
VYSOKÉ UČENÍ
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NEXT GENERATION VUT: Zvyšování kvality a relevance vzdělávání na VUT CZ.02.02.XX/00/23_022/0009052

Magnetism

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MINISTRY OF EDUCATION, YOUTH AND SPORTS

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Magnetic fields

The magnetic field is generated around permanent magnets or around moving electric charges (current-carrying conductors).

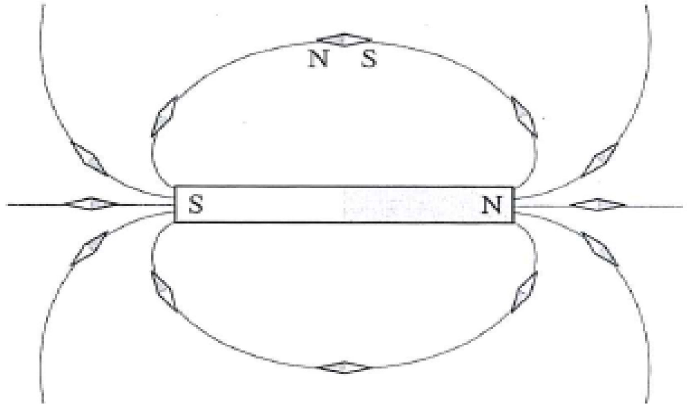
A permanent magnet bar always has 2 poles, which cannot be separated from each other even after splitting. The north pole is called **N** (north) and the south pole is called **S** (south). The opposite poles attract each other, the congruent poles repel each other.

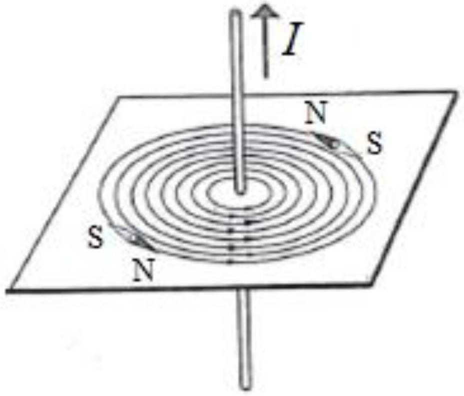
A magnetic field can be stationary – unchanging or non-stationary or variable.

Graphically, we can depict the magnetic field using magnetic induction lines, which are spatially oriented curves whose tangents at a given point in space have the direction of the axis of a very small magnet located at this point.

Magnetic induction lines are always closed curves that do not intersect anywhere.

The orientation of the induction lines is from the north pole **N** to the south pole **S**.



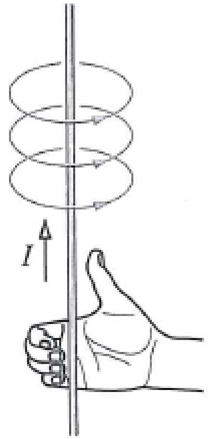


Magnetic field of a bar magnet and conductors with current.

The Right-Hand Rule

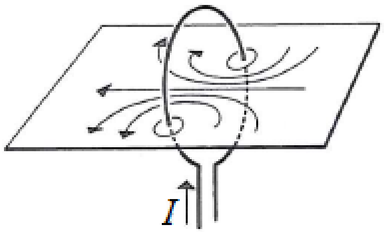
The orientation of magnetic induction lines around the current-flowing conductors is determined using the Ampère's right-hand rule:

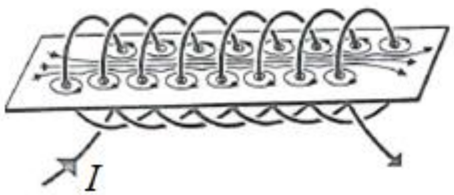
Indicate the grasp of the wire in the right hand so that the thumb shows the agreed direction of the current in the conductor, then the fingers show the orientation of the induction lines.



A wire flowing with current. Ampere's rule for determining the orientation of magnetic induction.

Magnetic field of loops and coils



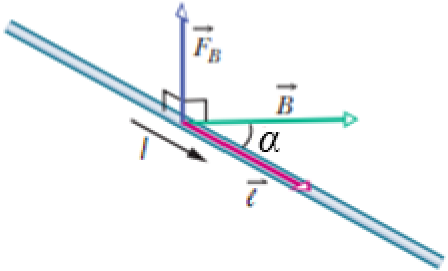


Magnetic induction

A straight line conductor of length ℓ flowing with an electric current I and placed in a homogeneous magnetic field is exerted by a magnetic force $\vec{\boldsymbol{F}}_{\boldsymbol{m}}$, the magnitude of which is

$$F_m = BI\ell \sin\alpha,$$

where α is the angle between the conductor and the magnetic induction lines, and B is the magnitude of magnetic induction $\vec{B}$ that characterizes the magnetic field.



The force $\vec{F}_m$ is perpendicular to the plane given by the conductor and the magnetic induction vector $\vec{B}$, $\alpha \in < 0, \pi >$.

From the previous relation, it follows that for the direction of the current perpendicular to the induction lines, the magnitude of magnetic induction is

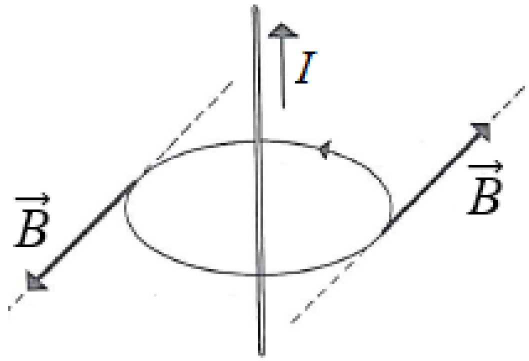
$$B = \frac{F_m}{I\ell}$$

The unit of magnetic induction $\vec{\boldsymbol{B}}$ is tesla (T), $\mathrm{T} = \mathrm{N.A^{-1}.m^{-1}}$.

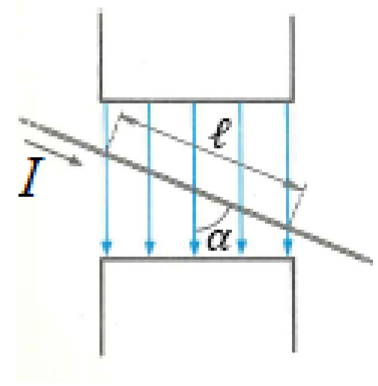
A magnetic field has an induction of 1 T (one tesla) if it exerts a force of 1 N on a conductor of active length of 1 m, perpendicular to the induction lines, if a current of 1 A flows through it.

Direction of magnetic induction

The direction of magnetic induction $\vec{B}$ is identical to the direction of the tangent to the magnetic induction line. The induction lines in a homogeneous magnetic field are parallel.

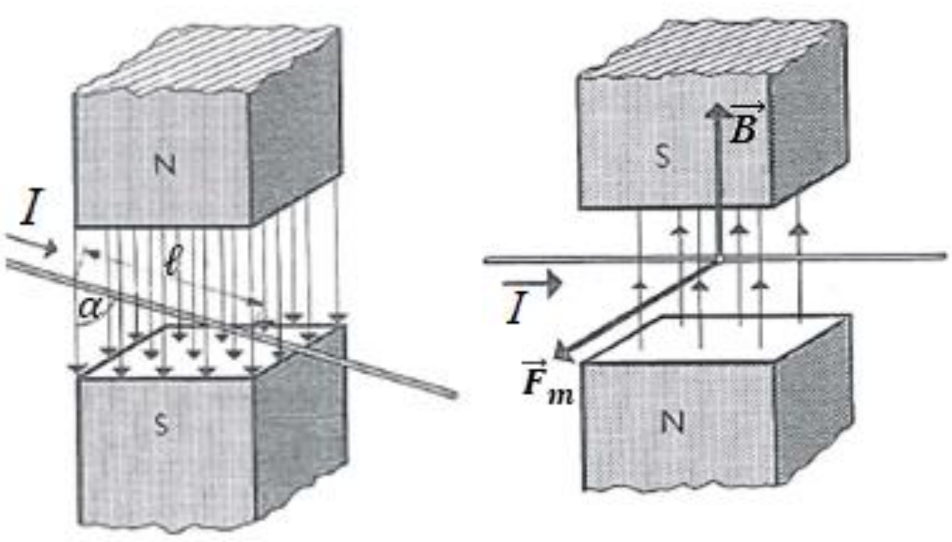


The direction of magnetic induction around the conductor.

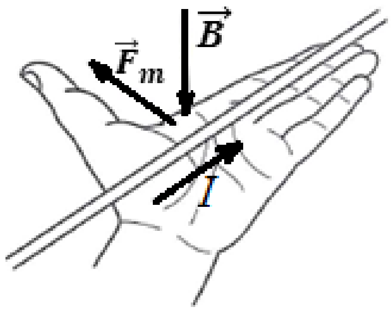


Homogeneous magnetic field.

Conductor with current in a magnetic field



A conductor with a current in a homogeneous magnetic field.



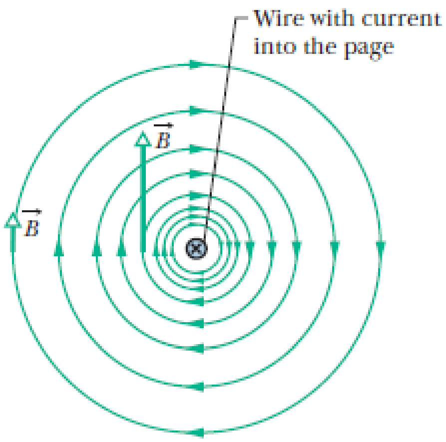
Fleming's left-hand rule for orientation $\vec{F}_m$.

Magnetic field of conductors with current

There is a magnetic field around every current conductor.

The magnitude of magnetic induction of this field is directly proportional to the current I in the conductor, but it also depends on the distance from the conductor, the shape of the conductor and the magnetic properties of the environment in which the conductor is located.

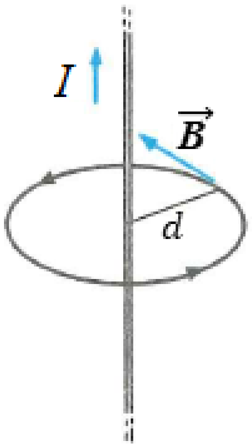
In a straight conductor flowing with a current I, the induction lines have the **shape of concentric circles** distributed in a plane perpendicular to the conductor with the center in the axis of the conductor.



Magnetic field of conductors with current

The magnitude of magnetic induction B at distance d from a long straight conductor is

$$B = \mu \frac{I}{2\pi d},$$



where the permeability of the environment is $\mu = \mu_r \mu_0$.

μ_0 is the permeability of the vacuum, or magnetic constant,

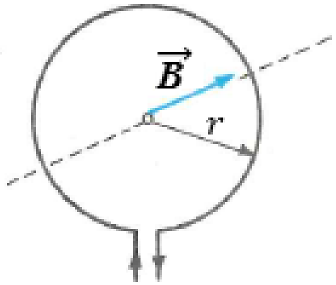
μ_r is the relative permeability of the environment.

Vacuum permeability is a constant whose value is defined exactly: $\mu_0 = 4\pi \cdot 10^{-7}\ \mathrm{N.A^{-2}}$.

For air, it is $\mu_r \doteq 1$.

Magnetic field of conductors with current

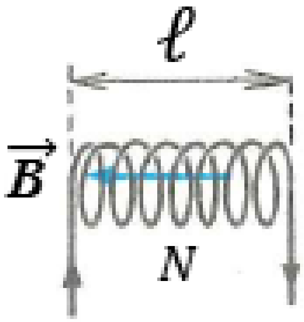
At the center of a planar circular thread with radius r is the magnitude of magnetic induction



$$B = \mu \frac{I}{2r} .$$

The magnetic induction **inside a long cylindrical coil (solenoid)** densely wound by a thin wire has a homogeneous magnetic induction size

$$B = \mu_0 \frac{NI}{\ell},$$



where I is the current in the coil and N is the number of turns per length of the ℓ coil.

The proportion $n = \frac{N}{\ell}$ indicates the thread density.

Magnetic properties of substances

According to their magnetic properties, we divide substances into

Diamagnetic	$\mu_r < 1$	slightly weaken the magnetic field (Au, Cu, Hg, inert gases, etc.)
Paramagnetic	$\mu_r > 1$	slightly intensify the magnetic field (Al, Pt, Mn, O, Na, K, etc.)
Ferromagnetic	$\mu_r \gg 1$	significantly amplify the magnetic field (Fe, Ni, Co, steel, etc.)

In practice, ferromagnetic substances are especially important, as they amplify the magnetic field of the vacuum by 100 to 100000 times.

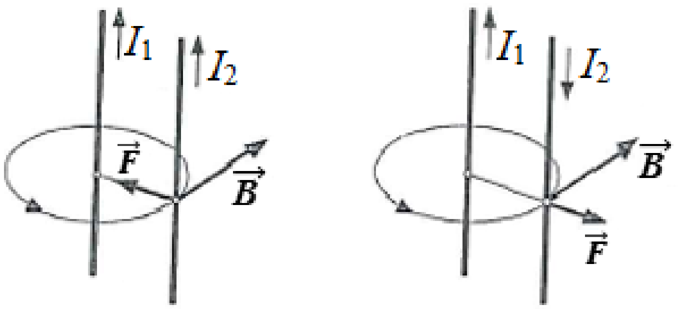
Force action between two parallel conductors with current

Two parallel straight conductors of length ℓ, whose distance is d, act on each other with a magnetic force of $\vec{F}_m$ of magnitude

$$F_m = \mu \frac{I_1 I_2}{2\pi d} \ell,$$

I_1 and I_2 are currents flowing through conductors.

In the case of a congruent direction of currents, the conductors are attracted, in the case of a discordant direction, they are repelled. This force was used to define the ampere as a unit of current.



Mutual force of two straight conductors with current

Definition of ampere as a unit of current

Ampere is a constant current which, when passing through two straight parallel conductors of infinite length, negligible cross-section, placed in a vacuum at a distance of 1 m from each other, causes a force of $2 \cdot 10^{-7}$ N per 1 m of conductor length between the conductors.

Particles with a charge in a magnetic field

A force of magnitude is applied to a straight conductor of length ℓ with current in a homogeneous magnetic field.

$$F_m = BI\ell \sin\alpha .$$

If we understand electric current as the flow of electrons with a total charge $Q = N \cdot e$, which move at a velocity of magnitude v in the direction of the conductor, then the current $I = Q/t$.

The path ℓ is traveled by the particle in time t according to the relation $\ell = v \cdot t$, then the magnitude

$$F_m = B\frac{Q}{t} v\, t \sin\alpha .$$

Particles with a charge in a magnetic field

A particle with a charge Q that moves in a magnetic field with a velocity $\vec{v}$ is exerted by a magnetic force $\vec{F}_m$, which is perpendicular to the magnetic induction vector $\vec{B}$ and to the velocity vector of the particle $\vec{v}$ at any given moment.

Its magnitude is

$$F_m = |Q| v B \sin \alpha,$$

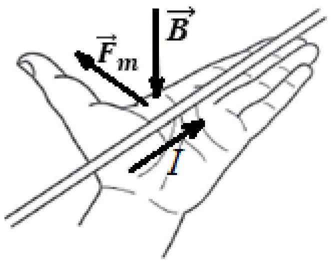
where α is the angle formed by the vectors $\vec{B}$ and $\vec{v}$.

Per one free electron a force of magnitude

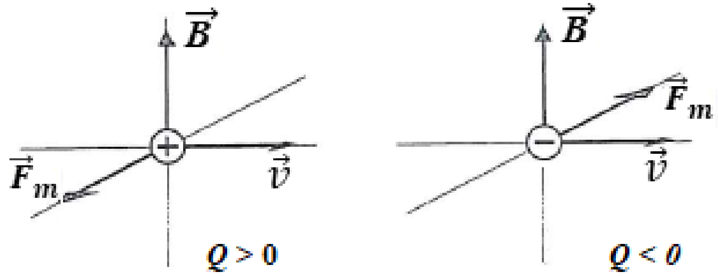
$$F_m = e v B \sin \alpha.$$

Direction of the applied magnetic force

The direction of the applied magnetic force is determined using Fleming's left-hand rule.



The direction of the magnetic force acting on a charged particle that moves in a magnetic field can be determined according to the figure.



Particles with a charge in a magnetic field

Since the F_m is always oriented perpendicular to the direction of motion of the particle, the magnetic force does not work, and the kinetic energy of the particle does not change, and hence only the direction of the velocity vector of the particle changes.

If a particle with a charge enters a homogeneous magnetic field perpendicular to the induction lines (α = 90°, so sinα = 1), it moves further along a circle in a plane perpendicular to the induction lines.

The following applies: $F_m = |Q|vB\sin\alpha = \frac{mv^2}{r}$,

What is the radius of a circular path $r = \frac{mv}{QB}$.

Lorentz force

If a particle with a charge Q moves simultaneously in both an electric and magnetic field, it is subjected to a force that is the vector sum of the electric force F_e and the magnetic force F_m.

The electric force is $\vec{F}_e = Q \cdot \vec{E}$ and the magnetic force is determined by the vector product

$$\vec{F}_m = Q \cdot (\vec{v} \times \vec{B}).$$

Therefore, the $\vec{F}_L = \vec{F}_e + \vec{F}_m$.

This resultant force $\vec{F}_L$ is called the Lorentz force.

Examples

The following values of physical constants were used in the examples:

Vakua Permeability $\mu_0 = 4\pi \cdot 10^{-7}\ \mathrm{N.A^{-2}}$,

Elemental charge $e = 1{,}602 \cdot 10^{-19}\ \mathrm{C}$.

Examples

1. A charged particle moves at a velocity of $2 \cdot 10^6$ m.s^{-1} along a circle with a radius of 10 m in a plane perpendicular to the direction of the induction lines of a magnetic field with a magnetic induction of 2,5 T. The kinetic energy of the particle is 0,05 J. Calculate the magnitude of the electric charge of the particle.

Solution:

A force acts on a particle moving in a magnetic field $F_m = |Q|vB \sin\alpha$.

Angle $\alpha = 90°$, so $\sin\alpha = 1$ and the magnetic force has a magnitude $F_m = |Q|vB$.

This force causes the path of the particle to curve – it is centripetal force. Therefore, the $F_d = F_m$, i.e. $\frac{mv^2}{r} = |Q|vB$.

Examples

From the relation for kinetic energy $E_k = \frac{1}{2}mv^2$, we express $mv^2 = 2E_k$ and after substituting it into the previous equation and adjusting it, we obtain for the magnitude of the charge $|Q| = \frac{2E_k}{vBr}$.

Numerically $|Q| = \frac{2\cdot 0{,}05\ \text{J}}{2\cdot 10^6\ \text{m.}s^{-1}\cdot 2{,}5\ \text{T}\cdot 10\ \text{m}} = 2\cdot 10^{-9}$ C.

The magnitude of the electric charge of a particle is $2\cdot 10^{-9}$ C.

Examples

2. A straight conductor with a length of 8,5 cm, which is entirely located in a homogeneous magnetic field, forms a constant angle of 30° with the magnetic induction vector.

What is the value of magnetic field induction if, when the current in the conductor increases by 2 A, the force acting on the conductor in the magnetic field increases by 0,17 N?

[$B = 2$ T]

3. A particle with a charge of $1{,}6 \cdot 10^{-19}$ C and a mass of $9{,}6 \cdot 10^{-31}$ kg moves in a homogeneous magnetic field with a magnetic induction of $9 \cdot 10^{-4}$ T along a circle with a radius of 2 cm. Determine the peripheral velocity of this particle. [$v = 3 \cdot 10^{6}$ m.s^{-1}]

Examples

4. Determine the kinetic energy of a particle carrying a charge of $1{,}6 \cdot 10^{-19}$ C, which moves in a circle with a radius of 1 m in a homogeneous magnetic field with a magnetic induction of $3 \cdot 10^{-4}$ T . The mass of the particle is $1{,}6 \cdot 10^{-27}$ kg.

[$E_k = 7{,}2 \cdot 10^{-19}$ J]

5. The solenoid with a length of 30 cm has 400 turns and a current of 0.6 A flows through it.

a) Determine the magnitude of magnetic field induction inside the solenoid.

b) How does this size change if we stretch the coil to twice the length?(Select $\mu = \mu_0$.)

[a) $B_1 = 1005{,}3 \cdot 10^{-6}$ T , b) $B_2 = \frac{1}{2} B_1$]

Faraday's Law of Electromagnetic Induction

Faraday's law of electromagnetic induction is an action caused by a non-stationary magnetic field. The changing magnetic field excites an induced electric field in the conductor, and if the conductor is part of a closed electrical circuit, an induced electric current is generated in it.

In a homogeneous magnetic field of magnetic induction B, the magnetic inductive flux passes through the surface with the content of S that is perpendicular to the induction vector

$$\Phi = BS.$$

Faraday's Law of Electromagnetic Induction

If the surface S is not perpendicular to the induction, then the magnetic inductive flux Φ by the surface S is determined by the relation

$$\Phi = BS\cos\alpha,$$

where α is the angle that the normal of the surface S forms with the direction of the induction vector $\vec{B}$, $\alpha \in < 0, \frac{\pi}{2} >$.

The unit of magnetic inductive flux is weber, $\mathrm{Wb} = \mathrm{T.m^2}$.

Faraday's Law of Electromagnetic Induction

When the magnetic inductive flux changes by the area bounded by the conductor by $\Delta\Phi$ over the time Δt, **an induced electromotive voltage $\boldsymbol{U_i}$** is generated in the conductor and the **induced electric current $\boldsymbol{I_i}$** flows through a closed circuit.

It has such a direction that its magnetic field counteracts the change in magnetic induction flux, which is its cause.

For medium values, the following applies $U_i = -\frac{\Delta\Phi}{\Delta t}$, $I_i = \frac{U_i}{R}$,

the minus sign is an expression of Lenz's law (see below). The voltage is numerically equal to the change in inductive flux in 1 second.

Faraday's Law of Electromagnetic Induction

If a conductor of length ℓ moves in a magnetic field perpendicular to the induction B with velocity v, we determine the induced voltage according to the relation

$$U_i = -\frac{\Delta\Phi}{\Delta t} = B\ell v.$$

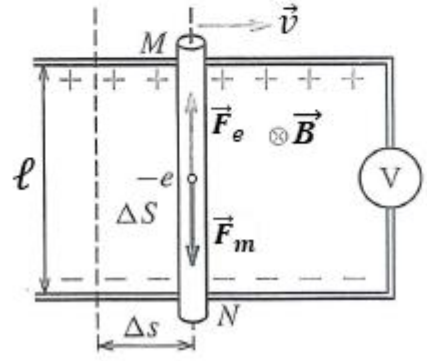
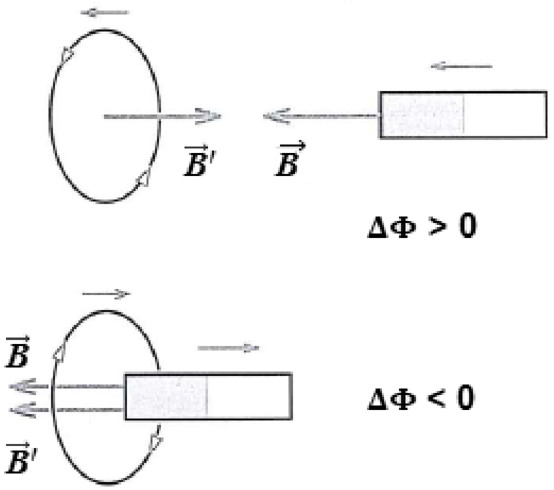


Illustration of Faraday's law of electromagnetic induction.

Lenz's Law

The direction of the induced electric current in an electrical circuit is determined by the so-called Lenz's law in such a way that its magnetic field counteracts the change in the magnetic field that caused the induced current.



Lenz's rule for determining the direction of induced current.

Inductance of the coil

If the current flowing through the coil changes by ΔI in time Δt, a voltage is induced in the coil $U_i = -\frac{\Delta\Phi}{\Delta t} = -L\frac{\Delta I}{\Delta t}$,

where L is the self inductance of the coil. Self inductance is measured in units of henry (H).

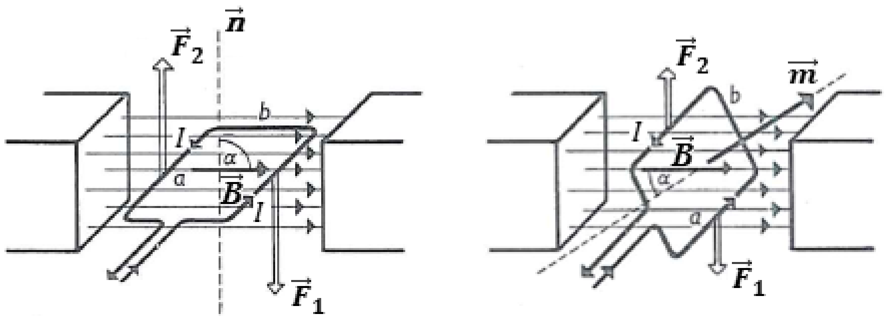
A conductor has an intrinsic inductance of 1 H at the ends of which a voltage of 1 V is induced by a current change of 1 A in 1 second.

Self-induction is dependent on inductance L, which is a quantity characteristic of each element of an electrical circuit. It manifests itself mainly in coils, other elements have a very low inductance.

Coil magnetic field energy

The energy of the magnetic field of a coil of inductance L, flowing through the current I is $E_m = \frac{1}{2} L I^2$.

Electromagnetic induction allows the direct conversion of mechanical energy into the energy of electric current.



Force action of a homogeneous magnetic field on a thread with current.

Examples

The following values of physical constants were used in the examples:

Vakua Permeability $\mu_0 = 4\pi \cdot 10^{-7}\ \mathrm{N.A^{-2}}$,

Elemental charge $e = 1{,}602 \cdot 10^{-19}\ \mathrm{C}$.

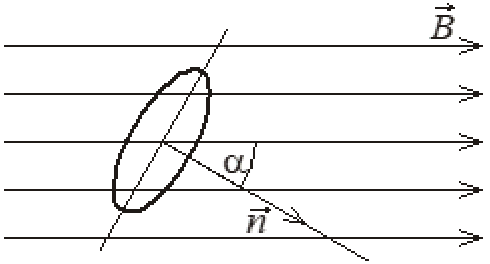
Examples

1. The magnetic induction of a homogeneous magnetic field has a magnitude of 1,4 T. Calculate the magnetic inductive flux with a circular surface with a radius of 10 cm if the plane of the surface forms an angle of 60° with the direction of magnetic induction.

Solution:

The magnetic inductive flux through the surface is determined by the relation $\Phi = BS\cos\alpha$, where α is the angle that the normal of the surface S forms with the magnetic induction vector.

From the figure it is clear that the angle $\alpha = 90° – 60° = 30°$.



Given that $S = \pi r^2$,

is magnetic induction flux by surface area $\Phi = B\pi r^2\cos(30°)$.

Examples

After inserting the values

$$\Phi = 1{,}4\ \mathrm{T} \cdot \pi \cdot 0{,}1^2\ \mathrm{m}^2 \cdot 0{,}866,$$

$$\Phi = 0{,}038\ \mathrm{Wb}.$$

The magnetic inductive flux through a circular surface is 0,038 Wb.

Examples

2. At what speed should a conductor of 20 cm in length move in the direction perpendicular to the induction lines of a homogeneous magnetic field with a magnetic induction of 0,1 T in order to induce a voltage of 0,01 V at its ends?

Solution:

At the ends of a conductor that moves in a magnetic field, a voltage is induced $U_i = B\ell v \sin\alpha$,

where ℓ is the length of the conductor, v is the magnitude of its velocity in the magnetic field o induction B and α is the angle that forms the velocity $\vec{v}$ with the magnetic induction vector $\vec{B}$. Thus, the speed of movement of the conductor is $v = \frac{U_i}{B\ell \sin\alpha}$.

Examples

After inserting the values

$$v = \frac{0{,}01\ \mathrm{V}}{0{,}1\ \mathrm{T} \cdot 0{,}2\ \mathrm{m} \cdot \sin(90^\circ)} = 0{,}5\ \mathrm{m.s^{-1}}$$

The wire should move at speed $0{,}5\ \mathrm{m.s^{-1}}$.

Unsolved examples

3. Determine the electric current that passes through a long cylindrical coil if the magnitude of magnetic induction in the coil is $3{,}14 \cdot 10^{-3}$ T. The turn density of the coil is $2{,}5 \cdot 10^{3}\ \mathrm{m}^{-1}$.

[I = 1 A]

4. A straight metal rod 20 cm long moves entirely in a homogeneous magnetic field at a speed of 4 $\mathrm{m.s}^{-1}$ perpendicular to the magnetic induction lines so that its length is still perpendicular to the direction of the magnetic induction lines and to the direction of the velocity vector. The magnetic field induction has a value of 0,5 T. Determine the electromotive voltage induced in the conductor.

[U_i = 0,4 V]

Unsolved examples

5. A straight conductor with a length of 1 m moves at a constant speed of 15 $\mathrm{m.s^{-1}}$ in a homogeneous magnetic field with a magnetic induction of 2 T. The conductor forms a constant angle of 90° with the magnetic induction vector.

The direction of motion of the conductor is perpendicular to both the conductor and the magnetic induction vector. Both ends of the conductor are connected by a resistance wire such that the total resistance of this closed circuit is 3 Ω.

Calculate the magnitude of the induced voltage and the current in the conductor.

[U_i = 30 V, I_i = 10 A]

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