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The Ph.D. Thesis is available in the library of the Faculty of Mechanical Engineering, Brno University of Technology

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INTRODUCTION

Coordinating marketing and production decisions still ranks as one of the most challenging practical and theoretical problems of operations management. Indeed, a number of producers have used innovative marketing strategies to gain effective control of their inventories. The thesis discusses the problems of simultaneously determining the quantity necessary to order and the related marketing decisions required for a product for which demand is random. Recently, there has been an increasing focus on the development of pricing as well as on marketing strategies and their further applications in industry. These pricing problems involve the determination of sales prices as a decision variable for a product under the relevant supply and demand constraints.

A simple newsvendor model platform may prove to be a relevant and principal “laboratory” for an increased understanding of how management science may be applied in order to solve these problems. The objective of such a stochastic single-period problem is to determine the quantity for a fixed period of time, maximizing the expected total profit. Although pricing is applied in the newsvendor problem (NP) in section 1 and in the transportation network design problem (TNDP) in section 2, it is not limited to these two problems but it can be applied in many other areas. See, e.g., its application in lot-size problems [7], integrated forward logistics network design with pricing for the collection of used products [6], or toll road pricing [41]. The author has already presented some ideas for coordination between production and pricing decisions; see [10, 13] for the NP-related stream and see [11, 15, 16] for the TNDP stream.

As the title *Mathematical programs for dynamic pricing - Demand based management* indicates, the thesis presents the above-mentioned selected demand-based problems that may be further extended to the concepts of dynamic pricing and marketing that drive their development [25, 32].

GOAL AND CONTRIBUTION OF THE THESIS

- The goal was to select illustrative (underlying) demand-based problems to provide insights into decision-making within various marketing situations.
- The NP was selected as such a “laboratory” principle. The NP with pricing (NPP) was reviewed and extended into the decision-dependent randomness case.
- Afterwards, the experience gained from the NPP was utilized and new results and insights into decision-making as well as managerial interpretations (e.g., within the effects of the parameters or the distribution) within NP with advertising (NPA) were established.
- Last but not least, the pricing and advertising decisions were coordinated; thus, the NP with joint pricing and advertising (NPPA) was introduced and investigated.
- The pricing results were then applied into the TNDP; its potential application in waste management (among others) was also discussed.
- A hybrid algorithm was proposed as a tool to solve similar mixed-integer (non-)linear problems; the portability of the algorithm usage is also verified and discussed.

1. NEWSVENDOR PRICING PROBLEM

The first mathematical formulation of price effects in inventory control problems was provided by Whitin [38] in 1955. Until this year, economic theoreticians had disregarded several important aspects, causing Whitin to focus on the hitherto-neglected demand aspects. He adapted the NP, where the unit selling price is a decision variable and where demand linearly depends on the selling price per unit, where the retailer knows a probability distribution of demand. Hence, he knows the amount demanded at any given price. In [24], Mills extended the NPP by a specifying mean demand as a function of the selling price. He refined Whitin's work by modeling the uncertainty in additive form: $D(p, \xi) = d(p) + \xi$, where $d(p)$ is a decreasing demand function of price p and ξ is a random variable defined within some range. Mills established that the optimal price under uncertain demand is never greater than the optimal price set in the equivalent deterministic monopoly models, called the riskless price. Both Whitin and Mills considered the single period form of the problem, where only a single price and an ordered amount need to be determined. In [19], they employed the infinite-period approximation to the n -period dynamic model. However, they did so under the assumption that a single constant price needed to be determined at the beginning of the planning horizon. They established that the optimal price under multiplicative uncertain demand is never smaller than the riskless price, i.e., an opposite outcome to Mills' outcome for additive uncertainty.

1.1. MODELING ISSUES

In the classical newsvendor model [8], the selling price is considered as exogenous, over which the newsvendor has no control [5]. Here, the pricing problem is approached as a price-setting newsvendor problem; two decision variables are defined: the selling price p and the ordered quantity x . The following parameters are considered: buying cost c , salvage value v , and shortage cost s . The demand $D(p, \xi)$ is both stochastic and price dependent, where ξ presents the random term. Thus, for each pair (p, ξ) the retailer knows the resulting demand, but he is unable to predict it in advance because he does not know the demand value as a reaction to the random term. The objective function is:

$$\pi(p, x, \xi) = \begin{cases} px - cx - s[D(p, \xi) - x], & \text{for } x < D, \\ pD(p, \xi) - cx + v[x - D(p, \xi)], & \text{for } x \geq D, \end{cases} \quad (1)$$

where x units are stocked at the beginning of the selling period for cost cx . If demand D is greater than x , then the revenue is px and $s[D(p, \xi) - x]$ denotes shortages multiplied by the shortage penalty cost per unit. Otherwise, if demand D is less or equal to an ordered quantity x , the income is only $pD(p, \xi)$ and $v[x - D(p, \xi)]$ denotes leftovers multiplied by salvage value per-unit. The goal is to maximize the expected value of the objective function. For this stochastic optimization program the so-called *expected objective reformulation* (see [29]) will be further used.

1.2. DEMAND FUNCTION AND RANDOMNESS

This section is focused on the demand function $D(p, \xi)$. Price independent randomness is further assumed in demand. More specifically, the demand function satisfies

$$D(p, \xi) \equiv D(p, \xi_a, \xi_m) = d(p)\xi_m + \xi_a, \quad (2)$$

where ξ is a two-dimensional vector, and $\xi = (\xi_m, \xi_a)$ and ξ_m, ξ_a are the random variables. In principle, the stochastic demand curve given by (2) should capture a real situation: when the price rises then the demand decreases, i.e., $d(p)$ is assumed as strictly decreasing, i.e., $d'(p) = \frac{dd(p)}{dp} < 0$, that presents the dependency between demand and price; the expected value of demand tends toward zero at sufficiently high prices, since demand cannot be negative. The assumption of monotonicity satisfies all common items; only special luxury items are excluded (i.e., Veblen paradox) [42]. Moreover, $d(p)$ is assumed to be a continuous function and twice differentiable. So, $d(p)$ is defined on a closed interval $[c, \bar{p}]$, where $d(\bar{p}) = 0$, see [42].

For the purpose of this thesis, two special cases of the demand function (2) are mentioned. Mills in [24] defined the *additive demand* function, where $\xi_m = 1$ (or $P(\xi_m = 1) = 1$) and so $D(p, \xi) = d(p) + \xi_a$. Furthermore, for this case it is assumed that $E[\xi_a] = 0$. Another special case of the demand function is the *multiplicative demand* case defined by Karlin and Carr [19], where $\xi_a = 0$ (or $P(\xi_a = 0) = 1$) and so $D(p, \xi) = \xi_m d(p)$. Furthermore, here it is assumed that $E[\xi_m] = 1$. For more details on special cases see [4].

Then, expectation of D is specified for both cases as $E[D(p, \xi)] = d(p)$. Moreover, $F(\cdot)$ represents the cdf and $f(\cdot)$ the pdf. It is also reasonable to consider that F is invertible.

1.3. RISKLESS PROBLEM

In the riskless theory demand is considered as strictly a function of the price p , which means that no random factor ξ is considered, see [24]. The related objective function is denoted by $\Psi(p)$ and so the problem is: $\max_p \{\Psi(p) | p \geq c\}$, where

$$\Psi(p) = (p - c)d(p), \quad (3)$$

i.e., the profit for a given price in the certainty-equivalent problem. The optimal riskless price p_{Ψ}^* can be determined by solving the first order condition $\frac{d\Psi(p)}{dp} = 0$.

Yao, Chen and Yan [42] define a class of demand functions with an *increasing price elasticity* (IPE), i.e., functions that satisfy $\frac{de}{dp} \geq 0$, where $e = -\frac{pd'(p)}{d(p)}$ denotes the *price elasticity* of $d(p)$ that gives the percentage change in demand in response to a one percent change in price [43]; i.e., for the retailer, it is less desirable to raise the price; in other words, if the price increases by a certain percentage, demand decreases by a larger percentage. The following lemma guarantees uniqueness of the first order condition solution.

Lemma 1.1. If $d(p)$ has IPE then the riskless profit Ψ is quasi-concave in p in the interval $[c, \bar{p}]$. The optimal riskless price p_{Ψ}^* can be uniquely determined by solving $\frac{d\Psi}{dp} = 0$ [42].

See [42] for proof. In [42], the authors provide some typical examples with IPE property.

1.4. ADDITIVE DEMAND CASE

This case involves the demand function $D(p, \xi)$ modeled as follows:

$$D(p, \xi) \equiv d(p) + \xi_a, \quad (4)$$

see subsection 1.2 for more (modeling) details. The objective function (1) can be rewritten by substituting (4) and defining the *stocking factor* z , $z = x - d(p)$, as:

$$\pi(p, z, \xi_a) = \begin{cases} p[z + d(p)] - c[z + d(p)] - s(\xi_a - z), & \text{for } z < \xi_a, \\ p[d(p) + \xi_a] - c[z + d(p)] + v(z - \xi_a), & \text{for } z \geq \xi_a. \end{cases} \quad (5)$$

This variable transformation (from x to z) is often used in the newsvendor-related literature (e.g., in [27]); it will be further shown to simplify the computations. It also provides an alternative interpretation of the stocking decision: if the choice of z is greater than the realized value of the random variable ξ_a , then leftovers occur, otherwise shortages occur.

The convention put forth by [27] is further used, with the following quantities for expected leftovers $\Lambda(z)$ and expected shortages $\Theta(z)$:

$$\Lambda(z) = E[(z - \xi_a)^+] = \int_A^z (z - t)f(t)dt, \quad (6)$$

$$\Theta(z) = E[(\xi_a - z)^+] = \int_z^B (t - z)f(t)dt. \quad (7)$$

Then, the expected profit is expressed by:

$$\Pi(p, z) = (p - c)[z + d(p)] - (p - v) \int_A^z (z - t)f(t)dt - s \int_z^B (t - z)f(t)dt. \quad (8)$$

Considering (6) and (7), the loss function can be expressed as: $L(p, z) = (c - v)\Lambda(z) + (p + s - c)\Theta(z)$, where if z is chosen too high, an overage cost $(c - v)$ appraises each of the $\Lambda(z)$ expected leftovers, and if z is chosen too low, an underage cost $(p + s - c)$ appraises each of the $\Theta(z)$ expected shortages and the expected profit can hence be expressed by:

$$\Pi(p, z) = \Psi(p) - L(p, z), \quad (9)$$

the riskless profit, which would occur in the absence of uncertainty, see (3), less the expected loss that occurs as a result of the presence of uncertainty (see [26, 27]).

Through integration by parts, the expected profit can be expressed from (8) as:

$$\Pi(p, z) = (p - c)[z + d(p)] - (p - v) \int_A^z F(t)dt - s \int_z^B [1 - F(t)]dt. \quad (10)$$

Whitin [38] established the sequential method for first determining the optimal value of z as a function of p by using the famous fractile rule for determining z when p is fixed, i.e., the result of the common NP. Let the subscript * denote optimality. By solving the first ordered condition $\frac{\partial \Pi(p, z)}{\partial z} = 0$, the following expression is observed:

$$z^* \equiv z(p) = F^{-1}\left(\frac{p + s - c}{p + s - v}\right). \quad (11)$$

Substituting (11) into (10) is p^* obtained by solving $\frac{d\Pi(z(p), p)}{dp} = 0$. Then:

$$\frac{d\Pi(z(p), p)}{dp} = d(p) + (p - c)\frac{dd(p)}{dp} + \frac{c - v}{p + s - v}z^* + \int_A^{z^*} tf(t)dt.$$

The second derivative w.r.t. p is:

$$\frac{d^2\Pi(z(p), p)}{dp^2} = (p - c)\frac{dd^2(p)}{dp^2} + 2\frac{dd(p)}{dp} + \frac{1}{f(z^*)}\frac{(c - v)^2}{(p + s - v)^3}$$

1.4.1. Linear pricing function

For the linear demand $d(p) = b - ap$, $a, b > 0$, Zabel [43] developed the following sequential solution method. The optimal selling price can be determined as:

$$p^* \equiv p(z) = p_{\Psi}^* - \frac{\Theta(z)}{2a} \quad (12)$$

where $p_{\Psi}^* = \frac{b+ac}{2a}$ (see subsection 1.3). Equation (12) implies that $p^* \leq p_{\Psi}^*$. Then the boundary condition for p^* is $c \leq p^* \leq p_{\Psi}^*$. The second partial derivative is $\frac{\partial^2 \Pi(p, z)}{\partial p^2} = -2a$ and so Π is concave in p for a given z . Then, z^* can be found by searching through the resulting optimal trajectory to maximize $\Pi(p, z^*)$ [27, 43].

The optimal strategy is to stock $x^* = d(p^*) + z^*$ units to sell at the price p^* per unit.

1.5. MULTIPLICATIVE DEMAND CASE

The demand function is defined as (according to subsection 1.2 and [19, 27]):

$$D(p, \xi) \equiv d(p)\xi_m. \quad (13)$$

Substituting (13) and $z = \frac{x}{d(p)}$ (the stocking factor) to model (1), it can be obtained:

$$\pi(p, z, \xi_m) = \begin{cases} pzd(p) - czd(p) - sd(p)(\xi_m - z), & \text{for } z \leq \xi_m, \\ p\xi_m d(p) - czd(p) + vd(p)(z - \xi_m), & \text{for } z > \xi_m. \end{cases}$$

It can be seen that the effect of z is the same as for the additive demand case model (5).

The following is defined: expected leftovers $d(p)\Lambda(z)$, expected shortages $d(p)\Theta(z)$ and the loss function: $L(p, z) = d(p)[(c - v)\Lambda(z) + (p + s - c)\Theta(z)]$, where $\Lambda(z)$ and $\Theta(z)$ are defined by expressions (6) and (7). Expected profit $\Pi(p, z)$ is again expressed by (9), where $L(p, z)$ assesses an overage cost $(c - v)$ for each of the $d(p)\Lambda(z)$ expected leftovers when z is chosen too high and an underage cost $(p + s - c)$ for each of the $d(p)\Theta(z)$ expected shortages when z is chosen too low. The expected profit can be expressed as:

$$\Pi(p, z) = (p - c)zd(p) - (p - v)d(p) \int_A^z F(t)dt - sd(p) \int_z^B [1 - F(t)]dt. \quad (14)$$

The optimal stocking factor z^* can be expressed by similar steps as in the additive case; in addition, z^* is observed identically to that of the additive case, i.e., (11). Substituting z^* into (14) is p^* obtained by solving $\frac{d\Pi(p, z(p))}{dp} = 0$. The related derivative is:

$$\begin{aligned} \frac{d\Pi(p, z(p))}{dp} &= z^*d(p) + (p - c)z^* \frac{dd(p)}{dp} - (p - v) \frac{dd(p)}{dp} \int_A^{z^*} F(t)dt - d(p) \int_A^{z^*} F(t)dt \\ &\quad - s \frac{dd(p)}{dp} \int_{z^*}^B [1 - F(t)]dt. \end{aligned}$$

The second derivative w.r.t. p is:

$$\begin{aligned} \frac{d^2\Pi(p, z(p))}{dp^2} &= \frac{d^2d(p)}{dp^2} \left[(p + s - c)z^* - (p + s - v) \int_A^{z^*} F(t)dt + s \int_A^B F(t)dt - sB \right] \\ &\quad + 2 \frac{dd(p)}{dp} \left[z^* - \int_A^{z^*} F(t)dt \right] + d(p) \frac{1}{f(z^*)} \frac{(c - v)^2}{(p + s - v)^3}. \end{aligned}$$

Remark 1.1. Considering (9), the necessary optimal condition of the price is as follows:

$$\frac{dd(p)}{dp}[p - c - l(p, z)] - d(p)[1 - \Theta(z)] = 0.$$

1.5.1. Isoelastic pricing function

Let the isoelastic demand curve $d(p)$ be defined as $d(p) = ap^{-b}$, where $a > 0, b > 1$, and, moreover, $A > 0$ (see [27]). The related partial derivative w.r.t. p is:

$$\frac{\partial \Pi(p, z)}{\partial p} = (b - 1) \frac{d(p)}{p} [1 - \Theta(z)] \left\{ p_{\Psi}^* + \frac{b}{b - 1} \left[\frac{(c - v)\Lambda(z) + s\Theta(z)}{1 - \Theta(z)} \right] - p \right\},$$

where p_{Ψ}^* is the optimal riskless price maximizing the riskless profit $\Psi(p)$: the derivative needed is $\frac{d\Psi(p)}{dp} = -(b - 1)ap^{-b-1}[p - \frac{bc}{b-1}]$, where $(b - 1)ap^{-b-1} > 0$ for $p < \infty$ and so the maximum of function is $\Psi(p)$ is $p_{\Psi}^* = \frac{bc}{b-1}$. Further, because $1 - \Theta(z) \geq A > 0$, for a given z , the unique optimal price p^* as a function of z can be established:

$$p^* \equiv p(z) = p_{\Psi}^* + \frac{b}{b - 1} \left[\frac{(c - v)\Lambda(z) + s\Theta(z)}{1 - \Theta(z)} \right]. \quad (15)$$

For a proof see [27]. From (15) the boundary condition for p^* can be established: $p^* \geq p_{\Psi}^*$.

1.6. SOME UP-TO-DATE RESULTS

In order to review some selected up-to-date results, the following definitions are needed [40, 42]: the *failure rate function* of the random variable $r(\xi) = \frac{f(\xi)}{(1-F(\xi))}$, the *generalized failure rate function* $g(\xi) = \xi \cdot r(\xi)$ as well as the related property called *generalized strict increasing failure rate (GSIFR)*: $g'(\xi) > 0$ for all ξ . For more details, see [42]. To the author's best knowledge, the GSIFR class of distributions and $d(p)$ -functions with IPE property are the major and most up-to-date class in the recent literature.

It can be shown that if the mean demand has IPE and the distribution has GSIFR, then Π is quasi-concave in p in the range $[c, \bar{p}]$ and thus the first order condition $\frac{d\Pi(z(p), p)}{dp} = 0$ has a unique solution [40, 42].

The list of mean demand and random factor distributions used in recent literature references for the additive demand case provided by [42] is also reviewed in the thesis.

2. TRANSPORTATION NETWORK DESIGN PROBLEM WITH PRICING

In addition to the network-design decision variables, which represent the inclusion of additional edges, pricing variables are included herein. If the network operator has the possibility of decision on price(s) charged to the customers, the demand profile of the customers can adapt and this can lead to overall more sensible network designs. Since suppliers have imperfect information about the demands of their customers in real-world problems, a scenario-based approach to the uncertain demand is used.

Thus, the entire section 2 concerns the problem of determining the pricing and production decisions (i.e., transportation and network design) of a single continuously divisible item in TNDP over a single period for a stochastic price-dependent demand. More specifically, in subsection 2.1 it is considered that demand is linearly price-dependent, while it is considered that demand is a nonlinear (isoelastic) function of the price in subsection 2.2. Remind paper [27] for the linear as well as nonlinear case and [42] for an even more general pricing approach which was mentioned in subsection 1.6. See Figure 1 for testing example of the TNDP that is used through the network design part of the work.

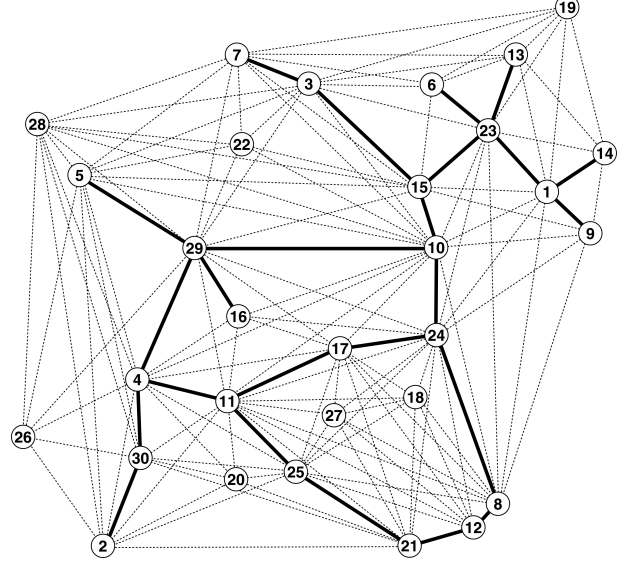


Figure 1: Testing example.

2.1. WS REFORMULATION OF STOCHASTIC TNDP WITH LINEAR PRICING

Here, the demand is a linear function of price. More specifically, the demand is considered to be decreasing, continuous, and defined on a closed interval [27]. Thus, the demand function is defined as (for each scenario s and each customer i): $b_{i,s}(p_{i,s}) = \beta_{i,s} - \alpha_{i,s}p_{i,s}$, where $\alpha_{i,s}$ and $\beta_{i,s}$ capture the uncertainty in the demand function $b_{i,s}$ that may differ for each customer i in each scenario s . The scenario-based approach assumes to have enough observations of the parameters (for each customer, one observation presents one particular market situation). Then, the selling price $p_{i,s}$ is the decision variable (as it is described below). For concrete examples see results and figures presented in the dissertation thesis.

Then, the following WS (stochastic) mixed-integer bilinear program is defined:

$\forall s \in S$:

$$\begin{aligned}
\max \quad & \sum_{i \in I} \sum_{e \in E} A_{i,e} x_{e,s} p_{i,s} - \sum_{e \in E} c_e x_{e,s} - \sum_{e_n \in E_n} d_{e_n} \delta_{e_n,s} - \sum_{i \in I} (r_i^- y_{i,s}^- + r_i^+ y_{i,s}^+) \\
\text{s.t.} \quad & \sum_{e \in E} A_{i,e} x_{e,s} = b_{i,s} - y_{i,s}^+ + y_{i,s}^-, \quad \forall i \in I, \\
& \sum_{e \in E} A_{j,e} x_{e,s} = b_j, \quad \forall j \in J, \\
& \sum_{e \in E} A_{k,e} x_{e,s} = b_k, \quad \forall k \in K, \\
& x_{e_n} \leq \delta_{e_n,s} \sum_{j \in J} (-b_j), \quad \forall e_n \in E_n, \\
& y_{i,s}^+ \leq b_{i,s}, \quad \forall i \in I, \\
& x_{e,s} \geq 0, \quad \forall e \in E, \\
& \delta_{e_n,s} \in \{0, 1\}, \quad \forall e_n \in E_n, \\
& y_{i,s}^+, y_{i,s}^- \geq 0, \quad \forall i \in I, \\
& p_{i,s} \geq p_i^{\min}, \quad \forall i \in I, \\
& p_{i,s} \leq p_i^{\max}, \quad \forall i \in I, \\
& b_{i,s} = \beta_{i,s} - \alpha_{i,s} p_{i,s}, \quad \forall i \in I,
\end{aligned} \tag{16}$$

where the following notation is used:

- the decision variables are:

$$\begin{aligned} p_{i,s} & : \quad \text{the unit selling price of the product at a node } i \text{ in scenario } s, \\ x_{e,s} & : \quad \text{the amount of a given product to be transported on edge } e \text{ in scenario } s, \\ \delta_{e_n,s} & \in \{0, 1\} : 1 \text{ if new edge } e_n \text{ is built in scenario } s, 0 \text{ otherwise,} \end{aligned}$$

- the second-stage variables (that relate to the particular scenarios):

$$\begin{aligned} y_{i,s}^+ & : \quad \text{shortages for customer } i \text{ in scenario } s, \text{ where } y_{i,s}^+ = \max\{b_{i,s} - \sum_e A_{i,e}x_e, 0\}, \\ y_{i,s}^- & : \quad \text{leftovers for customer } i \text{ in scenario } s, \text{ where } y_{i,s}^- = \max\{\sum_e A_{i,e}x_e - b_{i,s}, 0\}, \end{aligned}$$

- parameters:

$$\begin{aligned} \alpha_{i,s} & : \quad \text{the slope of the linear demand function } b_{i,s} \text{ for customer } i \text{ in scenario } s, \\ \beta_{i,s} & : \quad \text{the intercept of the linear demand function } b_{i,s} \text{ for customer } i \text{ in scenario } s, \\ A_{v,e} & : \quad \text{incidence matrix, } A_{v,e} \begin{cases} 1 & \text{if edge } e \text{ leads to node } v, \\ -1 & \text{if edge } e \text{ leads from node } v, \\ 0 & \text{otherwise,} \end{cases} \\ b_{i,s} & : \quad \text{the demand in node } i \text{ for scenario } s; b_{i,s} > 0 \forall i \in I, \forall s \in S, \\ b_j & : \quad \text{the production in node } j; b_j < 0 \forall j \in J, \\ b_k & : \quad \text{the (zero) demand/production in node } k; b_k = 0 \forall k \in K, \\ c_e & : \quad \text{unit transporting cost on edge } e, \\ d_{e_n} & : \quad \text{cost of building of a new edge } e_n, \\ p_i^{\min} & : \quad \text{a price lower bound for customer } i, \\ p_i^{\max} & : \quad \text{a price upper bound for customer } i, \\ q_s & : \quad \text{probability that scenario } s \text{ occurs, where } 0 \leq q_s \leq 1, \forall s \in S, \\ & \quad \text{and } \sum_s q_s = 1, \\ r_i^+ & : \quad \text{unit penalty cost for shortages (unsatisfied demand) at customer node } i, \\ r_i^- & : \quad \text{unit penalty cost for leftovers (redundant units) at customer node } i. \end{aligned}$$

- and index sets:

$$\begin{aligned} E & : \quad \text{set of edges, } e \in E, \\ E_n & : \quad \text{set of newly built edges, } e_n \in E_n, E_n \subset E, \\ I & : \quad \text{set of customers (or locations with a non-zero demand), } i \in I, \\ J & : \quad \text{set of production locations (or warehouses), } j \in J, \\ K & : \quad \text{set of traffic nodes, } k \in K, \\ S & : \quad \text{set of all possible scenarios, } s \in S, s = 1, 2, \dots, m, \\ V & : \quad \text{set of all nodes (vertices) in the network, } v \in V, V = I \cup J \cup K. \end{aligned}$$

Note, that $b_{i,s}$ are the the linear demand functions capturing the decision-dependent stochastic (scenario-based) demand parameter.

2.1.1. Computational results and discussion

The aforementioned model (16) was programmed in GAMS and solved through the use of CPLEX and XA solvers for small test instances obtaining acceptable results. See the dissertation thesis and author's papers [31, 14, 15] for more details on the algorithm.

Comparing the obtained results with the results published in [14], it can be seen that the total number of the designed edges (the 0-1 variables) as well as its variability have decreased due to the pricing strategy used. Moreover, pricing usually leads to a significant improvement in the objective function value (it should never obtain the worst solution).

Even if the linear pricing does not capture reality very well, it helps us to understand how the pricing technique works; see the obtained results provided in the dissertation.

2.2. WS REFORMULATION OF STOCHASTIC TNDP WITH NONLINEAR PRICING

This subsection presents a scenario-based WS stochastic MINLP, which models the design of a transportation network under nonlinear price-sensitive stochastic demand. The author follows up on previous modeling ideas presented in subsection 2.1 (or [15]), where a MILP with linear price-dependent stochastic demand was modeled. Therefore, the authors also modified the previously used hybrid algorithm [15, 31].

2.2.1. WS stochastic TNDP model with isoelastic pricing function

Consider a price-setting firm that faces a price-dependent demand function, $b_{i,s}(p_{i,s})$, describing the dependency between price $p_{i,s}$ and demand $b_{i,s}$ for each customer denoted by i and for each possible scenario s . For most goods the elasticity (the responsiveness of quantity demanded to price) is negative, so it can be convenient to write the constant elasticity demand function with a negative sign on the exponent, in order for the coefficient to take on a positive value: $b_{i,s}(p_{i,s}) = \alpha_{i,s} p_{i,s}^{-\beta_{i,s}}$; such isoelastic function should capture (the most common) real-world situations; see also subsection 1.5.1.

The following notation is changed comparing to that used in the model (16):

- the (scenario-based) parameters:

$\alpha_{i,s}$: effectiveness of the pricing function $b_{i,s}$ for customer i in scenario s , $\alpha_{i,s} > 0$,
 $\beta_{i,s}$: elasticity of the demand function $b_{i,s}(p_{i,s})$ for customer i in scenario s , $\beta_{i,s} > 1$.

Thus, the stochastic TNDP with isoelastic pricing is formulated using the WS approach:

$\forall s \in S$:

$$\begin{aligned}
\max \quad & \sum_{i \in I} \sum_{e \in E} A_{i,e} x_{e,s} p_{i,s} - \sum_{e \in E} c_e x_{e,s} - \sum_{e_n \in E_n} d_{e_n} \delta_{e_n,s} - \sum_{i \in I} (r_i^- y_{i,s}^- + r_i^+ y_{i,s}^+) \\
\text{s.t.} \quad & \sum_{e \in E} A_{i,e} x_{e,s} = b_{i,s} - y_{i,s}^+ + y_{i,s}^-, \quad \forall i \in I, \\
& \sum_{e \in E} A_{j,e} x_{e,s} = b_{j,s}, \quad \forall j \in J, \\
& \sum_{e \in E} A_{k,e} x_{e,s} = b_{k,s}, \quad \forall k \in K, \\
& x_{e_n,s} \leq \delta_{e_n,s} \sum_{j \in J} (-b_j), \quad \forall e_n \in E_n, \\
& y_{i,s}^+ \leq b_{i,s}, \quad \forall i \in I, \\
& x_{e,s} \geq 0, \quad \forall e \in E, \\
& \delta_{e_n,s} \in \{0, 1\}, \quad \forall e_n \in E_n, \\
& y_{i,s}^+, y_{i,s}^- \geq 0, \quad \forall i \in I, \\
& p_{i,s} \geq p_i^{\min}, \quad \forall i \in I, \\
& p_{i,s} \leq p_i^{\max}, \quad \forall i \in I, \\
& b_{i,s} = \alpha_{i,s} p_{i,s}^{-\beta_{i,s}}, \quad \forall i \in I.
\end{aligned} \tag{17}$$

2.2.2. Computational results

The problem (17) is mixed-integer nonlinear, but it seems that the exact solvers deal with a linearized version of the problem. Such nonlinear problems (especially large-scale) often require a heuristic approach. Therefore, a hybrid algorithm is proposed in the thesis.

The dissertation thesis presents visualizations of the example; see also Figure 2. The thickness of the lines represents the frequencies of usage in m scenarios, and hence, the probabilities that variables x_e related to the edges are non-zeros. The fixed lines are drawn as dashed lines to emphasize the role of the edges generated by the WS computations. It may also be seen that the stochastic demand usually requires new edges to bring about the necessary adaptation in the results. In comparison with the HN solutions (cf. [31]), it can be done in a more flexible and cheaper way. Figure 2 also shows that only suboptimality has been reached by computations for some scenarios, as extra unnecessary edges are switched on by the GA runs (e.g., 5-28).

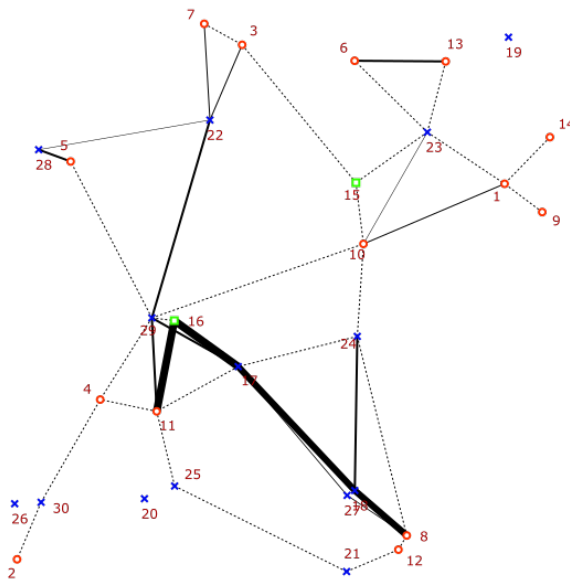


Figure 2: Visualization of results for the hybrid algorithm for 100 scenarios.

2.2.3. Discussion

The entire section 2 presents a stochastic programming approach to the TNDP with stochastic price-dependent demands while subsection 2.2 deals with the isoelastic form of the price-demand function. The developed mixed-integer nonlinear model is solved with the original hybrid algorithm involving GA for the solution of the WS network design problem. The previously introduced hybrid algorithm (see [15, 31]) has been modified and successfully tested. This reconfirms authors' conclusions in [31] about the portability of the approach to other problems.

In author's further research, it is planned to compare (or improve) the proposed hybrid algorithm with similar ideas dealing with differential evolution, specifically with multi-chaotic success-history based parameter adaptation for differential evolution [12], which is a novel version of the standard GA that, hopefully, may achieve better computational results for the MINLP problems. Moreover, some obvious suboptimalities (see, e.g., Figure 2) produced by the GA can easily be eliminated by appending a local search procedure to the GA run.

Similar mixed integer (nonlinear) stochastic programs may appear in many application areas, including NDP [28], traffic networks [9] or waste management problems [12]. Therefore, the suggested hybrid algorithm can be modified and widely applied.

3. NEWSVENDOR PROBLEM WITH ADVERTISING

A typical NP reveals that the quantity ordered maximizes the expected profit. In the setting given herein, the newsvendor is faced with advertising-sensitive stochastic demand where the demand-related random element depends on advertising decisions. It is assumed, that a suitable advertising strategy can lead to increases in sales. Note that the assumption of a fixed price corresponds to an instance of the buyers effectively representing mere price-takers.

Although, NP has been studied for decades, it still serves as a suitable tool to illustrate many new marketing situations [5, 17, 33, 39]. In this chapter, it is used notation that is frequently utilized for the newsvendor problem with pricing (NPP, see section 1) in order to show how marketing aspects interact with production decisions in the newsvendor problem with advertising (NPA). Moreover, inspired by [27], who presented findings on NPP for the linear price-demand function in an additive demand model, as well as the hyperbolic price-demand function for a multiplicative demand case (higher prices cause a decrease in demand), suitable (and more complex) functions related to various advertising situations for additive and multiplicative cases (an increase in advertising expenditure brings about higher sales) are presented. Using suitable notations and procedures, it is the aim to further the understanding of the matter and to present new results.

Note, that this chapter is mostly based on material that was published in [10].

3.1. PROBLEM FORMULATION AND DEMAND FUNCTION

In the classical NP or in the NPP (section 1), the newsvendor's marketing effort, which can be used to enhance the demand, is not taken into consideration at all [5]. Therefore, the following situation is assumed: First, the retailer has to decide about an amount a to advertise for a product to be sold and simultaneously has to buy and stock x units of the product for a unit cost c . Then, the selling period begins. If demand D is greater than x , all stocked units are sold for revenue px , where p is a unit price, $p > c$. In this case, a loss given by a unit shortage penalty cost s for all shortages, $D - x$, is considered. Otherwise, if demand D is less or equal to x , the revenue is only pD and leftovers, $x - D$, are salvaged through a unit salvage value v , $v < c$. Then, the objective (profit) function is denoted by $\pi(a, x, \xi_m, \xi_a)$ being defined as follows:

$$\pi(a, x, \xi_a, \xi_m) = \begin{cases} px - cx - s[D(a, \xi_a, \xi_m) - x] - a, & x < D, \\ pD(a, \xi_a, \xi_m) - cx + v[x - D(a, \xi_a, \xi_m)] - a, & x \geq D. \end{cases} \quad (18)$$

The decision variables are the order quantity x and the amount spent on the advertising a , while the demand $D(a, \xi_a, \xi_m)$, which depends on the amount a and is affected by the random elements ξ_a , ξ_m , is not completely known when the decisions are made.

One of the keys to understanding the marketing problems lies in the relation between demand and advertising response function. Therefore, the next section focuses on the demand function together with the definition of its related uncertainty. Then, the basic concepts of the advertising response function are presented.

3.1.1. Demand function and randomness

The demand is further modeled using a (response) function, which can be affected by the advertising expenditure and which somehow depends on a random element.

Inspired by many papers on the NPP [19, 24, 27, 42] and some on the NPA [20, 35, 36], it is further assumed that advertising-related randomness is independent of the demand, which helps to avoid complexities.

Let the demand function be denoted as $D(a, \xi_a, \xi_m)$ now and let it satisfy

$$D(a, \xi_a, \xi_m) = d(a)\xi_m + \xi_a,$$

where ξ_a, ξ_m are independent continuous random variables. Further, one special case of demand function $D(a, \xi_a, \xi_m)$ is presented: the multiplicative demand case (section 3.3), while the additive demand case is analyzed in the dissertation thesis. In order to define the multiplicative demand case, let $P(\xi_a = 0) = 1$ and let the random variable ξ_m be defined on the domain $[A_m, B_m]$ and satisfy $E[\xi_m] = 1$.

This section examines the effects of the multiplicative form on the optimal advertising strategy for the concave and the S-shaped functions. Before the multiplicative demand is modeled and examined in section 3.3, assumptions and properties of the advertising response function $d(a)$ are introduced.

3.2. ADVERTISING RESPONSE FUNCTION

The response function describes the sales effect of additional amounts of advertising, even though it sometimes illustrates the amount of advertising needed to trigger buying [18]. Two (general) functions that are often used are further assumed: a) the concave response function [18, 21] and b) the S-shaped response function [18].

Although the S-shaped function is very important from the marketing literature perspective, it has not yet been considered by researchers dealing with operational research in the discussed context. According to the marketing literature trying to approximate the advertising situations, the S-shaped function is defined as a bounded real function defined for all nonnegative input values with a positive derivative at each point, which is first convex and then concave. It means, in the beginning, when advertising budgets are low, sales do not respond significantly to advertising. It supposedly takes time for advertising wear-in. It can be seen the point of increasing returns, as sales really begin to respond to increased advertising, as the advertising budget exceeds some minimum critical-level threshold. Finally, the curve begins to slope downward again, as once again the diminishing-returns phase appears, see [3] and [1].

Let the response function $d(a)$ be continuous, nonnegative, twice-differentiable and increasing on its domain $[0, a_{max}]$ in the advertising expenditure a [20, 22]. Moreover, since $d(0) > 0$ holds, $d(a)$ is positive.

3.3. MULTIPLICATIVE DEMAND MODEL

Let the demand function $D(a, \xi_a, \xi_m)$ be defined in the multiplicative form (see subsection 3.1.1) and let $F(\cdot)$ denote a cdf and $f(\cdot)$ be a pdf of ξ_m . In order to assure that demand is positive, it is required that $A_m > 0$. Then, the demand is in the multiplicative form

$$D_M(a, \xi_m) = d(a)\xi_m, \tag{19}$$

see [19] for similar ideas in the NPP. The objective function (18) can be rewritten by substituting (19) and utilizing the 'stocking factor' defined as

$$z = \frac{x}{d(a)}, \tag{20}$$

where $z \geq 0$. A similar variable transformation of the objective has already been used to simplify the calculations in the NPP. It provides an interpretation of the stocking decision: if the choice of z is greater than the realized value of random variable ξ_m , then leftovers occur, otherwise shortages occur [27]. An important managerial interpretation for z demonstrates that, although z is defined differently for each of the two mentioned demand cases, its meaning is consistent for both: z represents a stocking factor that [27] defined as a surrogate for safety factor by [34]. Then, the NPA is as follows:

$$\pi(a, z, \xi_m) = \begin{cases} pzd(a) - czd(a) - sd(a)[\xi_m - z] - a, & \text{for } z < \xi_m, \\ p\xi_md(a) - czd(a) + vd(a)[z - \xi_m] - a, & \text{for } z \geq \xi_m. \end{cases}$$

The expected profit $\Pi(a, z)$ can be expressed as:

$$\begin{aligned} \Pi(a, z) = E[\pi(a, z, \xi_m)] &= d(a) \int_{A_m}^z [pt + v(z - t)]F(t)dt \\ &+ d(a) \int_z^{B_m} [pz - s(t - z)]F(t)dt - czd(a) - a \end{aligned} \quad (21)$$

Defining the *riskless profit* [24, 27], which would occur in the absence of uncertainty, as

$$\Psi(a) = (p - c)d(a) - a, \quad (22)$$

and the so-called *expected loss per unit* as

$$l(z) = (c - v)\Lambda(z) + (p + s - c)\Theta(z), \quad (23)$$

where $d(a)\Lambda(z)$ denotes *expected leftovers* and $d(a)\Theta(z)$ *expected shortages*, the expected profit given by (21) can be rewritten as

$$\Pi(a, z) = \Psi(a) - L(a, z) = d(a)[p - c - l(z)] - a. \quad (24)$$

Note that $L(a, z) = d(a)l(z)$ is the *expected loss* that occurs as a result of the presence of uncertainty [27, 34] and $p - c - l(z)$ denotes the so-called *per-unit expected benefit*, i.e., margin minus expected loss. If z is chosen too high, in (24) or in (23), respectively, an overage cost $(c - v)$ appraises each of the $d(a)\Lambda(z)$ expected leftovers, and, if z is chosen too low, an underage cost $(p + s - c)$ appraises each of the $d(a)\Theta(z)$ expected shortages. The equivalence of expressions (21) and (24) can be obtained by a sequence of straightforward substitutions.

3.3.1. Optimal stocking quantity

To maximize $\Pi(a, z)$ over two variables, the first order condition (with respect to z) is considered; then, an expression for optimal z is expressed as $F(z^*) = \frac{p+s-c}{p+s-v}$. It is comparatively easy to show that $\Pi(a, z)$ is concave in z on $[0, \infty)$: $\frac{\partial^2 \Pi(a, z)}{\partial z^2} = (v - p - s)F(z)d(a)$, where $v - p - s < 0$. Moreover, assuming that F is invertible, the optimal and unique z^* can be expressed as in (11) which is the standard NP/NPP result [38, 30].

3.3.2. Optimal advertising expenditure

Substituting (11) into (24) leads to the following expected profit expression:

$$\Pi(a, z^*) = d(a)[p - c - l(z^*)] - a, \quad (25)$$

where $l(z)$ is given by (23). Notice in (25) that, if $p - c - l(z^*) < 0$, the expected profit $\Pi(a, z^*)$ is, under our assumptions about $d(a)$ in subsection 3.1.1, negative and strictly decreasing in a , which does not capture any real situation [35], similarly if $p - c - l(z^*) = 0$. This leads to the following assumption.

Assumption 3.1. The per-unit expected benefit must be positive, i.e., $p - c - l(z^*) > 0$.

This assumption simply means that the expected profit per unit (price p minus cost c minus expected loss per unit $l(z^*)$) is greater than zero. Otherwise, if a loss is expected, the only good strategy is to "do nothing" ($x = a = 0$ and so $z = 0$). As Assumption 3.1 depends on the expected loss function $l(z)$, it also depends on the distribution F . The more shortages or leftovers are expected (caused, for example, by greater variance of the distribution), the greater is the expected loss $l(z)$. Similarly to other parameters from the expected loss function given by (23), e.g., as s increases, so does $l(z)$. Since the assumption is crucial for further analysis, more detailed insights for the uniform distribution is provided in the dissertation thesis.

Further, the expected profit expression given by (25) is assumed. Solving the first order condition of $\Pi(a, z^*)$ with respect to a , leads to the following remark.

Remark 3.1. The optimal advertising expenditure a^* must satisfy the (necessary) optimality condition, which is given by:

$$\frac{dd(a)}{da} = \frac{1}{p - c - l(z^*)}. \quad (26)$$

3.3.3. Monotonicity

Consider the second derivative of the expected profit given by (25):

$$\frac{d^2\Pi(a, z^*)}{da^2} = \frac{d^2d(a)}{da^2} [(p - c - l(z^*))]. \quad (27)$$

Then, due to Assumption 3.1, the following lemma is obtained from (27).

Lemma 3.1. The intervals of concavity and convexity of the expected profit $\Pi(a, z^*)$ with respect to a are identical with the intervals of concavity and convexity of $d(a)$.

The following assumption, together with Assumption 3.1, will further help us to guarantee optimality uniqueness for selected types of demand functions (i.e. for the concave and the S-shaped function). The assumption results from expression (24), or (26) respectively.

Assumption 3.2. The demand function $d(a)$ satisfies that $\lim_{\Delta a \rightarrow 0+} \frac{d(\Delta a) - d(0)}{\Delta a} > \frac{1}{p - c - l(z^*)}$ and $\lim_{\Delta a \rightarrow 0+} \frac{d(a_{max}) - d(a_{max} - \Delta a)}{\Delta a} < \frac{1}{p - c - l(z^*)}$.

Remark 3.2. In such case, where the function $d(a)$ is defined on a higher range than $[a, a_{max}]$, the conditions can be rewritten to: $\frac{dd(0)}{da} > \frac{1}{p - c - l(z^*)}$ and $\frac{dd(a_{max})}{da} < \frac{1}{p - c - l(z^*)}$.

Concave response function

Suppose that the demand/response function $d(a)$ is strictly concave in domain of a [21]. Then, the following theorem can be deduced.

Theorem 3.2. If the response function $d(a)$ is strictly concave, then, under assumptions 3.1 and 3.2, the expected profit $\Pi(a, z^*)$ is strictly concave in a and so the globally optimal advertising expenditure a^* is unique and is given by solution of (26) with respect to decision variable a .

Proof. Since the response function $d(a)$ is considered to be strictly concave in its domain then under Assumption 3.1 and Lemma 3.1, then $\Pi(a, z^*)$ is also strictly concave in a , see (27). Moreover, under Assumption 3.2, the expected profit $\Pi(a, z^*)$ is to be increasing at the initial point and decreasing at the end point. Then, the critical point determined from the optimality condition (26) is unique and is the optimal advertising amount a^* . \square

S-shaped response function

Theorem 3.3. If the response function $d(a)$ is S-shaped, then, under assumptions 3.1 and 3.2, the expected profit $\Pi(a, z^*)$ is strictly quasi-concave in a and so the globally optimal advertising expenditure is unique and is given by (26).

Proof. Since the response function is supposed to be S-shaped, under Assumption 3.1, the expected profit $\Pi(a, z^*)$ is also first convex and then concave in a . Moreover, using Assumption 3.2, the expected profit $\Pi(a, z^*)$ increases at the initial point and so it will increase until it reaches its maximum. In other words, $\Pi(a, z^*)$ is strictly quasi-concave in a . Then, from the optimality condition (26), one critical point a^* can be expressed that presents the optimal advertising amount, which always lies in the concave range. \square

In order to solve the original problem of maximizing the expected value of the objective function given by (18) with respect to decision variable x , a final step is to determine an optimal order quantity x^* from (20). The pair $[a^*, x^*]$ then presents the optimal solution of the original NPA given by (18) for the multiplicative demand case defined by (19) for the expected objective function case, see (21) and (24).

3.3.4. Comparison with riskless problem

Consider the advertising decision without demand uncertainty and note that the profit of such a deterministic problem is called riskless profit, $\Psi(a)$, given by (22). Solving the first order condition of $\Psi(a)$ leads to the following necessary optimality condition:

$$\frac{dd(a)}{da} = \frac{1}{p-c}, \quad (28)$$

which must be satisfied by the optimal riskless advertising a_{Ψ}^* .

Remark 3.3. If the response function $d(a)$ is either concave or S-shaped, then, under Assumption 3.2, the necessary optimal condition (28) is also sufficient for the optimal riskless advertising a_{Ψ}^* as (27) and Lemma 3.1 can be adequately applied.

Based on the optimality condition (28), the following theorem can be proved under assumptions 3.1 and 3.2 considering concave and S-shaped functions.

Theorem 3.4. For the multiplicative demand model, the optimal advertising a^* is always less than or equal to the optimal riskless advertising a_{Ψ}^* .

Proof. Using expressions (26) and (28), it can be expressed that $\frac{1}{p-c} \leq \frac{1}{p-c-l(z^*)} \Rightarrow \frac{dd(a_{\Psi}^*)}{da} \leq \frac{dd(a^*)}{da}$. For both functions, concave and S-shaped, the optimal advertising a^* , if it exists and is greater than zero, belongs to the concave part of $d(a)$. Then, for the concave part of $d(a)$, $\frac{dd(a)}{da}$ is decreasing and so if $\frac{dd(a_{\Psi}^*)}{da} \leq \frac{dd(a^*)}{da}$ then $a_{\Psi}^* \geq a^*$. \square

Remark 3.4. Recall that the optimal price for multiplicative uncertain demand is not less than the riskless price in the NPP [27].

Even though there are similar structures of the the expected profit functions, which is (9) in the NPP case, while the NPA equivalent is given by (24), the demand function is defined differently: $d(p)$ is decreasing in p in the pricing case, but $d(a)$ increases in a in the advertising case. Therefore, it is not surprising that the observation on the effect of uncertainty given by Theorem 3.4 is opposite to its NPP equivalent (see Remark 3.4).

4. NEWSVENDOR PROBLEM WITH JOINT PRICING AND ADVERTISING

Coordination between advertising, pricing, and production decisions still falls within the very challenging problems associated with operations and computational management; see, e.g., [2, 23]. Typically, marketing decisions (e.g., pricing and advertising) are made independently of knowledge of production and logistics constraints, possibly leading to the sub-optimality of the decisions. Here, marketing decisions are seen together with logistic decisions aiming for improved company performance.

In the aforementioned models the retailer/distributor cannot jointly adjust the price and the advertising amount (marketing effort) to stipulate the market demand. Here, a decision-maker may adjust the current selling price in order to increase or reduce the demand in most cases; moreover, he has the opportunity to influence the final demand by choosing appropriate marketing activities, e.g., providing shelf spaces, promotional displays, advertising, after-sales service support, and other demand-enhancing activities [5]. In a word, price and marketing efforts can be used to affect the final sale of the products ordered, and hence to exert influence on the initial ordering decision [5].

This thesis follows two papers on the NPPA: [5] and [37]. In the paper [37], they deal with the multiplicative demand form, where the demand $D(a, p)$ is nonnegative, twice-continuous differentiable, strictly concave, and is defined on $[0, \infty) \times [0, \infty)$; conceivably, $D(a, p)$ is strictly increasing and concave in the advertising premium, while it is strictly decreasing and convex in the sale price. In the paper [5], which combines the NPPA with its risk-averse form, two demand cases are investigated: the marketing-dependent price-multiplicative case and the marketing-dependent price-additive case; see section 4.1.

4.1. PROBLEM FORMULATION AND DEMAND FUNCTION

The NPPA model is similar to the NPA model (18); the crucial difference is that p is the decision variable, so the newsvendor faces stochastic demand $D(a, p, \xi_a, \xi_m)$. Thus, the newsvendor simultaneously decides on: the advertising amount a , the selling price p , and the amount x of a product to be stocked and sold. Replacing $D(a, \xi_a, \xi_m)$ with $D(a, p, \xi_a, \xi_m)$ in the NPA model (18), the NPPA model is expressed as

$$\pi(a, p, x, \xi_a, \xi_m) = \begin{cases} px - cx - s[D(a, p, \xi_a, \xi_m) - x] - a, & \text{for } x < D, \\ pD(a, p, \xi_a, \xi_m) - cx + v[x - D(a, p, \xi_a, \xi_m)] - a, & \text{for } x \geq D. \end{cases} \quad (29)$$

4.1.1. Demand function and randomness

Let the demand function be denoted as $D(a, p, \xi_a, \xi_m)$ and let it satisfy

$$D(a, p, \xi_a, \xi_m) = d_1(a)[d_2(p) + \xi_a]\xi_m,$$

where ξ_a, ξ_m are independent continuous random variables. [5] refers to two special cases of the demand function $D(a, p, \xi_a, \xi_m)$: a) the marketing-dependent price-multiplicative (MDPM) case and b) the marketing-dependent price-additive (MDPA) case.

In the MDPM case, let $P(\xi_a = 0) = 1$ and let the random variable ξ_m be defined on the domain $[A_m, B_m]$ and satisfy $E[\xi_m] = 1$. In the MDPA case, let $P(\xi_m = 1) = 1$ and let the random variable ξ_a be defined on the domain $[A_a, B_a]$ and satisfy $E[\xi_a] = 0$. Then, for both cases, the expectation of D is specified as: $E[D(a, p, \xi_a, \xi_m)] = d_1(a)d_2(p) \equiv d(a, p)$.

In this thesis, the MDPM case is further investigated.

4.2. MDPM DEMAND MODEL

Let the demand function $D(a, p, \xi_a, \xi_m)$ be defined in the MDPM form and let $d(a, p)$ denote a general demand function. Then, the demand is in the MDPM form

$$D_M(a, p, \xi_m) = d(a, p)\xi_m. \quad (30)$$

The objective function (29) can be rewritten by substituting (30) and defining the “stocking factor” as $z = \frac{x}{d(a, p)}$:

$$\pi(a, p, z, \xi_m) = \begin{cases} pzd(a, p) - czd(a, p) - sd(a, p)(\xi_m - z) - a, & \text{for } z < \xi_m, \\ p\xi_md(a, p) - czd(a, p) + vd(a, p)(z - \xi_m) - a, & \text{for } z \geq \xi_m. \end{cases}$$

The objective is to maximize the expected profit $\Pi(a, p, z)$; therefore, the expected profit, where $\Pi(a, p, z) = E[\pi(a, p, z, \xi_m)]$, is expressed as:

$$\Pi(a, p, z) = \Psi(a, p) - L(a, p, z) = \Psi(a, p) - d(a, p)l(p, z),$$

where $\Psi(a, p) = (p - c)d(a, p) - a$ and $l(p, z) = (c - v)\Lambda(z) + (p + s - c)\Theta(z)$. Remember that $d(a, p)\Lambda(z)$ denotes expected leftovers and $d(a, p)\Theta(z)$ expected shortages.

4.2.1. Optimal stocking quantity

To maximize $\Pi(a, p, z)$ over three variables, according to the previous experience from the NPP and NPA, the optimal stocking quantity is expressed as first. Taking $\frac{\partial \Pi(a, p, z)}{\partial z} = 0$, the following expression for the optimal stocking quantity z^* is expressed as in (11). This quantity corresponds to the standard NP, NPP as well as the NPA results.

4.2.2. Optimal price

Taking $\frac{\partial \Pi(a, p, z)}{\partial p} = 0$, it can be observed: $\frac{\partial d(a, p)}{\partial p}[p - c - l(z, p)] - d(a, p)[\Theta(z) - 1] = 0$. Substituting $d(a, p)$ with its multiplicative form $d_1(a)d_2(p)$, the following is expressed:

$$\frac{dd_2(p)}{dp}[p - c - l(z, p)] - d_2(p)[\Theta(z) - 1] = 0. \quad (31)$$

Theorem 4.1. Under our assumptions (i.e., if $d(p)$ has IPE and cdf has GSIFR), $\Pi(a, p, z)$ is quasi-concave in p , and optimal price of the MDPM model is unique and is always equal to that of the multiplicative form in the NPP.

Proof. The proof is obvious comparing two optimal price conditions, the NPP condition (1.1) and the NPPA condition (31); see also [5]. \square

Optimal p and z can then be found similarly as in the NPP case; see section 1.

4.2.3. Optimal advertising expenditure

Taking $\frac{\partial \Pi(a, p^*, z^*)}{\partial a} = 0$, it can be observed: $\frac{\partial d(a, p^*)}{\partial a} = \frac{1}{p^* - c - l(z^*, p^*)}$. Substituting the demand function $d(a, p^*)$ with its multiplicative form $d_1(a)d_2(p^*)$, the following is expressed:

$$\frac{dd_1(a)}{da} = \frac{1}{d_2(p^*)[p^* - c - l(z^*, p^*)]}. \quad (32)$$

Optimal advertising depends on the choice of p as well as $d_2(p^*)$; see (32). Moreover, with increasing p (or p^* , respectively) the optimal value of $d_2(p^*)$ decreases. Therefore, optimal advertising for the NPPA depends on specification not only $d_1(a)$ but also $d_2(p)$. Remember that p^* is equivalent to that of the NPP (see Theorem 4.1).

4.2.4. Comparison with riskless problem

Let the riskless objective function be defined as

$$\Psi(a, p) = (p - c)d_1(a)d_2(p) - a.$$

Taking $\frac{\partial \Psi(a, p)}{\partial a} = 0$, an optimality condition for a_{Ψ}^* is expressed as:

$$\frac{dd_1(a)}{da} = \frac{1}{d_2(p)(p - c)}. \quad (33)$$

Then, an equivalent result to that of NPA is observed: if the response function $d_1(a)$ is either concave or S-shaped, then, under Assumption 3.2, the necessary optimal condition (33) is also sufficient for the optimal riskless advertising a_{Ψ}^* .

Based on the optimality condition (33), the following theorem can be proven under assumptions 3.1 and 3.2 considering concave and S-shaped functions.

Theorem 4.2. For the MDPM, the optimal advertising a^* is always less than or equal to the optimal riskless advertising a_{Ψ}^* .

Proof. See proof of Theorem 3.4. \square

Theorem 4.3. For the isoelastic pricing function in the MDPM, the optimal pricing p^* is always greater than or equal to the optimal riskless pricing p_{Ψ}^* .

Proof. See [19, 27] for similar observations and related proofs. \square

CONCLUSIONS

The thesis is based on two underlying demand-based problems: the NP and the TNDP. Both problems are used throughout the work as tools for examining how marketing decisions may affect particular decisions.

As to the NP branch, the thesis subsequently presents NP, NPP, NPA, and NPPA. From a managerial perspective, most of the findings given herein might seem somewhat theoretical. However, a problem that simultaneously comprises advertising and pricing decisions should be of greater practical importance. Clearly, the same holds for the indication that uncertainty does not necessarily lead to greater advertising expenses. It is the intention to continue to pursue such a direction in future research, with the expectation of achieving results that are both theoretically and practically relevant. The results add valuable managerial insight into these problems, which may be of higher importance in future volatile and globalized markets. The newsvendor platform is simple and does not cover the most complex practical situations. Still, the need for more research in this direction is evident.

The above-mentioned results and models are further utilized in a more complex demand-based problem: the TNDP. In particular, the TNDP with pricing provides a suitable tool for other marketing extensions; although the author et al. have already found a possible application for the theoretical problem, it seems that the practical need is rapidly increasing nowadays. Therefore, in the thesis, a computational tool was also developed; the portability of the algorithm for various problems is also discussed.

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ABSTRACT

The thesis deals with the development, modeling, and analysis of demand-based problems containing marketing, operations, and logistics decisions. The problems may be further extended to the concepts of dynamic pricing and marketing that drive the development. Two demand-based problems are presented in the thesis: a) the newsvendor problem, due to its simple structure as a suitable tool for illustrating how facets of marketing may affect decision-making concerning operational problems, and b) the transportation network design problem, where some results and knowledge gained from the newsvendor problem are applied. In the setting presented, the newsvendor is subsequently faced with pricing, advertising, and joint pricing and advertising-sensitive stochastic demand. A demand-related random element comprises the particular marketing decision(s) of a specific form (e.g., multiplicative or additive). It is assumed that a real pricing strategy is captured with a nonlinear decreasing demand function while a suitable advertising strategy results in increased sales. The properties of the obtained optimal decisions for particular models are discussed. The pricing-related results are applied to the stochastic transportation problem, where the stochastic demand is modeled using wait-and-see and here-and-now deterministic (scenario-based) reformulations. A hybrid algorithm composed of a heuristic (genetic) algorithm and an optimization software tool is proposed for solving of a mixed-integer linear as well as a mixed-integer nonlinear problem. Potential applications, especially in waste management, are also discussed at the end of the thesis.

ABSTRAKT

Tato disertační práce se zabývá vývojem, modelováním a analýzou poptávkově orientovaných úloh, které zahrnují marketingová, operační a logistická rozhodnutí. Úlohy jsou zvoleny tak, aby mohly být dále rozšířeny o koncept tzv. dynamického oceňování a jiných dynamických marketingových rozhodnutí. V práci jsou využity dvě základní poptávkově orientované úlohy: a) úloha kolportéra novin, která je zvolena pro její jednoduchou formu a která tak slouží jako nástroj pro ilustrativní ukázky rozhodovacích procesů v podobných typech úloh, a b) úloha návrhu dopravní sítě, kde jsou využity některé výsledky a znalosti získané při řešení úlohy kolportéra novin. Kolportér (či obecně maloobchodník) čelí náhodné poptávce, která může být postupně ovlivněna oceňováním, marketingovými (tj. reklamními) rozhodnutími a nakonec jejich kombinací. Poptávka obsahuje tedy náhodnou složku, která je pomocí přístupů stochastické optimalizace modelována ve specifickém tvaru (tj. aditivní či multiplikativní tvar). Závislost cena-poptávka je zachycena pomocí nelineární klesající poptávkové funkce, zatímco (vhodná) reklama vede ke zvýšení poptávky (běžně rostoucí s-krivka či konkávní funkce). Výsledky získané při řešení úlohy kolportéra novin s oceňováním jsou následně využity v úloze návrhu dopravní sítě. Tato stochastická úloha je modelována (reformulována) pomocí dvou přístupů stochastické optimalizace: wait-and-see přístup a here-and-now přístup. Jelikož tato implementace vede na lineární či nelineární celočíselnou (navíc scénářovou) úlohu, jsou v práci zmíněny taky výpočetní nástroje. Autor pro řešení používá (původní) tzv. hybridní algoritmus, což je kombinace heuristického (genetického) algoritmu a nástroje optimalizačního softwaru. Potenciální aplikace sestavených modelů, obzvláště v oblasti odpadového hospodářství, jsou diskutovány v závěrečné části disertační práce.