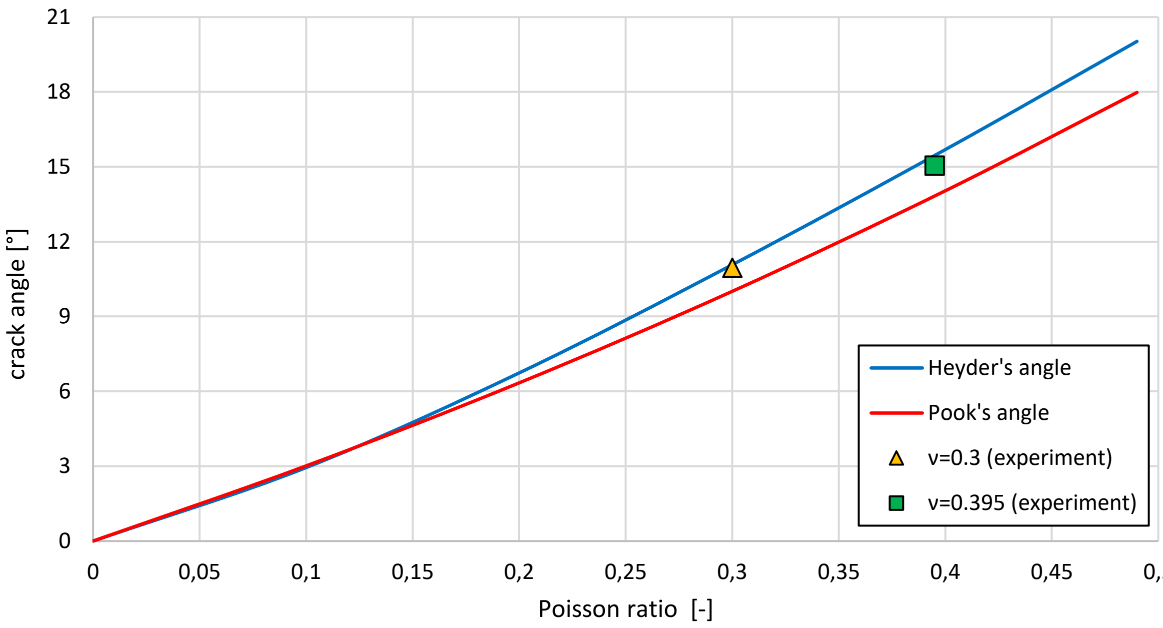

Fig. 8 Fatigue crack front angle dependency on Poisson's ratio according to Pook's expression and Heyder's results

5 CONCLUSIONS

Numerical model of the standard SENB specimen was created in order to study the stress field in the region close to vertex point. It was found, that stress singularity exponent is dependent on Poisson's ratio and decreases in the vicinity of the vertex point in the case of the straight crack. Values of the stress singularity exponent in the vertex point for various Poisson's ratio, estimated by FEM, are in a very good accordance with already published results [4], [5], [8].

Effect of singularity causes the decrease of the fatigue crack propagation rate in the region close to the free surface, which is followed by a curvature of the fatigue crack. Once the critical angle γ_r between the free surface and the crack front is reached, the process of fatigue crack shaping is finished and the FCPR is constant along the whole crack front. Critical angle γ_r is a function of Poisson's ratio and can be estimated by simple Pook's expression [7].

A series of experimental work on steel and aluminium alloy SENB specimens, subjected to cyclic loading with load ratio $R = 0.8$, was presented. The measurement of the angle of the crack front curvature γ was performed on both materials. Average angle on steel specimen is $\gamma = 11.0$ °, average angle on aluminium alloy specimen is $\gamma = 15.1$ °. Both angles lie in the range between Pook's estimation [7] and Heyder's results [10].

The presented results are the first step to model numerically fatigue crack curvature and help to describe accurately three-dimensional stress field around crack front.

REFERENCES

[1] T. L. Anderson. Fracture Mechanics: Fundamentals and Applications, 3rd ed. Boca Raton, FL : Taylor & Francis, **2005**.

[2] S. Suresh. Fatigue of Materials, 2nd ed. Cambridge: Cambridge University Press, **1998**.

[3] J. Sladek, V. Sladek. Stress intensity factors for corner quarter-elliptical crack in a finite thickness plate. *J. Mech. Eng.* **1999** (50), No. 6, 365 - 386.

[4] Z. P. Bažant, L. F. Estenssoro. Surface singularity and crack propagation. *Int. J. Solids Struct.* **1979** (15), No. 5, 405 – 426.

[5] J. P. Benthem. The quarter-infinite crack in a half space; Alternative and additional solutions. *Int. J. Solids Struct.* **1980** (16), No. 2, 119 – 130.

[6] P. Hutař, L. Náhlík, Z. Knésl. The effect of a free surface on fatigue crack behaviour. *Int. J. Fatigue* **2010** (32), No. 8, 1265 – 1269.

[7] L. P. Pook. Some implications of corner point singularities. *Eng. Fract. Mech.***1994** (48), No. 3, 367 – 378.

[8] W. S. Burton, G. B. Sinclair, J. S. Solecki, J. L. Swedlow. On the implications for LEFM of the three-dimensional aspects in some crack/surface intersection problems. *Int. J. Fract.*, **1984** (25), No. 1, 3 – 32.

[9] P. Hutař, L. Náhlík, Z. Knésl. Quantification of the influence of vertex singularities on fatigue crack behavior. *Comput. Mater. Sci.* **2009** (45), No. 3, 653 – 657.

[10] M. Heyder, K. Kolk, G. Kuhn. Numerical and experimental investigations of the influence of corner singularities on 3D fatigue crack propagation. *Eng. Fract. Mech.* **2005** (72), No. 13, 2095 – 2105.